ગુજરાત રાજ્યના શિક્ષણવિભાગના પત્ર-ક્રમાંક
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PHYSICS

Standard 12

(Semester IV)

PLEDGE

India is my country.
All Indians are my brothers and sisters.
I love my country and I am proud of its rich and varied heritage.
I shall always strive to be worthy of it.
I shall respect my parents, teachers and all my elders and treat everyone with courtesy.
I pledge my devotion to my country and its people.
My happiness lies in their well-being and prosperity.

રાજ્ય સરકારની વિનામૂલ્યે યોજના હેઠળનું પુસ્તક

Gujarat State Board of School Textbooks
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PREFACE

The Gujarat State Secondary and Higher Secondary Education Board has prepared new syllabi in accordance with the new national syllabi prepared by N.C.E.R.T. based on N.C.F. 2005 and core-curriculum. These syllabi are sanctioned by the Government of Gujarat.

It is a pleasure for the Gujarat State Board of School Textbooks, to place before the students this textbook of **Physics, Standard 12, (Semester IV)** prepared according to the new syllabus.

Before publishing the textbook, its manuscript has been fully reviewed by experts and teachers teaching at this level. From the following suggestions given by teachers and experts, we have made necessary changes in the manuscript before publishing the textbook.

The Board has taken special care to ensure that this textbook is interesting, useful and free from errors. However, we welcome any suggestions, from people interested in education, to improve the quality of the textbook.

Dr. Bharat Pandit
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Date : 05-08-2015

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FUNDAMENTAL DUTIES

It shall be the duty of every citizen of India :*

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities; to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers and wild life, and to have compassion for living creatures;

(h) to develop a scientific temper, humanism and a spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

(k) to provide opportunities for education by the parent or the guardian, to his child, or a ward between the age of 6-14 years as the case may be.

* Constitution of India : Section 51 A

I N D E X

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About This Textbook...

We have pleasure in presenting this textbook of physics of Standard 12 to you. This book is on the syllabi based on the courses of National Curriculum Framework (NCF), Core-Curriculum and National Council of Educational Research and Training (NCERT) and has been sanctioned by the State Government keeping in view the National Education Policy.

The State Government has implemented the semester system in science stream. The semester system will reduce the educational load of the students and increase the interest towards study.

In this Textbook of Physics for Standard 12, Eight chapters are included, looking into the depth of the topics, time which will be available for classroom teaching, etc.

The real understanding of the theories of physics is obtained only through solving related problems. Hence, for the new concept, solved problems are given. One of the positive sides of the book is that at the end of each chapter extended summary is given. On the basis of this one can see the whole contents of the chapter at a glance.

Keeping in view the formats of various entrance test conducted on all India basis, we have included MCQs, Short questions, objective questions and problems in this book. At the end of the book, hints for solving the problems are also included so that students themselves can solve the problems.

This book is published in quite a new look in four-colour printing so that the figures included in the book are much clear. It has been observed, generally, that students do not preserve old text-books, once they go to the higher standard. In the semester system, each semester has its own importance and the look of the book is also very nice so the students would like to preserve this book and it will become a reference book in future.

The previous textbook got excellent support from students, teachers and experts. So a substantial portion from that book is taken in this book either in its original form or with some changes. We are thankful to that team of authors. We are also thankful to the teachers who remained present in the Review workshop and gave their inputs to make this textbook error-free.

Proper care has been taken by authors, subject advisors and reviewers while preparing this book to see that it becomes error-free and concepts are properly developed. We welcome suggestions and comments for the importance of the textbook in future.

Authors/Editors

1

ELECTROMAGNETIC INDUCTION

1.1 Introduction

Electricity and magnetism were considered separate and nonrelated branches for a long time. In the early decades of the nineteenth century, experiments on electric current carried out by Oersted, Ampere and a few others established the fact that electricity and magnetism are interrelated to each other. They found that moving electric charges (i.e. electric current) produce magnetic field. As for Illustration, a current carrying wire deflects a magnetic needle placed in its vicinity. This phenomenon raises the questions like: Is the converse effect possible ? Can moving magnets (i.e. magnetic field) produce electric currents or not ? Does the nature permit such a relation between electricity and magnetism ?

Around 1830, experiments conducted by Michael Faraday in England, and Joseph Henry in USA, demonstrated conclusively that electric current was induced in closed coil under the influence of changing magnetic flux. The phenomenon in which electric current is induced in a conductor by varying magnetic flux is called electromagnetic induction.

Practically the phenomenon of electromagnetic induction is of great importance. The historical experiments of Michael Faraday and Henry have led directly to the development of electric generators and transformers. Today's civilization owes its progress to a great extent to the discovery of electromagnetic induction.

In the present chapter we shall study about Faraday's experiments, induced current and induced emf; and phenomena like self-induction, mutual induction, eddy currents based on it.

1.2 Faraday's Experiments

The discovery and understanding of electromagnetic induction are based on a series of experiments performed by Faraday. We shall study some of these experiments.

Experiment 1 : As shown in figure 1.1, Faraday took a ring of soft iron in his historic experiment. An insulated conducting coil was wound on one side of the ring and connected to a battery.

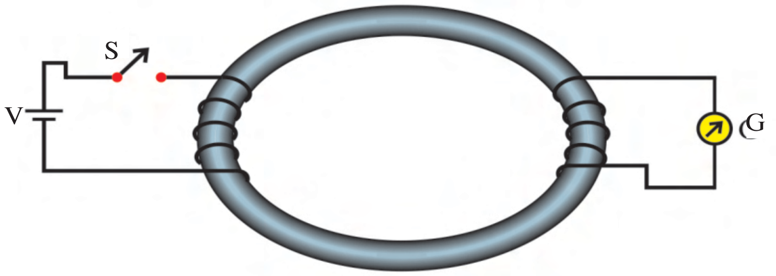


Figure 1.1 Faraday's Experiment

On the opposite side of this coil, another conducting coil was wound and connected with a sensitive galvanometer. The coil connected to a battery acts like a solenoid. When electric current is passed through the coil (i.e. solenoid), it produces a magnetic field.

Galvanometer measures electric current passing through the other coil. As the ring is of the soft iron, the magnetic field lines produced remain confined in the ring itself. Almost all magnetic field lines pass through the ring and hence through the inner part of second coil and form closed loops. In other words, the ring connects two coils through the magnetic field lines.

When Faraday passed a steady electric current through the left coil, no effect was observed in the galvanometer. Faraday was slightly disappointed, but Faraday's intuition worked at that moment. During minute observations of every moment, he observed that the galvanometer shows a momentary deflection whenever the battery is switched on or off. In both the cases, the deflections of galvanometer were in opposite directions.

From his observation that galvanometer showed no deflection when a steady current is passed, Faraday concluded that the steady current is not important but change of current plays an important role in this experiment.

Experiment 2 : In his second experiment, Faraday arranged two bar magnets in V shape as in the figure 1.2.

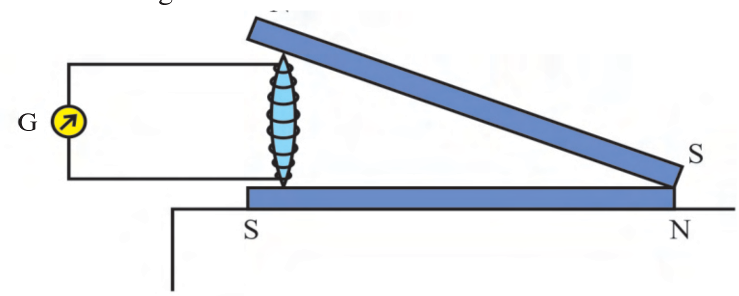


Figure 1.2 Faraday's Experiment of Two Barmagnets

At the other open end of V shape he kept one soft iron rod with an insulated copper wire wound around it. A galvanometer was connected with the conducting wire.

He observed that the galvanometer shows deflection on moving the end of upper magnet up and down. As the magnet moves nearer to the rod, magnetic flux linked with the coil increases. When the magnet touches the iron rod flux associated with the coil becomes maximum and as the magnet moves away from the rod, magnetic flux in the coil decreases.

From this experiment Faraday concluded that to induce electric current in a coil, change in magnetic flux is required and not the flux itself.

Experiment 3 : As shown in figure 1.3, an insulated conducting coil C_1 is connected to a galvanometer G.. When the North Pole (N) of a bar magnet is moved towards the coil, the pointer in the galvanometer shows deflection, indicating the presence of electric current in the coil.

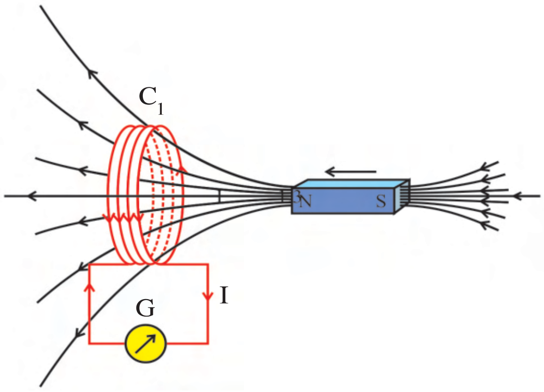


Figure 1.3 Faraday's Experiment of Bar Magnet and Coil

Galvanometer shows the deflection as long as the bar magnet is in motion. The galvanometer does not show any deflection when the magnet is held stationary.

When the magnet is pulled away from the coil, the galvanometer shows deflection in the opposite direction, which indicates reversal of the current's direction.

Moreover, when the South Pole (S) of the bar magnet instead of North Pole (N) is moved towards or away from the coil, the deflections in the galvanometer are opposite to that observed in the case of North Pole.

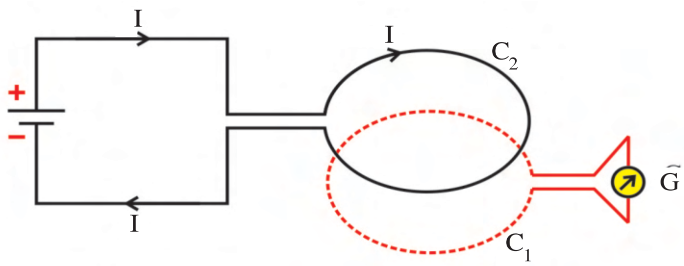


Figure 1.4 Faraday's Experiment on Two Coils

Further, the induced current is found to be larger when the magnet is pushed towards or away from the coil faster.

Instead, when the bar magnet is held fixed and the coil C_1 is moved towards or away from the magnet, the same results are observed.

When a bar magnet is replaced by a current carrying coil C_2 as shown in Figure 1.4, and relative motion is produced between the two coils C_1 and C_2 (nearest or farthest) then also the galvanometer connected with the coil C_1 shows deflection.

Further, if any one of the coils C_1 or C_2 is given a rotation with respect to each other, then also deflection is observed in the galvanometer.

The results of this experiment shows that :

(1) The relative motion between the magnet and the coil (or between the two coils) is responsible for generation (induction) of electric current in the coil.

(2) If the relative motion between the magnet and the coil is increased/decreased, more/less current is induced.

(3) The direction of induced current is reversed, if the direction of relative motion is reversed.

(4) If the magnet and the coil (or two coils) are moving with same speed in the same direction (if their relative velocity is zero), no current is induced in the coil.

Note : In the above experiment, electric current is induced due to the relative motion between the coil and the magnet and relative motion between the two coils respectively. However, Faraday showed that this relative motion is not an absolute requirement.

Faraday named the current produced in the other coil as the "induced current".

Here, the current generated in the other coil indicates that emf is produced in it which gives energy for the motion of charges. Faraday called this emf as "induced emf" and this phenomenon as electro magnetic induction.

Now, electric field is also produced in the second coil due to emf generated in it. Just as electric field is established in a wire by applying potential difference across its two ends, there is an electric field established in the second coil. Thus, we obtained the electric field due to the changing magnetic field with time. This fact is of basic importance in Farday's discovery.

Faraday's discovery fulfilled the dream of converting "mechanical energy into electrical energy".

1.3. Magnetic Flux

Magnetic flux is defined in the same way as electric flux is defined. The magnetic flux linked through any surface placed in a magnetic field is the number of magnetic field lines crossing this surface normally. Magnetic flux is a scalar quantity, denoted by ϕ.

Magnetic flux through a plane of surface area A placed in a uniform magnetic field $\vec{B}$ (Figure 1.5). can be written as,

$$\phi = \vec{B} \cdot \vec{A}$$

$$= BA\cos\theta \qquad (1.3.1)$$

where θ = Angle between $\vec{B}$ and $\vec{A}$.

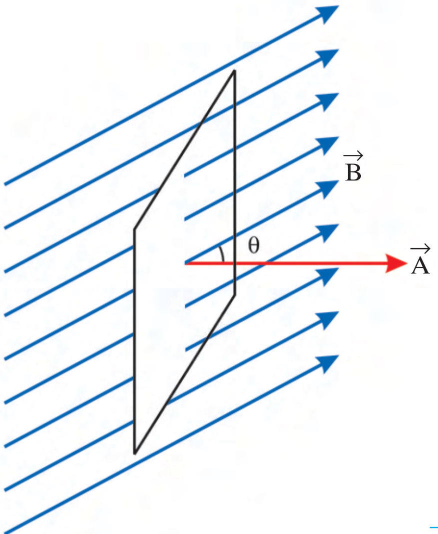


Figure 1.5 A Plane of Surface are $\vec{A}$ Placed in a Uniform Magnetic Field $\vec{B}$

Equation (1.3.1) can be extended to curved surfaces and non uniform fields too.

If the magnetic field has different magnitudes and directions at various parts of a surface as shown in figure 1.6, then the magnetic flux through the surface is given by,

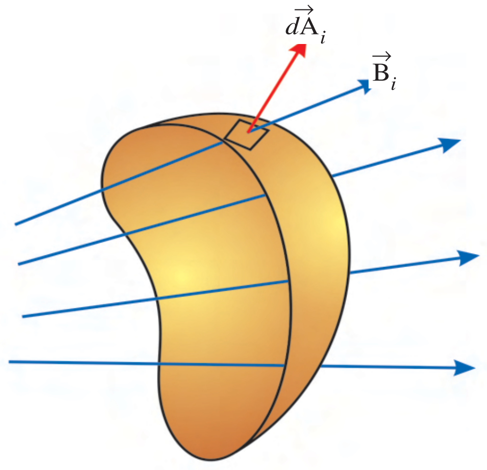


Figure 1.6 Magnetic field $\vec{B}_i$ at the i^{th} area Element $d\vec{A}_i$

$$\Phi = \vec{B}_1 \cdot d\vec{A}_1 + \vec{B}_2 \cdot d\vec{A}_2 + \vec{B}_3 \cdot d\vec{A}_3 + ...$$

$$\Phi = \sum_{\substack{\text{all} \\ \text{area elements}}} \vec{B}_i \cdot d\vec{A}_i \qquad (1.3.2)$$

where, $d\vec{A}_i$ is the area vector of i^{th} area element and $\vec{B}_i$ is the magnetic field at the area element $d\vec{A}_i$.

The SI unit of magnetic flux is weber (Wb) or Tm^2.

If the normal drawn to a plane points outward in the direction of the field ($\theta = 0$), then magnetic flux is positive. If the normal points in the opposite direction of the field ($\theta = \pi$), then flux is negative.

1.4. Lenz's Law

In article 1.2 we discussed about the induced emf but did not discuss about how much emf will be produced in which direction under the given conditions. In 1934, German physicist Lenz deduced a rule, known as Lenz's law which gives the direction (polarity) of the induced emf. We will first study this law and then Faradays law which gives the magnitude of induced emf.

As shown in figure 1.7, suppose a bar magnet is moved towards a conducting coil with its north pole (N) facing the coil. In this case the magnetic flux linked with the coil continuously changes and hence emf is induced in it. As a result the induced current flows through the coil which gives rise to the magnetic field and hence the coil acts like a magnet. Right now the direction of this current is not known to us.

In this situation, suppose the electric current flows in clockwise direction while looking at the coil normally from the same side as that of the bar magnet, then the side of the coil facing the magnet will act like a South Pole (S).

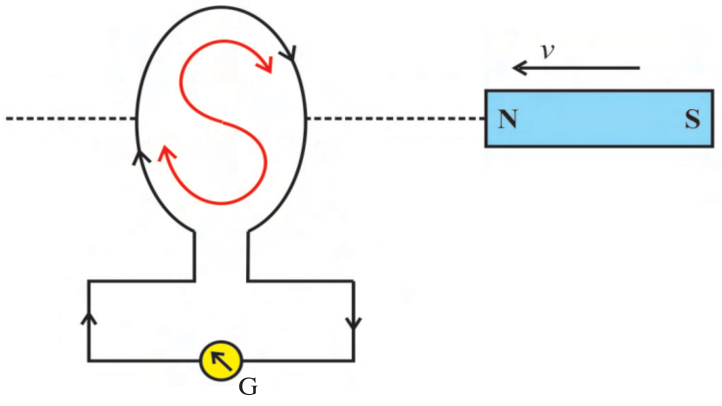


Figure 1.7

If this assumption is true, then by giving a gentle push to the magnet, the magnet will be attracted by the South Pole (S) of the coil and hence its speed will increase. As a result of this, the rate of change of flux linked with the coil increases and hence induced current will also increase. This makes the south pole of the coil more stronger which attracts the magnet towards itself with greater force. In this manner, the magnet will be accelerated more and more towards the coil (the velocity and kinetic energy of a magnet will continuously increase) and hence the induced current will also continuously increase. If an external resistance R is connected with the coil, the joule heat produced in it will continuously increase according to I^2Rt. In this case, no other mechanical work is done except giving a gentle push on the magnet even though heat energy I^2Rt is continuously being produced without expending any energy. According to the law of conservation of energy, we cannot produce energy without any cost. Hence our assumption that the "end of the coil facing the magnet acts like a South Pole (S) when the North Pole of the magnet is moved towards the coil" becomes wrong.

Now, according to the only other alternative left, this end of the coil, in the above case must behave like a North Pole (N), i.e. the current must flow in anticlockwise manner when viewed normally from the side of magnet, which opposes the change-here increase- in the flux.

If this is true, then there will be a repulsive force between the north pole of a magnet and induced north pole of the coil. Hence, mechanical work is required to be done by applying continuous external force in order to maintain the motion of the magnet towards the coil against this repulsive force. If this happens, the heat energy (I^2Rt) produced in the coil can be said to have produced at the cost of this mechanical work. This is consistent with the law of conservation of energy.

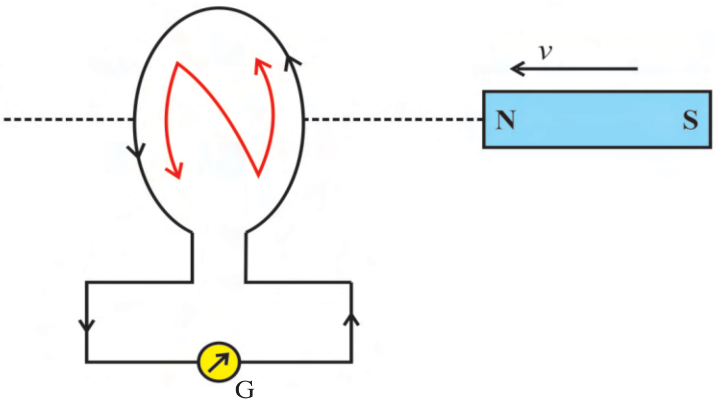


Figure 1.8 Direction of Induced Current

This discussion shows that, "induced emf (or induced current) is produced in such a direction that the magnetic field produced due to it opposes the very cause (here motion of the magnet) that produces it".

The above statement is known as Lenz's law which gives the direction of induced emf. Induced emf opposes the very cause which produces it.

1.5 Faraday's Law

From the experimental observations, Faraday arrived at a conclusion that an emf is induced in a coil when magnetic flux through the coil changes with time. Experimental observations discussed in article 1.2 shows the common fact that, the change of magnetic flux through a closed circuit (coil) induces emf in it. Faraday stated experimental observations in the form of a law called Faraday's law of electromagnetic induction which gives the magnitude of induced emf. The law is stated below.

"The magnitude of the induced emf produced in a closed circuit (or a coil) is equal to the negative of the time rate of change of magnetic flux linked with it".

Suppose the magnetic flux linked with each turn of the coil is ϕ at time t. The flux changes by $\Delta\phi$ in time interval Δt about this time is t. Then by Faraday's law,

Average induced emf = The negative of the time rate of change of magnetic flux during this time interval

$$\therefore \langle \varepsilon \rangle = -\frac{\Delta\phi}{\Delta t} \qquad (1.5.1)$$

Here, the negative sign indicates the presence of Lenz's Law.

$\therefore$ The instantaneous induced emf at time t,

$$\varepsilon = \lim_{\Delta t \to 0} \left(-\frac{\Delta\phi}{\Delta t} \right)$$

$$\varepsilon = -\frac{d\phi}{dt} \quad (1.5.2)$$

Now, if the coil is made up of N turns and flux linked with each turn is ϕ then the total magnetic flux linked with the coil (flux linkage) $\Phi = N\phi$.

Moreover, if the rate of change of flux associated with each turn is the same, then the rate of change of flux for the coil of N turns $= -\frac{d}{dt}(N\phi) = -N\frac{d\phi}{dt}$.

The total induced emf in a coil of N turns,

$$\varepsilon = -N\frac{d\phi}{dt} \quad (1.5.3)$$

1.6 Motional emf

The magnetic flux ($\phi = BA\cos\theta$) linked with a coil can be varied by many ways.

(1) The magnet can be moved with respect to the coil.

(2) The coil can be rotated in a magnetic field. (by changing angle θ between $\vec{A}$ and $\vec{B}$)

(3) The coil can be placed inside the magnetic field in a specific position and the magnitude of the magnetic induction ($\vec{B}$) can be changed with time.

(4) The magnet can be moved inside a non-uniform magnetic field.

(5) By changing the dimension of a coil (that is, by shrinking it or stretching it) in a magnetic field.

In all the cases mentioned above, the magnetic flux linked with the coil changes and hence emf is induced in the coil.

"The induced emf produced in a coil to due the change in magnetic flux linked with a coil due to some kind of motion is called motional emf"

One simple scheme of producing motional emf is shown in figure 1.9.

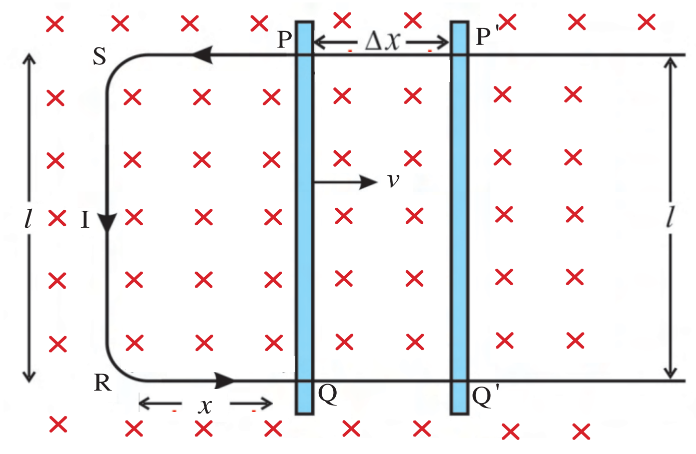


Figure 1.9 Motional emf

In figure 1.9, a U-shaped conducting wire is placed inside the uniform magnetic field $\vec{B}$ which is directed normally into the plane of paper in such a way that the plane formed by the conductor remains perpendicular to the magnetic field.

The conducting rod PQ is moved with a constant velocity v over the two arms of the U–shaped conductor. Assume that there is no loss of energy due to friction.

Here, the velocity of the rod is maintained constant by applying the force having same magnitude as that of the force which is acting opposite to the motion of the rod.

Suppose the normal distance between two arms of U-shaped conductor is RS $= l$ and RQ $=$ SP $= x$.

Note that the velocity of the rod is perpendicular to the magnetic field as well as its own length.

As the conducting rod PQ moves over the arms of U-shaped conductor, the area enclosed by a closed circuit PQRS changes and hence flux associated with a closed circuit also changes.

As a result emf is induced across two ends of a conducting rod PQ and induced current flows through the circuit.

Let PQ be the position of the rod at time t. In this situation the magnetic flux linked with the loop PQRS is,

$\phi = BA$

ϕ = (Magnetic induction) × (Area of PQRS)

$$\phi = Blx \qquad (1.6.1)$$

As the rod goes on sliding, the value of x also changes with time. The rate of change of flux will give the induced emf in a rod.

Using Faraday's law, the induced emf,

$$\varepsilon = -\frac{d\phi}{dt}$$

$$\varepsilon = -\frac{d}{dt}(Blx) = -Bl\frac{dx}{dt} = -Blv \qquad (1.6.2)$$

where, $\frac{dx}{dt} = v$ (velocity of rod)

This equation gives the value of induced emf produced in the circuit shown in figure 1.9. Here the motion of the rod is responsible for the generation of induced emf and hence this emf is called motional emf.

Thus, we are able to produce induced emf by moving a conductor instead of varying the magnetic field. (i.e. by changing the magnetic flux enclosed by the closed circuit)

In fact, if the rod is moves in a magnetic field without the U-shaped conductor, then also emf will be produced across the conductor which can be understood with the help of the following illustration.

The origin of the generation of induced emf : A conducting rod PQ moves in a magnetic field with its plane perpendicular to it as shown in figure 1.10. The positive ions and electrons in the rod will also move along with it (like a passenger in the train with a rod) in the direction of motion of the rod.

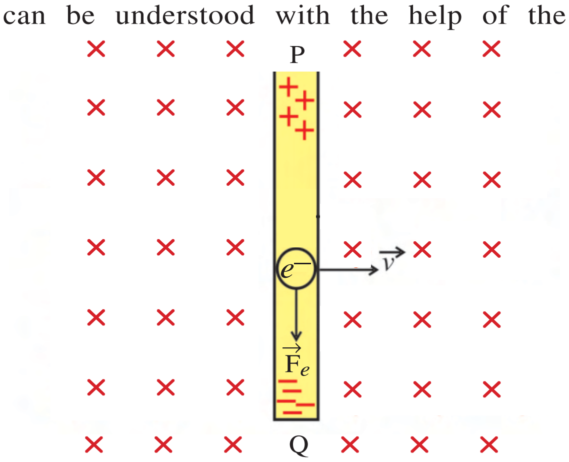


Figure 1.10 Induced emf in a Rod Moving in Magnetic Field

In the present case, they move with velocity $\vec{v}$ perpendicularly to the magnetic field $\vec{B}$. Hence. they experience Lorentz force $\vec{F} = q(\vec{v} \times \vec{B})$. Direction of this force can be determined by using right hand screw rule which is normal to the plane formed by $\vec{v}$ and $\vec{B}$.

Here, the positive ions will experience force from Q to P but as they remain fixed at their lattice points, they will not move under the influence of this force.

Now, according to the above equation, the force acting on electrons will be from P to Q. Since electrons are free to move, they deposit at Q end of the rod and make it negatively charged. Because of this positive charge of the ions opens up at P end and hence the resultant positive charge appears at P end.

Thus, end Q of the rod becomes negative and end P becomes positive and hence the rod behaves as a battery of emf $\varepsilon = Bvl$.

Conversion of Mechanical Energy into Electrical Energy : In the Illustration of U-shaped conductor, a rod is moving with velocity $\vec{v}$ perpendicular to the magnetic field $\vec{B}$ pointing into the plane of paper so that the lower end of the rod becomes negative and upper end becomes positive.

Here, the circuit gets completed and conventional current I flows in the direction PSRQ. Now the rod carrying electric current and moving through the magnetic field experiences a force according to $\vec{F} = I\vec{l} \times \vec{B}$.

If the resistance of the rod is R, the current flowing through a closed circuit is, $I = \frac{\varepsilon}{R} = \frac{Bvl}{R}$.

The force BIl acting on the rod, is opposite to the direction of velocity $\vec{v}$ of the rod. Thus, to continue the motion of the rod a force BIl must be applied towards right side. Such a force is called "Lenz force".

Here, mechanical power = Force × Velocity

$$P_m = Fv$$

$$P_m = BIlv$$

$$P_m = B\left(\frac{Bvl}{R}\right)lv = \frac{B^2v^2l^2}{R} \qquad (1.6.3)$$

Electrical power generated in the circuits, $P_e = \varepsilon I$

$$P_e = (Bvl)I$$

$$P_e = (Bvl)\left(\frac{Bvl}{R}\right) = \frac{B^2v^2l^2}{R} \qquad (1.6.4)$$

Equations (1.6.3) and (1.6.4) show that the electrical power generated is equal to the mechanical power spent i.e the mechanical work done in continuing the motion of the rod is obtained in the form of electrical energy. Here, we have ideally considered heat dissipation as zero.

From Faraday's law the magnitude of the induced emf,

$$|\varepsilon| = \frac{\Delta\Phi}{\Delta t}$$

However, $|\varepsilon| = IR = \frac{\Delta Q}{\Delta t}R$

$$\text{Thus, } \Delta Q = \frac{\Delta\Phi \text{ (Net change in Magnetic Flux)}}{R \text{ (Resistance)}} \qquad (1.6.5)$$

which gives the relation between the induced charge flow through the circuit and the change in magnetic flux. Note that induced charge does not depend on the rate of change of magnetic flux.

Illustration 1 : A conducting circular loop of surface area 2.5×10^{-3} m^2 is placed perpendicular to a magnetic field which varies as B = (0.20 T) sin [(50π s^{-1})t]. Find the charge flowing through any cross section during the time t = 0 to t = 40 ms. Resistance of the loop is 10 Ω.

Solution : Face area of the loop $A = 2.5 \times 10^{-3}$ m^2

Resistance of the loop R = 10 Ω

Magnetic field changes as $B = B_0 \sin\omega t$

where $B_0 = 0.20$ T and $\omega = 50\pi$ s^{-1}

The flux passing through the loop at time t is $\phi = AB_0\sin\omega t$

The induced emf is $\varepsilon = -\frac{d\Phi}{dt} = -AB_0\omega\cos\omega t$

Induced current $I = \frac{\varepsilon}{R} = \frac{-AB_0\omega}{R}\cos\omega t$

$$= -I_0\cos\omega t$$

where, $I_0 = \frac{AB_0\omega}{R}$

The current changes sinusoidally with the time period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50\pi\,s^{-1}} = 40 \times 10^{-3}$ s

The charge flowing through any cross section during time t = 0 to t = 0.04 s is,

$$Q = \int_0^{0.04} I dt = -I_0 \int_0^{0.04} \cos\omega t \; dt$$

$$\therefore \; Q = -\frac{I_0}{\omega}[\sin\omega t]_0^{0.04}$$

$$\therefore \; Q = 0$$

Illustration 2 : A conducting circular loop is placed in a uniform magnetic field of 0.04 T with its plane perpendicular to the field. Somehow, the radius of the loop starts shrinking at a constant rate of $2\frac{mm}{s}$. Find the induced emf in the loop at an instant when the radius becomes 2 cm.

Solution : Let the radius of the loop be r at time t.

The magnetic flux linked with the loop at this instant is,

$$\phi = AB = \pi r^2 B$$

Here, $\frac{dr}{dt} = 2$ mms^{-1} $= 2 \times 10^{-3}$ ms^{-1}

When radius of the loop becomes $r = 2$ cm $= 2 \times 10^{-2}$ m, the induced emf in the loop,

$$\varepsilon = \frac{d\phi}{dt}$$

$$\varepsilon = \frac{d}{dt}(\pi r^2 B)$$

$$\varepsilon = 2\pi B r \frac{dr}{dt}$$

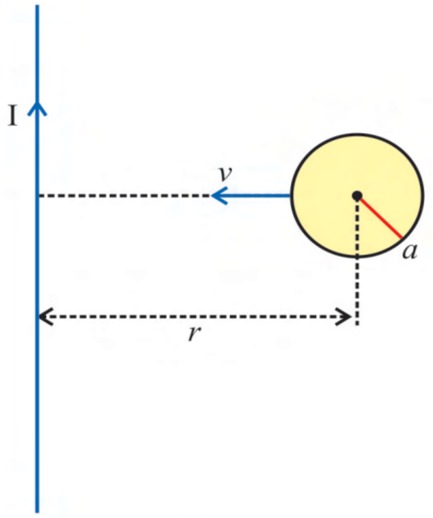
$$\varepsilon = 2\pi\ (0.04)\ (2 \times 10^{-2})\ (2 \times 10^{-3})$$

$$= 3.2\pi \times 10^{-6}\ V$$

$$= 3.2\pi\ \mu V$$

Illustration 3 : As shown in figure, a long wire kept vertically on the plane of paper carries electric current I. A conducting ring moves towards the wire with velocity v with its plane coinciding with the plane of paper. Find the induced emf produced in the ring when it is at a perpendicular distance r from the wire. Radius of the ring is a and $a << r$.

Solution : Magnetic field at a distance r from the wire carrying current is, $B = \frac{\mu_0 I}{2\pi r}$.



$\therefore$ Magnetic flux linked with the ring,

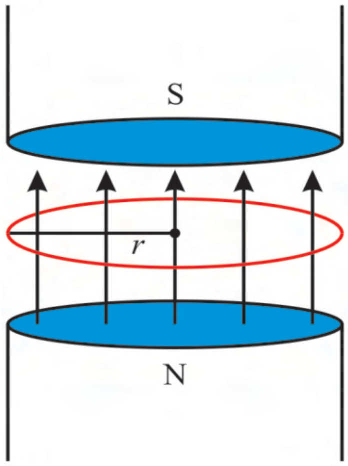
$$\phi = B(\pi a^2) = \frac{\mu_0 I}{2\pi r} \times \pi a^2 = \frac{\mu_0 I a^2}{2r}$$

$$\therefore \text{ Induced emf } \varepsilon = -\frac{d\phi}{dt} = -\frac{d}{dt}\left(\frac{\mu_0 I a^2}{2r}\right)$$

$$= \frac{\mu_0 I a^2}{2}\left(\frac{1}{r^2}\right)\frac{dr}{dt}$$

$$\therefore \varepsilon = \frac{\mu_0 I a^2}{2r^2} v \qquad (\because \frac{dr}{dt} = v)$$

Illustration 4 : A conducting ring of radius r is placed perpendicularly inside a time–varying magnetic field as shown in figure. The magnetic field changes with time according to $B = B_0 + \alpha t$ where B_0 and α are positive constants. Find the electric field on the circumference of the ring.



Solution : The magnetic field linked with the ring at time t,

$$B = B_0 + \alpha t$$

$$\therefore \phi = B(\pi r^2) = (B_0 + \alpha t)\pi r^2 \qquad .....(1)$$

From Faraday's law, emf produced in a ring,

$$\therefore \varepsilon = -\frac{d\phi}{dt}$$

$$= -\frac{d}{dt}\ [(B_0 + \alpha t)\pi r^2]$$

$$\therefore \varepsilon = -\alpha\ \pi r^2 \qquad .....(2)$$

Now by definition, emf is the work done by the electric field for one complete revolution of a unit positive charge on the circumference of the ring. If $\vec{E}$ is the electric field intensity, the work done is,

$= \int \vec{E} \cdot \vec{dl}$ since $\vec{E}$ and $\vec{dl}$ are in the same direction,

$\int \vec{E} \cdot \vec{dl} = E \int dl$

$= E(2\pi r)$ .....(3)

Comparing equations (2) and (3),

$E(2\pi r) = \alpha \pi r^2$ (neglecting negative sign)

$\therefore \ E = \frac{\alpha r}{2}$

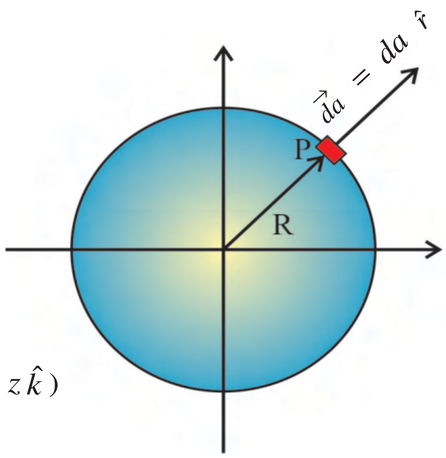
Note : See that the magnetic field goes on changing with time but the electric field in the ring remains constant. Though, this is not a general result. When the magnetic field changes non-linearly with time, the result will not be the same.

Illustration 5 : A field is given by $\vec{A} = x\hat{i} + y\hat{j} + z\hat{k}$. Can this field be used to obtain induced emf ?

[**Hint :** A field must be magnetic one to obtain induced emf.]

Solution : If a given field is magnetic, its surface integration over any closed surface (flux passing through the closed surface) must be zero. For this we will consider the surface of a sphere of radius R whose center is at the origin of the coordinate system.

In figure $\vec{da} = da\ \hat{r}$ represents the vector of a surface element on the surface of this sphere. If coordinates of point P are (x, y, z)



$\vec{R} = x\hat{i} + y\hat{j} + z\hat{k}$

$\therefore \ \hat{r} = \frac{\vec{R}}{|\vec{R}|} = \frac{x}{R}\hat{i} + \frac{y}{R}\hat{j} + \frac{z}{R}\hat{k}$

Surface integration over given surface is,

$$\int_{\text{surface of sphere}} \vec{A} \cdot \vec{da} = \int_{\text{surface of sphere}} (x\hat{i} + y\hat{j} + z\hat{k}) \cdot \frac{da}{R}(x\hat{i} + y\hat{j} + z\hat{k})$$

$$= \frac{1}{R}\int (x^2 + y^2 + z^2)\, da = R\int da$$

$$= R \times 4\pi R^2$$

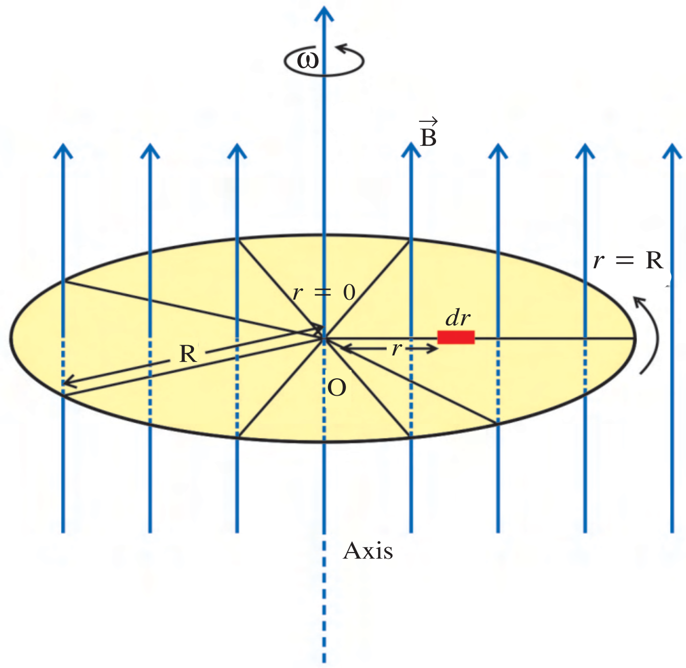
$$= 4\pi R^3 \neq 0$$

Thus the surface integral of given field is not zero over a closed surface and hence it cannot be magnetic field. Therefore, induced emf cannot be produced.

Illustration 6 : A wheel having n conducting concentric spokes is rotating about its geometrical axis with an angular velocity ω, in a uniform magnetic field B perpendicular to its plane. Prove that the induced emf generated between the rim of the wheel and the center is $\frac{\omega B R^2}{2}$, where R is the radius of the wheel. It is given that the rim of the wheel is conducting.

Solution : As shown in the figure, consider a small element dr on any spoke at a distance r from the center.

Linear velocity of this element $v = r\omega$



emf induced in a small element dr is,

$$d\varepsilon = Bvl$$

$$= B(r\omega)dr$$

Total emf induced along the entire length of any spoke is,

$$\varepsilon = \int_{r=0}^{r=R} d\varepsilon = \int_{r=0}^{r=R} B(r\omega)dr$$

$$\varepsilon = B\omega\int_0^R rdr = B\omega\left[\frac{r^2}{2}\right]_0^R$$

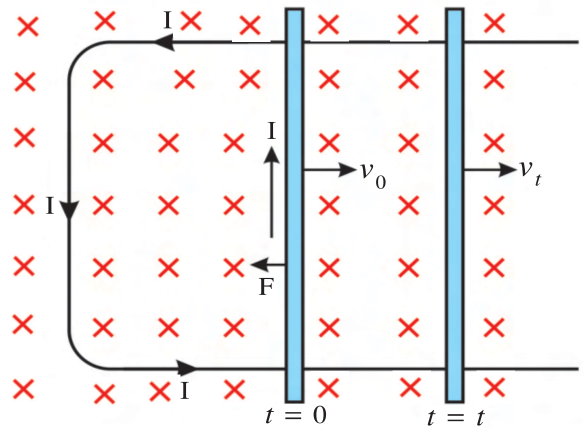
$$\therefore \varepsilon = \frac{1}{2}B\omega R^2$$

Application of the right hand screw rule with equation $\vec{F} = -e(\vec{v} \times \vec{B})$ shows that free electrons in a spoke will experience force towards the center of the wheel. Therefore, the free electrons accumulate at the center of the wheel leaving the rim positively charged. Thus, the end of the spoke at the center of the wheel behaves as a negative electrode and the end of the spoke on the rim behaves as a positive electrode.

If a rod of length L rotates with a uniform angular velocity ω about its perpendicular bisector and uniform magnetic field $\vec{B}$ exists parallel to the axis of rotation, what will be the potential difference between the two ends of the rod and between the center of the rod and the end ? Think !!

Illustration 7 : A U-shaped conducting frame is placed in a magnetic field B in such a way that the plane of the frame is perpendicular to the field lines. A conducting rod is supported on the parallel arms of the frame, perpendicular to them and is given a velocity v_0 at time $t = 0$. Prove that the velocity of the rod at time t will be given by $v_t = v_0 \exp\left(\frac{-B^2l^2}{mR}t\right)$.

Solution : As shown in figure, when a conducting rod is given a motion in magnetic field, emf is induced in it and hence induced current flows through the rod i.e. the rod carries current. Here, the force acting on the rod due to magnetic field ($F = BIl$) is opposite to the motion of the rod. Therefore, the rod will decelerate and its velocity decreases with time.



Emf induced in a rod at time t is,

$$\varepsilon = -Bv_t l$$

$$IR = -Bv_t l$$

$\therefore$ Induced current at time t, $I = \frac{-Bv_t l}{R}$

Here, the rod is moving in a magnetic field. Therefore, the force acting on the rod at time t according to Lenz's law is,

$$F = BIl$$

$$= B\left(\frac{-Bv_t l}{R}\right)l$$

$$\therefore F = \frac{-B^2 l^2 v_t}{R} \qquad \text{......(1)}$$

According to Lenz's law, this force is acting in the direction opposite to the motion of the rod, it produces deceleration $a = \frac{dv_t}{dt}$ in the rod.

From $ma = F$,

$$m\frac{dv_t}{dt} = \frac{-B^2 l^2 v_t}{R} \quad \text{(Using equation (1))}$$

$$\therefore \frac{dv_t}{v_t} = -\frac{B^2 l^2}{mR} dt$$

Integration on both the sides,

$$\int_{v_0}^{v_t} \frac{1}{v_t} dv_t = -\frac{B^2 l^2}{mR} \int_{t=0}^{t} dt$$

$$\left[ln\ v_t\right]_{v_0}^{v_t} = -\frac{B^2 l^2}{mR} [t]_0^t \qquad \text{......(2)}$$

$$\therefore ln\ v_t - ln\ v_0 = -\frac{B^2 l^2}{mR} t$$

$$\therefore ln\left(\frac{v_t}{v_0}\right) = -\frac{B^2 l^2}{mR} t$$

$$\therefore \frac{v_t}{v_0} = \exp\left[\frac{-B^2 l^2}{mR} t\right]$$

$$\therefore v_t = v_0 \exp\left[\frac{-B^2 l^2}{mR} t\right]$$

1.7 Eddy Currents

Whenever a solid conductor is kept in a region of varying magnetic field, the magnetic flux linked with the conductor changes and induced emf is produced by induction. As a result, circulatory currents are induced in the plane normal to the direction of flux. These currents are distributed throughout the conductor. These are known as Eddy currents because of their circulatory nature. Eddy currents were first observed by physicist Foucault in 1895. The direction of flow of these currents is determined by Lenz's law. When a conductor rotates in a uniform magnetic field, then also eddy currents are produced in it.

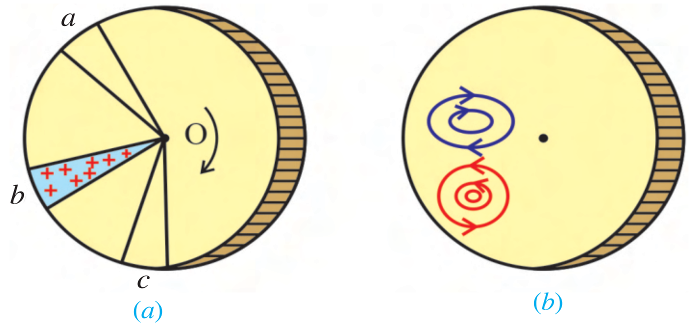


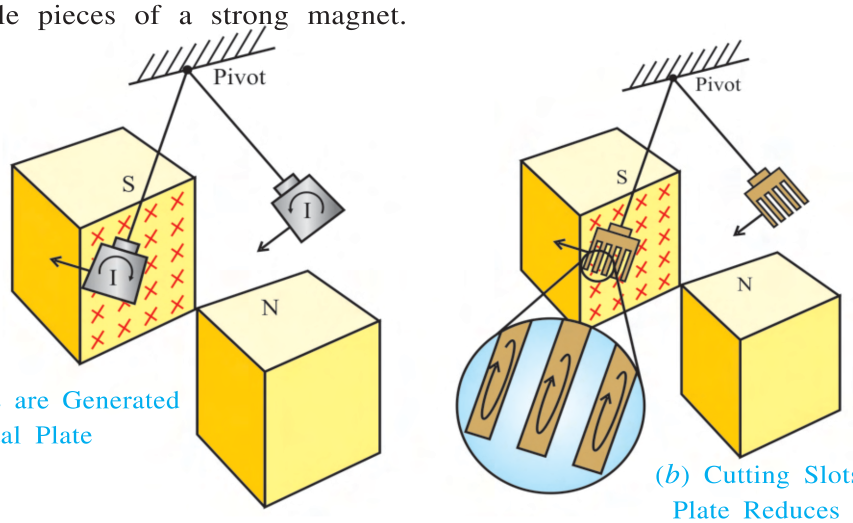
Figure 1.11 Eddy Currents in Disc

Suppose a magnetic field is applied to the portion of rotating disc in the direction perpendicular to the plane of disc. As shown in figure 1.11 (*a*), element O*b* of the disc is moving across the field, the electrons of this element will start moving under the influence of the force $\vec{F} = -e(\vec{v} \times \vec{B})$. Element O*a* and O*c* are not in the field. So, they provide return conducting path to the charges displaced along O*b*. In this way eddy currents are set up in the disc.

Arago performed a simple experiment to find the direction of eddy currents. A metal disc was pivoted horizontally so that it can rotate about vertical axis. A magnetic needle was freely suspended just above the plane of a disc without touching it, when the disc was rotated rapidly, it will cut the flux of magnetic field. The needle was found rotating in the direction of rotation of the disc due to induced current. When the direction of rotation of the disc was reversed, the needle was found to rotate in the reverse direction.

Consider a metal plate falling downward into a uniform magnetic field applied normal to the plane of paper and pointing inward. The electrons inside the plate will experience a force $[\vec{F} = -e(\vec{v} \times \vec{B})]$ because of the motion of the plate. Under the influence of this force, electrons move on the paths which offer minimum resistance and constitute eddy currents. These currents, according to Lenz's law, flow in such a direction that the magnetic field produced due to them opposes the motion of the conductor. Hence, the plate appears to fall with acceleration less than g in the presence of magnetic field.

As shown in figure 1.12 (*a*), a metallic plate is allowed to oscillate like a simple pendulum between two pole pieces of a strong magnet.



(*a*) Eddy Currents are Generated in the Metal Plate

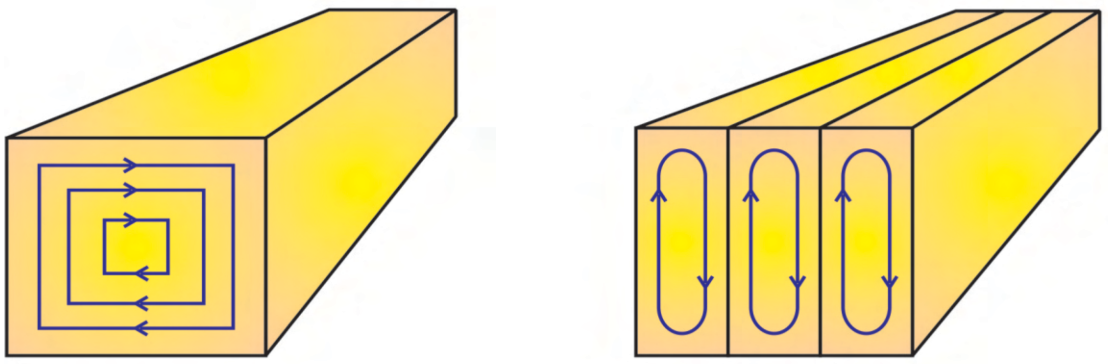
(*b*) Cutting Slots in the Metal Plate Reduces the Effect of Eddy Currents

Figure 1.12

It is observed that the oscillations of the plate is damped and in a short while the plate comes to rest in the magnetic field. Such a damping is called electromagnetic damping. Magnetic flux associated with the plate keeps on changing as the plate moves in and out of the region between magnetic poles. Eddy currents are produced in a plate due to change in magnetic flux. According to Lenz's law, these eddy currents oppose the motion of the plate in a magnetic field. The directions of eddy currents are opposite when the plate swings into the region between two poles and when it swings out of the region.

If rectangular slots are made in the metal plate as shown in figure 1.12 (*b*), area available to the flow of eddy currents become less. Thus, the length of the path of electrons is greatly increased in the plate. So magnitude of eddy current is decreased. As a result of this the effect of eddy current is reduced. Thus the pendulum plate with slots oscillates for a longer time because the effect of electromagnetic damping is reduced.

In the interior of the iron cores of the rotating armatures of motors and dynamos and also in the core of transformer, eddy currents are produced. Eddy currents are undesirable since they heat up the iron core and dissipate electrical energy in the form of heat energy. To reduce the effect of eddy currents, a laminated core instead of a single solid piece of iron is used (figure 1.13). The iron core is made up of several layers. These layers are separated by an insulating material (Varnish). In this way, eddy currents flow through the individual laminations instead of the whole core. Thus the length of the path of electron is greatly increased, as a result the strength of eddy currents can be minimized and energy loss is substantially reduced.



(*a*) Solid Piece of Metal

(*b*) Solid Piece Made Up of Insulated Layers

Figure 1.13

Applications of Eddy Currents

(1) Induction Furnace : When a metal specimen is placed in a rapidly changing magnetic field (produced by high frequency a.c.) eddy currents generated in the metal produce high temperatures sufficient to melt the metal. This process is used in the extraction of some metals from their ores.

Induction furnace can be used to produce high temperatures and can be utilized to prepare alloys.

(2) Speedometer : In a speedometer, a tiny magnet rotates according to the speed of the vehicle and produces the required changing magnetic field. The magnet rotates in an aluminium drum. Eddy currents are set up in the drum. The drum turns in the direction of the rotating magnet. A pointer attached to the drum indicates the speed of the vehicle on a calibrated scale.

(3) Electric Brakes : When a strong magnetic field is applied to the rotating drum attached to the wheel, eddy currents set up in the drum which exert a restoring torque on the drum so the motion of the drum stops. Using this fact, the eddy currents are used in braking system of trains so that the brakes can be applied smoothly.

(4) Electric Power Meters : The shiny metal disc in the electric power meter rotates due to the eddy currents. Electric currents are induced in the disc by magnetic fields produced by sinusoidally varying currents in a coil.

1.8 Self Induction

When an electric current is passed through an insulated conducting coil, it gives rise to a magnetic field in the coil so that the coil itself behaves like a magnet. The magnetic flux produced by the current in the coil is linked with the coil itself.

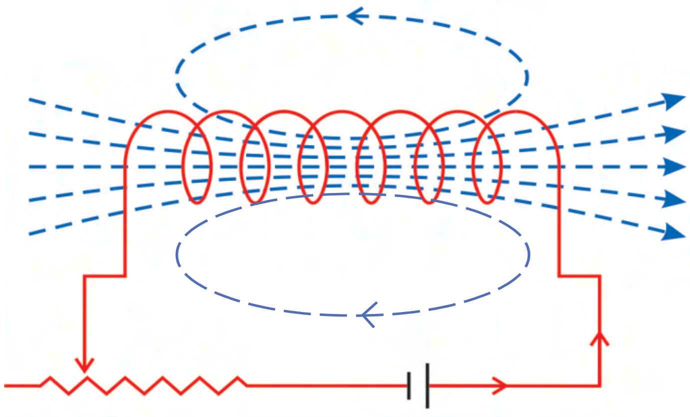


Figure 1.14 Self-Induction in a Coil

As the strength of the current in the coil is changed, the flux linked with the coil also changes. Under such circumstances an emf is induced in the coil too. Such emf is called a self-induced emf and this phenomenon is known as self-induction.

If the number of turn in a coil is N and the flux linked with each turn is ϕ, then the total flux linked through the coil = $N\phi$.

In this case, the total flux linked with the coil (which is called flux linkage) is directly proportional to the current I flowing through the coil.

$$N\phi \propto I$$

$$\therefore \quad N\phi = LI \qquad (1.8.1)$$

where the constant of proportionality L is called the self–inductance of a coil. From equation (1.8.1),

$$L = \frac{N\phi}{I} \qquad (1.8.2)$$

The self inductance L is a measure of the flux linked with coil per unit current.

The self-inductance L of a coil depends upon –

(1) The size and shape of the coil.

(2) The number of turns N.

(3) The magnetic property of the medium within the coil in which the flux exists.

If the coil is wound around a soft iron core, the flux linked with the coil increases because of very high permeability of soft iron. Therefore, the value of self-inductance L increases to a great extent.

Self-inductance L does not depend on current I.

Differentiating equation (1.8.1),

$N\phi = LI$ with respect to time t,

$$N\frac{d\phi}{dt} = L\frac{dI}{dt} \qquad (1.8.3)$$

In the case of self-induction, Faraday's law and Lenz's law holds good. Hence self-induced emf in the coil is,

$$\varepsilon = -N\frac{d\phi}{dt} \tag{1.8.4}$$

Self-induced emf is also called "back emf".

Equating equations (1.8.3) and (1.8.4), we get,

$$\varepsilon = -L\frac{dI}{dt} \tag{1.8.5}$$

If the rate of change of current $\left(\frac{dI}{dt}\right) = 1$ unit,

$\varepsilon = -L$

So the self-inductance of a circuit can be defined as under :

"The self-induced emf produced per unit rate of change of current $\left(\frac{dI}{dt} = 1\right)$ in the circuit is called self-inductance of the circuit."

Form equation $\varepsilon = -L\frac{dI}{dt}$,

Self inductance $L = -\frac{\varepsilon}{\left(\frac{dI}{dt}\right)}$

Unit of $L = \frac{\text{Unit of emf (V)}}{\text{Unit of rate of change of current (As}^{-1})} = VsA^{-1}$

The SI unit of self-inductance L is Henry H and its dimensional formula is $M^1L^2T^{-2}A^{-2}$.

Henry : If the rate of change of current $\left(\frac{dI}{dt}\right) = 1\ As^{-1}$ and the induced emf is $\varepsilon = 1V$, then the self-inductance of the circuit is said to be 1 H.

The component of the circuit (e.g. coil) which posseses self-inductance is called an inductor. The symbol for an inductor in a circuit is shown in figure 1.15.

When an inductor is connected in a circuit, the end of the inductor through which the current enters and increases with time is considered positive and the end through which the current leaves is considered as negative. Thus, direction of induced emf can be determined.

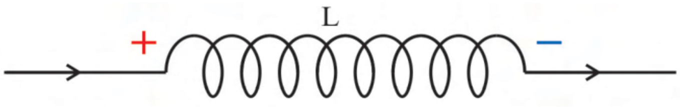


Figure 1.15 Symbol of Inductor

Magnetic Energy Stored in an Inductor : Suppose I is the current flowing through an inductor at time t and the rate of change of current in inductor is $\left(\frac{dI}{dt}\right)$.

Therefore, the induced emf between two ends of an inductor is, $\varepsilon = L\frac{dI}{dt}$. Here, the negative sign is ignored.

This self-induced emf is also called the back emf as it opposes any change in the current in a circuit. Physically, the self-inductance plays the role of inertia. It is the electro-magnetic analogue of mass (m) in mechanics. So, work needs to be done against the back emf (ε) in establishing the current. This work done is stored as magnetic potential energy.

For the current I at an instant in a circuit, the rate of work done is,

$$\frac{dW}{dt} = |\varepsilon|\ I \qquad (1.8.6)$$

Using equation (1.8.5),

$$\frac{dW}{dt} = LI\frac{dI}{dt} \qquad (1.8.7)$$

Total amount of work done in establishing the current I is,

$$W = \int_0^I dW$$

$$W = \int_0^1 LI\ dl$$

$$W = \frac{1}{2}LI^2 \qquad (1.8.8)$$

Thus, the electrical energy required to build up the current I is

$$W = \frac{1}{2}LI^2$$

This energy is stored in an inductor in the magnetic field linked with it.

This expression reminds us of $\frac{1}{2}mv^2$ for the (mechanical) kinetic energy of a particle of mass m. It shows that L is analogous to m. (i.e. L is the electrical inertia and opposes growth and decay of current in the circuit).

Illustration 8 : Find the value of the self-inductance of a very long solenoid of length l, having total number of turns equal to N, and cross-sectional area A.

Solution : The number of turns per unit length of the solenoid is $\frac{N}{l}$.

∴ The magnetic field at any point within the solenoid, on passing a current I will be

$$B = \frac{\mu_0 NI}{l}$$

The total flux linked with the entire solenoid will be,

$$\Phi = BAN$$

$$= \frac{\mu_0 NIA}{l}N$$

$$= \frac{\mu_0 N^2 IA}{l}$$

∴ Self-inductance, $L = \frac{\Phi}{I} = \frac{\mu_0 N^2 A}{l}$.

1.9 Mutual Induction

If two conducting coils are put close to each other, and a steady current is passed through one coil, magnetic flux links with the other coil. If the current flowing through the current carrying coil is changed, an emf is set up in the second coil according to Faraday's law. This phenomenon is called mutual induction.

Figure 1.16 shows two conducting coils near each other sharing a common central axis. Let the number of turns in coil 1 and 2 be N_1 and N_2 respectively.

Coil-1 is connected with battery, key and rheostat whereas coil-2 is connected to a sensitive galvanometer but contains no battery. When a steady current I_1 is passed through coil-1, it creates magnetic field (B_1) in coil-1. Some of the magnetic field lines of B_1 links with coil-2.

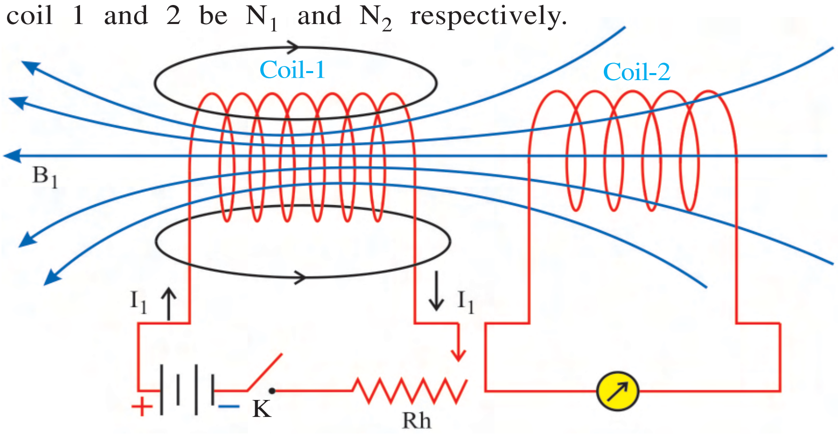


Figure 1.16 Mutual Inductance

For a given position of coil 1 and 2, it follows Bio-Savart law, that the flux Φ_2 linked with coil-2 will be proportional to the current I_1 in coil-1.

$$\Phi_2 \propto I_1$$

$$\therefore \ \Phi_2 = M_{21} I_1 \qquad (1.9.1)$$

If the current I_1 in coil-1 is changed, there will be a corresponding change in the flux Φ_2 linked with coil-2. An emf ε_2 given by Faraday's law is induced in coil-2 which is given by,

$$\varepsilon_2 = -\frac{d\Phi_2}{dt}$$

$$\varepsilon_2 = -\frac{d}{dt}(M_{21} I_1)$$

$$\varepsilon_2 = -M_{21} \cdot \frac{dI_1}{dt} \qquad (1.9.2)$$

The constant of proportionality M_{21} which appears in the equations (1.9.1) and (1.9.2) is termed as the mutual inductance of the system formed by two coils. It can be defined from equations (1.9.1) and (1.9.2).

Taking $I_1 = 1$ unit in the equation (1.9.1), we get $\Phi_2 = M_{21}$.

Thus, "The magnetic flux linked with one of the coils of a system of two coils per unit current flowing through the other coil is called mutual inductance of the system".

If the current is expressed in A and flux in Wb, then the unit of mutual inductance is WbA^{-1} = henry (H).

If, $\frac{dI_1}{dt} = 1$ unit in equation (1.9.2),

Then $\varepsilon_2 = -M_{21}$

Thus, "the emf generated in one of the two coils due to a unit rate of change of current in the other coil is called the mutual inductance of the system of two coils".

If we take $\frac{dI_1}{dt}$ in As^{-1} and ε_2 in V, then the unit of mutual inductance becomes $\frac{V}{As^{-1}} = VsA^{-1}$ = henry H. You can verify that the dimension of henry defined in either way are the same.

The mutual inductance M of a system of two coils depends upon their shape and size, their number of turns, distance between them, their relative orientation and the magnetic property of the medium on which they are wound.

Instead of coil-1, if we set up a current I_2 in coil-2 by means of a battery, this produces a magnetic flux Φ_1 that links with coil-1. If we change current I_2 flowing through coil-2, the emf induced in coil-1 by the argument given above is,

$$\varepsilon_1 = -M_{12}\frac{dI_2}{dt} \qquad (1.9.3)$$

The mutual inductance will be same in both the cases discussed above. i.e. $M_{21} = M_{12} = M$. This result is called the reciprocity theorem.

Illustration 9 : Two long solenoids are of equal length l and the smaller solenoid having a cross-sectional area a is placed within the larger solenoid in such a way that their axes coincide. Find the mutual inductance of the system.

Solution : When a current I_1 is flowing through the smaller solenoid, the magnetic field strength within it is given by $\frac{\mu_0 N_1 I_1}{l}$.

Where N_1 = Number of turns for the smaller solenoid.

The flux linked with the larger solenoid due to this field is

$$\Phi_2 = \frac{\mu_0 N_1 N_2 I_1 a}{l}$$ (Where, N_2 = number of turns of the larger solenoid.)

$$\therefore\ M_{21} = \frac{\Phi_2}{I_1} = \frac{\mu_0 N_1 N_2 a}{l} \qquad (1)$$

Now consider the situation in which a current I_2 is flowing through the outer solenoid, the field inside it is given by = $\frac{\mu_0 N_2 I_2}{l}$.

The flux due to this field linked with the inner solenoid is

$$\Phi_1 = \frac{\mu_0 N_1 N_2 I_2 a}{l}$$

$$\therefore\ M_{12} = \frac{\Phi_1}{I_2} = \frac{\mu_0 N_1 N_2 a}{l} \qquad (2)$$

From the equation (1) and (2) we find $M_{21} = M_{12} = M$.

Illustration 10 : A small square loop of wire of side l is placed inside a large square loop of wire of side L. (L >> l). The loops are coplanar and their centres coincide. Find the mutual inductance of the system.

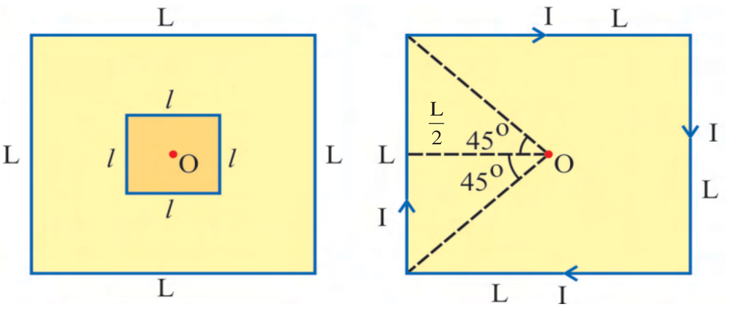
Solution : Let a current I pass through the large square loop of side L. Magnetic field at the centre O of the loop,

B = 4 × Magnetic field due to each side

$$B = 4 \times \frac{\mu_0 I}{4\pi\left(\frac{L}{2}\right)} (\sin 45^\circ + \sin 45^\circ)$$

$$B = \frac{2\mu_0 I}{\pi L}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)$$

$$\therefore\ B = \frac{2\sqrt{2}\mu_0 I}{\pi L}$$



Since l is very small compared to L, value of B can be considered uniform over the area A = πl^2 of the inner loop.

∴ Magnetic flux linked with the small square loop,

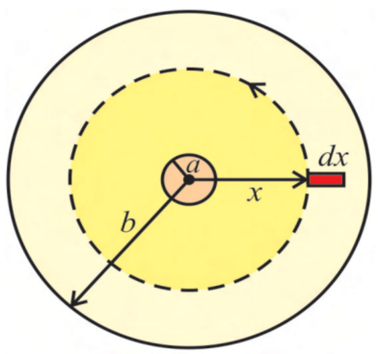
$$\Phi = BA = Bl^2 = \frac{2\sqrt{2}\mu_0 I l^2}{\pi L}$$

Mutual inductance of the system of two loops,

$$M = \frac{\Phi}{I} = \frac{2\sqrt{2}\mu_0 l^2}{\pi L}$$

Illustration 11 : Current I is passing through the central wire as well as the outer cylindrical layer of a co-axial cable in mutually opposite directions as shown in the figure. Find self inductance of this cable. The co-axial cable is normal to the plane of paper.

Solution : Magnetic field at a distance x from the central wire is,



$$B(x) = \frac{\mu_0 I}{2\pi x}$$

The magnetic flux passing through a strip of length l and width dx parallel to the axis is,

$$d\phi = B(x) l dx = \frac{\mu_0 I l}{2\pi x} dx$$

So the flux passing through the space of length l and breadth $(b - a)$, between the two wires will be,

$$\phi = \int_a^b d\phi = \frac{\mu_0 I l}{2\pi x} \int_a^b \frac{1}{x} dx$$

$$= \frac{\mu_0 I l}{2\pi} [\ln x]_a^b$$

$$= \frac{\mu_0 I l}{2\pi} \ln \frac{b}{a}$$

Now, self-inductance, $L = \frac{\phi}{I} = \frac{\mu_0 l}{2\pi} \ln \frac{b}{a}$

Illustration 12 : Prove that the energy density associated with the magnetic field of a very long solenoid is $\frac{B^2}{2\mu_0}$.

Solution : Magnetic induction linked with a solenoid is,

$$B = \frac{\mu_0 NI}{l} \quad (1)$$

Where, N = total number of turns of the solenoid,

l = length of the solenoid,

I = electric current.

Now, if self-inductance of the solenoid is L, the energy of magnetic field linked with it is,

$$U = \frac{1}{2}LI^2 \quad (2)$$

Keeping the value of I from equation (1) in equation (2),

$$U = \frac{1}{2}L\frac{B^2 l^2}{\mu_0^2 N^2} \quad (3)$$

But, self-inductance of a solenoid is,

$$L = \frac{\mu_0 N^2 A}{l} \quad (4)$$

where, A = Cross-sectional area of the solenoid.

Substituting the value of L from equation (4) in equation (3),

$$U = \frac{1}{2}\frac{\mu_0 N^2 A}{l}\frac{B^2 l^2}{\mu_0^2 N^2}$$

$$\therefore \; U = \frac{1}{2\mu_0} A l B^2$$

$\therefore$ Now energy density is the energy per unit volume of the solenoid,

$$\rho_B = \frac{U}{Al} = \frac{1}{2\mu_0}B^2$$

Note : We have proved that energy of a charged capacitor is stored in the electric field between its two plates and energy density of electric field is $\rho_E = \frac{1}{2}\varepsilon_0 E^2$. Though the equations of ρ_B and ρ_E are obtained in cases of a solenoid and a capacitor, they are valid for more general cases also. If electric and magnetic field exist in some region of space (for Illustration electromagnetic waves), the energy density associated with the fields,

$$\rho = \rho_E + \rho_B$$

$$\therefore \; \rho = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0}B^2$$

1.10 AC Generator

An important application of the phenomenon of electromagnetic induction is the generation of alternating currents (AC). Here, we shall discuss the principle of AC generator. One method to induce an emf or current in a loop is through a change in the loop's orientation or a change in its effective area. When a coil of surface area $\vec{A}$ rotates in a magnetic field $\vec{B}$, the effective area of the loop (the face perpendicular to the field) is $A\cos\theta$ (θ = angle between $\vec{A}$ and $\vec{B}$). This method of producing a flux change is the principle of operation of AC generator. An AC generator converts mechanical energy into electrical energy.

The basic components of an AC generator are shown in figure 1.17. It consists of a coil mounted on a rotor shaft. The axis of rotation of a coil (called armature) is perpendicular to the direction of magnetic field $\vec{B}$. When the coil (armature) is mechanically rotated in the uniform magnetic field ($\vec{B}$) by some external means, then the flux through the coil changes. So, an emf is induced in the coil. The ends of the coil are connected to an external circuit by means of slip rings A_1, A_2 and brushes B_1, B_2.

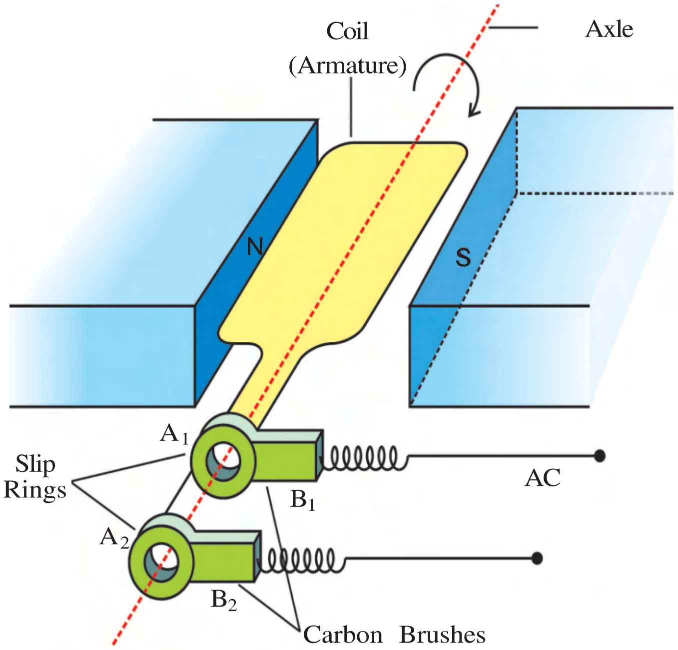


Figure 1.17 A.C. Generator

When the coil is rotated with a constant angular velocity ω in a uniform magnetic field $\vec{B}$, the angle θ between the magnetic field Vector $\vec{B}$ and the Area Vector $\vec{A}$ at any instant t is, $\theta = \omega t$ (assuming $\theta = 0$ at time $t = 0$).

As the coil having N turns is continuously rotating in a magnetic field, the magnetic flux $\Phi = NAB\cos\theta = NAB\cos\omega t$ associated with the coil keeps on changing with time.

The emf induced in the coil, according to Faraday's law is,

$$V = -\frac{d\Phi}{dt}$$

$$V = -\frac{d}{dt}(NBA\cos\omega t)$$

$$V = -NBA\frac{d}{dt}(\cos\omega t)$$

$$V = NBA\omega\sin\omega t \qquad (1.10.1)$$

where, $NBA\omega = V_m$ = Maximum induced emf in the coil.

$$\therefore\ V = V_m\sin\omega t \qquad (1.10.2)$$

Equation (1.10.2) suggests that the induced emf in the coil varies with time as per the function $\sin\omega t$. This emf is obtained between the brushes B_1 and B_2 which are in contact with the slip rings A_1 and A_2.

Since the value of the sine function varies between +1 and −1, the sign (polarity) of the induced emf changes with time. From figure (1.18), the emf has its extreme value when $\theta = \omega t = \frac{\pi}{2}$ or $\frac{3\pi}{2}$ as the rate of change of flux is maximum at these points. Emf will be zero for $\theta = \omega t = 0, \pi, 2\pi$.

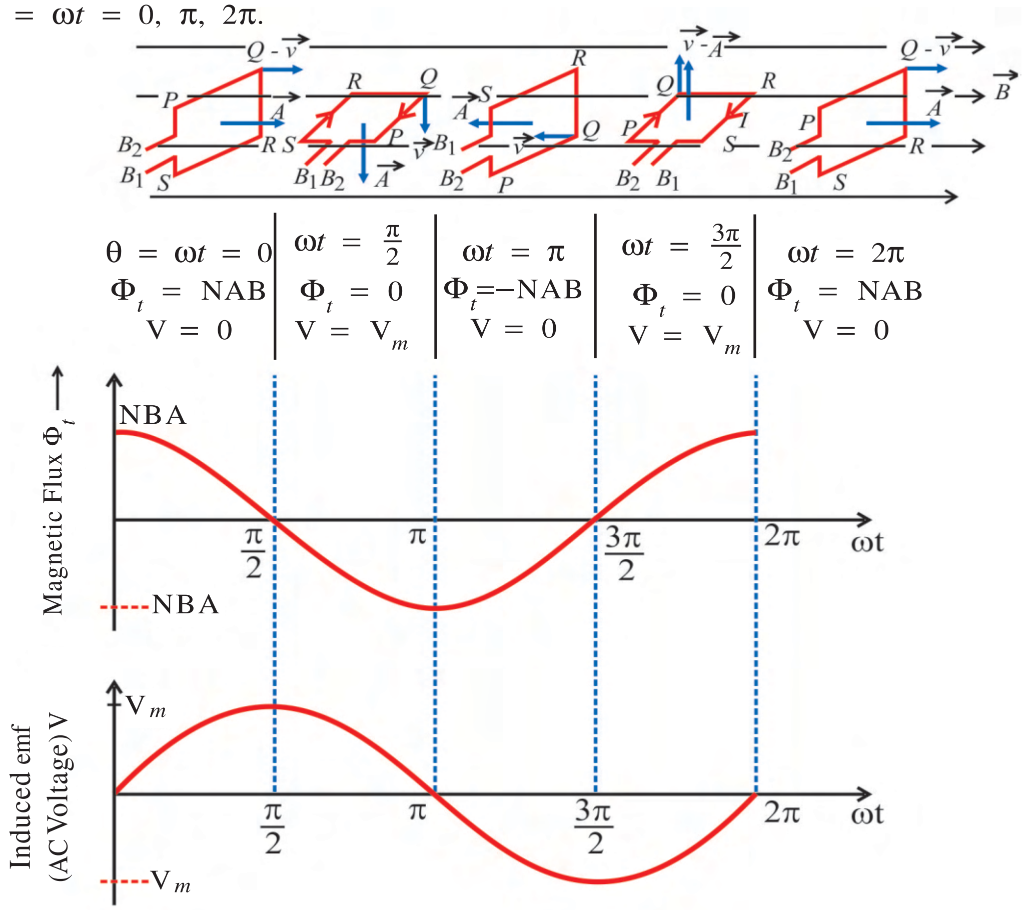


Figure 1.18 **Graph of Magnetic Flux (Φ_t) → ωt and AC Voltage (V) → ωt**

During the continuous rotation of the coil, the above mentioned situation is repeated. i.e. during a time interval of $\frac{\pi}{\omega} = \frac{T}{2}$, brushes B_1 and B_2 alternatively become positive and negative. Equation (1.10.2) gives the instantaneous value of the emf which varies between $+V_m$ and $-V_m$ periodically. Here, the voltage obtained between B_1 and B_2 is known as AC voltage (alternating voltage). B_1B_2 can be considered as AC voltage source.

Here, the direction of current changes periodically and therefore, such current is called alternating current (AC). Since $\omega = 2\pi f$, equation (1.10.2) can be written as,

$$V = V_m \sin 2\pi f t \qquad (1.10.3)$$

where, f = Frequency of revolution of the generator's coil.

In commercial generators the mechanical energy required for rotation of the armature is provided by water falling from a height (e.g. from dams). These are called "Hydro-electric Generators". Alternatively, water is heated to produce steam using coal or other sources. The steam at high pressure produces the rotation of the armature. These are called "Thermal Generators". Instead of coal, if a nuclear fuel is used, it is called "Nuclear Power Generator". Modern day generators produce electric power as high as 500 MW, i.e. one can light up 5 million 100 W bulbs ! In most generators, the coils are held stationary and it is the electromagnets which are rotated. In India the frequency of AC is 50 Hz and in certain countries like USA, it is 60 Hz.

Illustration 13 : The number of turns in the coil of an AC generator are 50 and its cross sectional area is 2.5 m^2. This coil is revolving in a uniform magnetic field of strength 0.3 T with a uniform angular velocity of 60 rad s^{-1}. The resistance of the circuit comprising the coil is 500 Ω.

(1) Find the maximum induced emf and maximum current produced in the generator.

(2) Calculate the flux passing through the coil, when current is zero.

(3) Calculate the flux passing through the coil, when the current is maximum.

Solution : N = 50, A = 2.5 m^2, ω = 60 $rads^{-1}$, B = 0.3 T, R = 500 Ω

(1) The induce emf generated in an AC generator,

$$V = NBA\omega\sin\omega t = V_m\sin\omega t$$

$$\therefore \text{ Maximum emf } V_m = NBA\omega = 50 \times 0.3 \times 2.5 \times 60$$

$$= 2250V = 2.25 \text{ kV}$$

$$\text{Maximum current } I_m = \frac{V_m}{R} = \frac{2250}{500} = 4.5 \text{ A}$$

(2) The impedence is purely resistive. Therefore, when current is zero, voltage is also zero.

$$V = \frac{d\Phi}{dt} = 0$$

$$\therefore \Phi = \text{Maximum}$$

$$\therefore \Phi_m = NBA = 50 \times 0.3 \times 2.5 = 37.5 \text{ Wb}$$

(3) At the time of maximum current, voltage V will also be maximum.

$$V = NBA\omega\sin\omega t = \text{Maximum}$$

$$\therefore \sin\omega t = 1$$

$$\therefore \omega t = \frac{\pi}{2}$$

$$\therefore \text{ Flux } \Phi = NBA\cos\omega t = NBA\cos\frac{\pi}{2} = 0$$

Thus, when current is maximum flux will be zero.

SUMMARY

1. **Magnetic Flux :** The number of magnetic field lines crossing a surface normally is called magnetic flux linked with the surface.

   Magnetic flux through a plane of surface area A placed in a uniform magnetic field $\vec{B}$ is,

   $$\phi = \vec{B}\cdot\vec{A} = BA\cos\theta$$

   where, θ = angle between $\vec{B}$ and $\vec{A}$.

2. **Electromagnetic Induction :** The phenomenon in which electric current (and emf) is induced in a conductor or a closed circuit by varying magnetic field is called electromagnetic induction.

3. Faraday's Law of Electromagnetic Induction : Whenever the magnetic flux linked with a closed circuit (or coil) changes, an emf is induced in it.

"The magnitude of the induced emf produced in a closed circuit (or a coil) is equal to the negative of the time rate of change of magnetic flux linked with it".

$$\varepsilon = -\frac{d\phi}{dt} \qquad \text{(For 1 turn)}$$

$$\varepsilon = -N\frac{d\phi}{dt} \qquad \text{(For N turns)}$$

4. Lenz's Law : Induced emf (or induced current) is produced in such a direction that the magnetic field produced due to it opposes the very cause (e.g. the motion of the magnet) that produces it.

Lenz's law gives the direction of induced emf.

5. Motional emf : "The induced emf produced in a coil due to the change in magnetic flux linked with a coil due to some kind of motion is called motional emf".

If a conducting rod of length l moves with velocity v in a magnetic field B perpendicular to both its length and the direction of magnetic field, then the emf induced across its two ends is given by,

$$\varepsilon = -Blv$$

Force required to move the rod with a constant velocity v is,

$$F = BIl = \frac{B^2l^2v}{R}$$

$$\text{Mechanical power } P = Fv = \frac{B^2l^2v^2}{R}$$

6. Relation between Induced Charge and Change in Magnetic Flux :

$$\text{Induced charge } \Delta Q = \frac{\Delta\Phi \text{ (Net Change in Magnetic Flux)}}{R \text{ (Resistance)}}$$

7. Methods of Generating Induced emf : The magnetic flux linked with a coil or a closed circuit can be changed and hence induced emf can be produced by following three methods.

(1) By changing the magnetic field $\vec{B}$.

(2) By changing the dimension (area A) of a coil.

(3) By changing the relative orientation (θ) of the coil in a magnetic field.

8. Eddy Currents : Whenever a solid conductor or a metallic plate is kept in a region of varying magnetic fields, the magnetic flux linked with it changes and circulatory currents are induced in it. These currents are called eddy currents. These currents are distributed throughout the conductor and their direction are determined by Lenz's law.

9. Electromagnetic Damping : When a pendulum made up of a metal plate is allowed to oscillate between two poles of a magnet, it performs damped oscillations due to eddy currents produced in it. Such a damping is called electromagnetic damping.

The effect of eddy currents can be minimized by making slots in the metal plate.

10. Self-Induction : When a current flowing through the coil is changed the magnetic flux linked with the coil itself changes. In such circumstances an emf is induced in the coil. Such emf is called self-induced emf and this phenomenon is called self-induction.

11. Self-Inductance : When a current I flows through a coil, flux linked with it,

$N\phi \propto I$

$N\phi = LI$

$L = \frac{N\phi}{I}$

The self-inductance L is a measure of the flux linked with the coil per unit current.

The self-inductance of a coil depends upon –

(1) The size and shape of the coil.

(2) The number of turns (N).

(3) The magnetic property of the medium within the coil in which the flux exists.

The self-induced emf in the coil is, $\varepsilon = -L\frac{dI}{dt}$

"The self-induced emf produced per unit rate of change of current $\left(\frac{dI}{dt} = 1\right)$ in the circuit is called the self-inductance of the circuit".

The SI unit of self-inductance L is henry H.

12. Henry : If the rate of change of current is $\left(\frac{dI}{dt} = 1As^{-1}\right)$ and the induced emf is $\varepsilon = 1V$, then the self-inductance of the circuit is 1H.

13. Self-Inductance of a Solenoid : $L = \frac{\mu_0 N^2 A}{l} = \mu_0 n^2 l A$

where, μ_0 = permeability of free space

l = length of solenoid

N = total number of turns in a solenoid

A = area of cross-section of a solenoid

$n = \frac{N}{l}$ = number of turns per unit length of solenoid.

When the solenoid is wound over a soft iron core of relative permeability μ_r, then the self-inductance of a solenoid is, $L = \mu_r \mu_0 n^2 l A$.

14. Mutual Inductance : In the system of two coils, if the current flowing through one coil is changed, an induced emf is produced in the neighbouring coil. This phenomenon is called mutual induction.

15. Mutual Inductance : In the system of two coils, when a steady current I_1 is passed through coil-1, the magnetic flux linked with coil-2.

$\Phi_2 \propto I_1$ $\quad\quad$ $\Phi_2 = M_{21} I_1$

Mutual inductance : (Definition-1) : "The magnetic flux linked with one of the coils of a system of two coils per unit currents flowing through the other coil is called mutual inductance of the system".

If the current I_1 in coil-1 is changed, there will be a corresponding change in the flux Φ_2 linked with coil-2. Therefore, an emf is induced in coil-2 according to Faraday's law given by,

$$\varepsilon_2 = -M_{21} \frac{dI_1}{dt}$$

Mutual Inductance : (Definition-2) : The emf generated in one of the two coils due to a unit rate of change of currents in the other coil is called the mutual inductance of the system of two coils".

The mutual inductance (M) of a system of two coils depends upon their shape and size, their number of turns, distance between them, their relative orientation and the magnetic property of the medium on which they are wound.

16. Mutual Inductance of a System of Two Co-axial Long Solenoids :

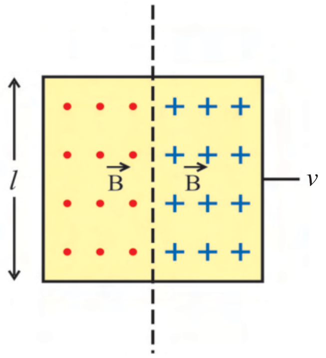
$$M = \frac{\mu_0 N_1 N_2 a}{l} = \mu_0 n_1 n_2 a l$$

where, n_1 and n_2 are the number of turns per unit length of two solenoids and a is the cross-sectional area of the inner solenoid.

EXERCISE

For the following statements choose the correct option from the given options :

1. A square conducting loop, whose plane is perpendicular to a uniform magnetic field, moves with velocity v normally to the magnetic field. If opposite sides of the loop, perpendicular to its velocity, remain in two mutually opposite uniform magnetic fields of strength B, the induced emf in this coil will be .... Length of each side is l.



(A) Bvl (B) $2Bvl$

(C) 0 (D) $\frac{Bvl}{2}$

2. The magnetic flux linked with a coil is changing with time t (second) according to $\phi = 6t^2 - 5t + 1$. Where ϕ is in Wb. At $t = 0.5$ s, the induced current in the coil is .... (the resistance of the circuit is 10 Ω)

(A) 1 A (B) 0.1 A (C) 0.1 mA (D) 10 A

3. A coil of surface area 100 cm^2 having 50 turns is held perpendicular to the magnetic field of intensity 0.02 Wbm^{-2}. The resistance of the coil is 2 Ω. If it is removed from the magnetic field in 1 s, the induced charge in the coil is ....

(A) 5 C (B) 0.5 C (C) 0.05 C (D) 0.005 C

4. When electric current in a coil steadily changes from +2 A to −2 A in 0.05 s, an induced emf of 8.0 V is generated in it. Then the self-inductance of the coil is .... H.

(A) 0.2 (B) 0.4 (C) 0.8 (D) 0.1

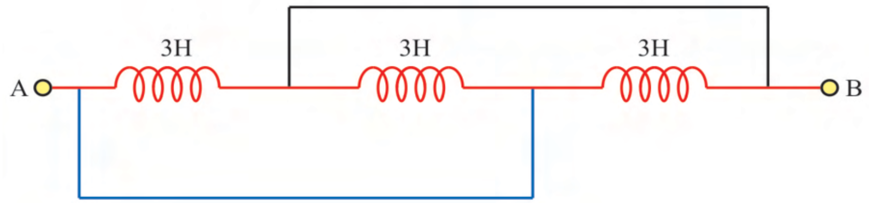
5. X and Y coils are joined in a circuit in such a way that when the change of current in X is 2 A , the change in the magnetic flux in Y is 0.4 Wb. The mutual induction of the system of two coils is .... H.

(A) 0.8 (B) 0.4 (C) 0.2 (D) 5

6. The mutual inductance of the system of two coils is 5 mH. The current in the first coil varies according to the equation $I = I_0 \sin\omega t$, where I_0 = 10A and $\omega = 100\pi$ rads^{-1}. The value of maximum induced emf in the second coil is ....

(A) 2π V (B) 5π V (C) π V (D) 4π V

7. Three pure inductances each of 3 H are connected as shown in figure. The equivalent inductance of this connection between points A and B is ....



(A) 1 H (B) 2 H (C) 3 H (D) 9 H

8. A square conducting coil of area 100 cm^2 is placed normally inside a uniform magnetic field of 10^3 Wbm^{-2}. The magnetic flux linked with the coil is .... Wb.

(A) 10 (B) 10^{-5} (C) 10^5 (D) 0

9. The distance between two extreme points of two wings of an aeroplane is 50 m. It is flying at a speed of 360 kmh^{-1} in horizontal direction. If the vertical component of earth's magnetic field at that place is 2×10^{-4} Wbm^{-2}, the induced emf between these two end points is .... V.

(A) 0.1 (B) 1.0 (C) 0.2 (D) 0.01

10. A wheel with 10 metallic spokes each 0.5 m long rotated with a speed of 120 rpm in a plane normal to the horizontal component of earth's magnetic field B_h at a place. If B_h = 0.4 G at the place, what is the induced emf between the axle and the rim of the wheel ? (1 G = 10^{-4} T)

(A) 0 V (B) 0.628 mV (C) 0.628 μV (D) 62.8 μV

11. The network shown in figure is a part of the circuit. (The battery has negligible resistance.)

At a certain instant the current I = 5 A and is decreasing at a rate of 10^3 As^{-1}. What is the potential difference between points B and A ?

(A) 5 V (B) 10 V (C) 15 V (D) 0V

12. A coil is placed in a time-varying magnetic field. Electrical power is dissipated in the form of Joule heat due to the current induced in the coil. If the number of turns were to be quadrupled and the wire radius halved, the electrical power dissipated would be ....

(A) halved (B) the same (C) doubled (D) quadrupled

13. The self-inductance of two solenoids A and B having equal length are same. If the number of turns in two solenoids A and B are 100 and 200 respectively, the ratio of the radii of their cross-section will be ....

(A) 2 : 1 (B) 1 : 2 (C) 1 : 4 (D) 4 : 1

14. A thin circular ring of area A is held perpendicular to a uniform field of induction B. A small cut is made in the ring and a galvanometer is connected across the ends such that the total resistance of the circuit is R. When the ring is suddenly squeezed to zero area, the charge flowing through the galvanometer is,

(A) $\frac{BR}{A}$ (B) $\frac{AB}{R}$ (C) ABR (D) $\frac{B^2A}{R}$

15. A magnet is moving towards a coil along its axis and the emf induced in the coil is ε. If the coil also starts moving towards the magnet with the same speed, the induced emf will be ....

(A) $\frac{\varepsilon}{2}$ (B) ε (C) 2ε (D) 4ε

16. A rod of 5 cm length is moving perpendicular to uniform magnetic field of intensity 2×10^{-4} Wbm^{-2}. If the acceleration of rod is 2 ms^{-2}, the rate of increase of the induced emf is ....

(A) 20×10^{-4} Vs^{-2} (B) 20×10^{-4} V (C) 20×10^{-4} Vs (D) 20×10^{-4} Vs^{-1}

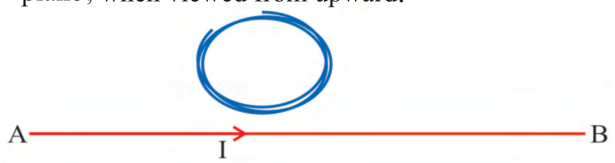
17. Current of 2A passing through a coil of 100 turns gives rise to a magnetic flux of 5×10^{-3} Wb per turn. The magnetic energy associated with coil is ....

(A) 5×10^{-3} J (B) 0.5×10^{-3} J (C) 5 J (D) 0.5 J

18. The flux linked per each turn of a coil of N turns changes from ϕ_1 to ϕ_2. If the total resistance of the circuit including the coil is R, the induced charge in the coil.

(A) $\frac{N(\phi_2 - \phi_1)}{t}$ (B) $\frac{N(\phi_2 - \phi_1)}{R}$ (C) $\frac{N(\phi_2 - \phi_1)}{Rt}$ (D) $N(\phi_2 - \phi_1)$

19. The current flows from A to B in a straight wire as shown in figure and it is decreasing with time. The induced current in the loop is placed near the wire in the same horizontal plane, when viewed from upward.



(A) In clockwise direction

(B) In anticlockwise direction

(C) Will not be produced

(D) Nothing can be said

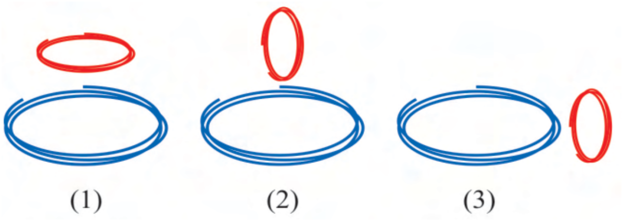
20. Two circular coils can be arranged in any of the three situations as shown in figure. Their mutual inductance (M) will be.

(A) Maximum in situation (1)

(B) Maximum in situation (2)

(C) Maximum in situation (3)

(D) The same in all situations.



21. In AC generator, induced emf is zero at time $t = 0$. The induced emf at time $\frac{\pi}{2\omega}$ is ....

(A) $+2 V_m$ (B) $+2 V_m$ (C) zero (D) $+2 V_m$

ANSWERS

1. (B)	**2.** (B)	**3.** (D)	**4.** (D)	**5.** (C)	**6.** (B)
7. (A)	**8.** (A)	**9.** (B)	**10.** (D)	**11.** (C)	**12.** (B)
13. (A)	**14.** (B)	**15.** (C)	**16.** (D)	**17.** (D)	**18.** (B)
19. (B)	**20.** (A)	**21.** (A)			

Answer the following questions in brief :

1. Give the statement of Lenz's law to determine the direction of induced emf.

2. State Faraday's law of electromagnetic induction.

3. What is the physical significance of negative sign appearing in the mathematical form of Faraday's law ?

4. Define motional emf.

5. What is Lenz's force ?

6. Will induced emf be produced in a wire kept in north-south direction allowed to fall freely ? Why ?

7. What are eddy currents ?

8. What is electromagnetic damping ?

9. How can the effects of eddy currents be reduced ?

10. Define self-inductance.

11. What will be the change in self-inductance if the current flowing in the coil is increased ?

12. Why self-inductance of a coil increases when the coil is wound on a soft iron core ?

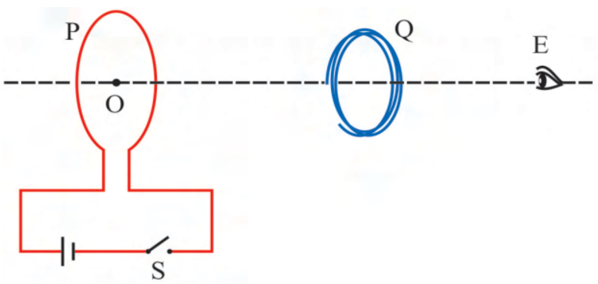
13. Why a metal plate falls downward with acceleration less than g in the presence of magnetic field ?

14. On what factors does the mutual inductance of a system of two coils depend ?

15. What is reciprocity theorem in the context of mutual inductance ?

16. A coil having N turns and resistance R Ω is connected to a galvanometer of resistance 4R Ω. This combination is moved in time t seconds from a magnetic flux Φ_1 Wb to Φ_1 Wb. What is the induced current in the circuit?

17. As shown in the figure, P and Q are two coaxial conducting loops separated by some distance. When the Switch S is closed, a clockwise current I_P flows in loop P (as seen by E) and an induced current I_Q flows in loop Q. In which direction this induced current I_Q will flow as seen by E ?



18. How can the magnetic flux associated with a coil or a closed circuit be changed ?

19. In an AC generator, the brushes in contact with the slip rings alternatively become positive and negative in the time interval of 5 ms. What is the frequency of the voltage generated ?

Answer the following questions :

1. Describe Faraday's historical experiment to induce current by winding insulated conducting coils on the ring of soft iron.

2. Discuss the results of Faraday's experiment performed with bar magnet and insulated conducting coil.

3. "Lenz's law is a special statement of law of conservation of energy." – Explain.

4. Obtain an equation for motional emf produced in a conducting rod which is moving on the two arms of U-shaped conductor perpendicular to magnetic field.

5. Using necessary figure (circuit), obtain an expression for self-induced emf produced in a coil.

6. Deduce an equation $U = \frac{1}{2}LI^2$ for an inductor.

7. Give two definitions of mutual inductance and write its unit.

8. Explain eddy currents.

9. Discuss the reason behind the production of induced emf in case of a conducting rod moving in the magnetic field with its velocity perpendicular to the magnetic field.

10. Explain conversion of mechanical energy into the electrical energy in case of conducting rod sliding over a U-shaped wire which is placed inside a magnetic field.

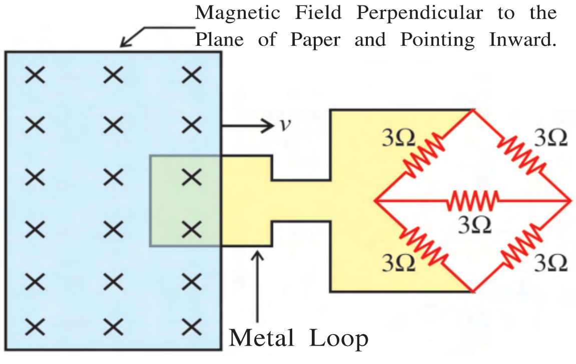
11. Give the applications of eddy currents.

12. With the help of neat diagram derive the expression for induced emf in an AC generator.

13. Give the characteristics of an induced emf in AC generator.

Solve the following examples :

1. A square metal wire loop of side 10 cm and resistance 1 Ω is moved with a constant velocity v in a uniform magnetic field of induction B = 2 Wbm^{-2} as shown in figure. The magnetic field is perpendicular to the plane of the loop and directed into the paper. The loop is connected to a network of resistors each of value 3 Ω. With what speed should the loop be moved so that a steady current of 1 mA flows in the loop.



[**Ans. :** 2×10^{-2} ms^{-1}]

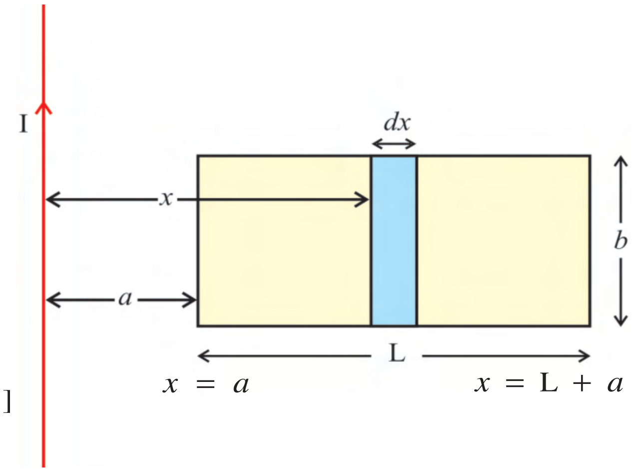
2. A coil having 200 turns has a surface area 0.12 m^2. A magnetic field of strength 0.10 Wbm^{-2} linked perpendicular to this area changes to 0.5 Wbm^{-2} in 0.2 s. Find the average emf induced in the coil. **[Ans. : 48 V]**

3. A coil of cross-sectional area A and having N turns lies in a uniform magnetic field B with its plane perpendicular to the field. In this position the normal to the coil makes an angle of 0^o with the field. The coil rotates at a uniform rate to complete one rotation in time T. Find the average induced emf in the coil during the interval when the coil rotates :

   (i) From 0^o to 90^o position, (ii) From 90^o to 180^o position, (iii) From 180^o to 270^o position and (iv) From 270^o to 360^o position.

   [Ans. : (i) $\frac{4NBA}{T}$ (ii) $\frac{4NBA}{T}$ (iii) $\frac{-4NBA}{T}$ (iv) $\frac{-4NBA}{T}$]

4. As shown in figure, a rectangular loop of length L and breadth b is placed near a very long wire carrying current I. The side of the loop nearer to the wire is at a distance a from the wire. Find magnetic flux linked with the loop.

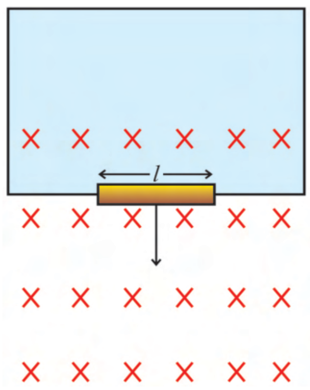


   [Hint : $\int \frac{1}{x} dx = ln\ x$]

   [Ans. : $\phi = \frac{\mu_0 I b}{2\pi} ln\left(\frac{L + a}{a}\right)$]

5. A conducting bar of 2 m length is allowed to fall freely from a 50 m high tower, keeping it aligned along the east-west direction. Find the emf. induced in the rod when it is 20 m below the top of the tower g = 10 ms^{-2}. Earth's magnetic field is 0.7×10^{-4} T and angle of dip = 60^o. **[Ans. : 1.4 mV]**

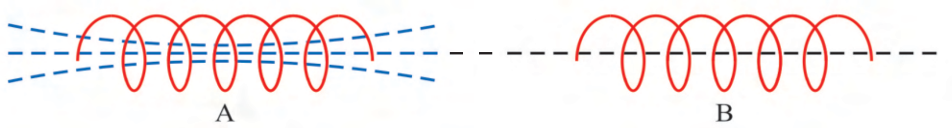
6. A conducting rod of length l, mass m and resistance R is falling freely through a uniform magnetic field $\vec{B}$ which is perpendicular to the plane of paper as shown in figure. Find terminal velocity (v_t) of this rod.



   [Ans. : $\frac{mgR}{B^2 l^2}$]

7. Find the equivalent inductance of two inductors having inductances L_1 and L_2 connected in parallel with the help of appropriate DC circuit. [**Ans. :** $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$]

8. Two coils placed near to each other have number of turns equal to 600 and 300 respectively. On passing a current of 3.0 A through coil A, the flux linked with each turn of coil A is 1.2×10^{-4} Wb and the total flux linked with coil B is 9.0×10^{-5} Wb. Find (1) self-inductance of A, (2) The mutual inductance of the system formed by A and B.



[**Ans. :** $L_A = 24$ mH; $M_B = 30\ \mu H$]

9. There are 1.5×10^4 turns in the winding of a toroidal ring. The radius of circular axis of the ring is 10 cm. The radius of cross-section of ring is 2.0 cm. Find inductance of the ring. [**Ans. :** 0.57 H]

10. A conducting loop of radius r is placed concentric with another loop of a much larger radius R so that both the loops are coplanar. Find the mutual inductance of the system of the two loops. Take R >> r. [**Ans. :** $\frac{\mu_0 \pi r^2}{2R}$]

•

2 ALTERNATING CURRENT

2.1 Introduction

Earlier, we have discussed about D.C. voltage and D.C. current. We also studied in Chapter-1 about the A.C. dynamo or the generator which is the device used for the production of A.C. voltage. In this chapter we will discuss about alternating current (A.C.). We use A.C voltage in our home, office or industries.

In this chapter we will analyse some simple A.C. circuits and then will study about an electrical device. A.C. voltage and current are taken as varying according to the sin ωt or cos ωt function. We should remember that they are not varying according to only sine or cosine functions. In future you will learn that they can change periodically with time in many other ways.

2.2 A.C. Circuit with Series Combination of Inductor, Capacitor and Resistor (L-C-R A.C. series circuit)

As shown in the figure 2.1, an inductor (L) having zero ohmic resistance, a capacitor with capacitance (C) and a resistor (R) with zero inductance are joined in series with the source of A.C. voltage.

Here the voltage from the source varies with time according to $V = V_m \cos\omega t$. (2.1.1)

At some time t, let the current passing in the circuit be = I

Charge deposited on the capacitor = Q

The rate of change of current = $\frac{dI}{dt}$

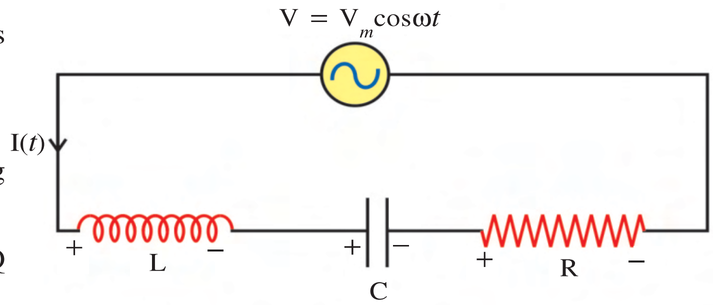


Figure 2.1 L-C-R A.C. Series Circuit

As a result, the potential difference between two ends of inductor is $V_L = L\frac{dI}{dt}$.

Potential difference between two ends of capacitor is $V_C = \frac{Q}{C}$.

The potential difference between two ends of resistor is $V_R = IR$.

According to Kirchoff's second law,

$V_L + V_C + V_R = V$.

$$L\frac{dI}{dt} + \frac{Q}{C} + IR = V_m \cos\omega t \qquad (2.2.2)$$

But, $I = \frac{dQ}{dt}$ and $\frac{dI}{dt} = \frac{d^2Q}{dt^2}$

$$\therefore L\frac{d^2Q}{dt^2} + \frac{Q}{C} + \frac{dQ}{dt}R = V_m \cos\omega t$$

$$\therefore \frac{d^2Q}{dt^2} + \frac{R}{L}\frac{dQ}{dt} + \frac{Q}{LC} = \frac{V_m}{L}\cos\omega t \qquad (2.2.3)$$

This is the differential equation for the charge Q, for this A.C. circuit. It resembles with the equatio,

$$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F_0}{m}\sin\omega t \qquad (2.2.4)$$

For the forced oscillations in mechanics which you studied in standard 11. This equation contains mechanical quantities while in the differential equation of LCR series A.C. circuit there are electrical quantities. Equations (2.2.3) and (2.2.4) are differential equations of similar type. As these equations have cosine and sine functions they are harmonic functions.

By comparing the above equations the equivalence between the mechanical quantities and electrical quantities can be seen in the following Table 2.1 :

Table 2.1 Equivalence between the Mechanical and Electrical Quantities

Number	Mechanical Quantity	Electrical Quantity
1	Displacement (y)	Electric charge (Q)
2	Velocity $\left(\frac{dy}{dt} = v\right)$	Electric current $\left(\frac{dQ}{dt} = I\right)$
3	Resistive coefficient (b)	Resistance R
4	Mass (m)	Inductance (L)
5	Force constant (k)	Inverse of capacitance $\left(\frac{1}{C}\right)$
6	Angular frequency $\left(\sqrt{\frac{k}{m}}\right)$	Angular Frequency $\left(\sqrt{\frac{1}{LC}}\right)$
7	Periodic Force	Periodic Voltage

Equation (2.2.3) is the differential equation for electric charge Q, in the A.C. circuit. The time-dependent function of Q, which satisfies the equation (2.2.3) is called the solution of equation (2.2.3). To obtain such solution complex functions are used. (Complex number and complex function are explained in the Appendix-A at the end of the chapter. It is only for information.)

2.3 Solution of the Differential Equation of Q for L-C-R Series A.C. circuit

Equation (2.2.2) can be written as $\frac{dI}{dt} + \frac{R}{L}I + \frac{1}{LC}\int I dt = \frac{V_m}{L}\cos\omega t$. (2.3.1)

Here we have taken $Q = \int I dt$

The solution of the above equation can be obtained by using complex number. Since $\cos\omega t$ is the real part of complex number $e^{j\omega t}$ the real part of solution which we shall obtain, will become the solution of equation (2.3.1). Moreover electric current I will have to be taken as a complex number. Expressing current I by complex current i,

$$\frac{di}{dt} + \frac{R}{L}i + \frac{1}{LC}\int i dt = \frac{V_m}{L}e^{j\omega t} \quad (2.3.2)$$

We should remember that R, L and C are only real numbers.

On the right hand side of equation (2.3.2) there is a harmonic function of time, hence the complex current I, would also be a harmonic function of time. Hence the solution of equation (2.3.2) can be written as,

$$i = i_m e^{j\omega t} \quad (2.3.3)$$

$$\therefore \frac{di}{dt} = i_m j \omega e^{j\omega t} \quad (2.3.4)$$

$$\text{and, } \int i\, dt = \frac{i_m e^{j\omega t}}{j\omega} \quad (2.3.5)$$

Using equations (2.3.3), (2.3.4) and (2.3.5) in equation (2.3.2), we get

$$i_m j \omega e^{j\omega t} + \frac{R}{L} i_m e^{j\omega t} + \frac{1}{LC}\frac{i_m e^{j\omega t}}{j\omega} = \frac{V_m}{L}e^{j\omega t}$$

$$\therefore i_m\left(j\omega + \frac{R}{L} + \frac{1}{j\omega LC}\right) = \frac{V_m}{L}$$

Multiplying both the sides by L and writing $\frac{1}{j} = \frac{j}{j^2} = -j$, we get

$$i_m\left(j\omega L + R - \frac{j}{\omega C}\right) = V_m$$

$$\therefore i_m = \frac{V_m}{R + j\omega L - \frac{j}{\omega C}} \quad (2.3.6)$$

Substituting this value of i_m in equation (2.3.3), we get,

$$i = \frac{V_m e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)} \quad (2.3.7)$$

This equation shows the relation between complex current i and complex voltage $V_m e^{j\omega t}$ and has the same form as the equation $I = \frac{V}{R}$ which represents Ohm's law. Thus Ohm's law is obeyed by the instantaneous voltage and current.

From this, it can be seen that whatever effect is produced by the resistance R on the current, similar effects produced by the inductor and the capacitor are obtained respectively by $j\omega L$ and $\frac{-j}{\omega C}$. That is, $j\omega L$ and $\frac{-j}{\omega C}$ can be called the effective resistances of the inductor and the capacitor. $j\omega L$ is called the inductive reactance and $\frac{-j}{\omega C}$ is called the capacitive reactance of the capacitor. Their symbols are Z_L and Z_C respectively. Their magnitudes are respectively ωL and $\frac{j}{\omega C}$ and their symbols are X_L and X_C. Thus,

$$Z_L = j\omega L \tag{2.3.8}$$

$$X_L = \omega L \tag{2.3.9}$$

$$Z_C = \frac{-j}{\omega C} \tag{2.3.10}$$

$$X_C = \frac{1}{\omega C} \tag{2.3.11}$$

The summation of Z_L, Z_C and R is called the impedance (Z) of the present series circuit. Its unit is Ohm.

$$\therefore\ Z = R + Z_L + Z_C \tag{2.3.12}$$

$$\therefore\ Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \tag{2.3.13}$$

Now equation (2.3.7) can be written as under :

$$i = \frac{V_m e^{j\omega t}}{Z} = \frac{\text{Voltage}}{\text{Effective resistance (Z)}} \tag{2.3.14}$$

This equation is Ohm's law with complex current, complex voltage and the impedance . Note that the impedance is also complex.

Now taking $Z = |Z|e^{j\delta}$, [See equation in Appendix-A]

$$i = \frac{V_m e^{j\omega t}}{|Z| e^{j\delta}} \tag{2.3.15}$$

$$= \frac{V_m}{|Z|} e^{j(\omega t - \delta)} = \frac{V_m}{|Z|}[\cos(\omega t - \delta) + j\sin(\omega t - \delta)] \tag{2.3.16}$$

$$\text{where, } |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{2.3.17}$$

$$\text{Now, } I = R_e(i) \tag{2.3.18}$$

$$\therefore\ I = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V_m \cos(\omega t - \delta)}{|Z|} \tag{2.3.19}$$

In this circuit the current varies with time according to the equation (2.3.19) while the voltage varies according to $V = V_m \cos\omega t$. This shows that the current in the circuit lags in phase behind the voltage by δ. This fact is shown in the figure 2.2.

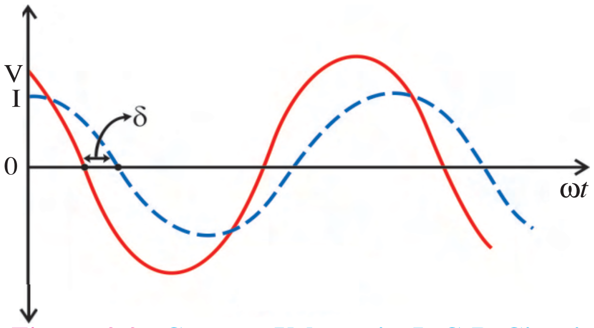


Figure 2.2 Current Voltage in L-C-R Circuit

The equation showing the complex impedance Z of the circuit is

$$Z = R + j\omega L - \frac{j}{\omega C} \tag{2.3.20}$$

The real part of this complex number is R, shown on the real axis in the figure 2.3. The imaginary part is shown on the imaginary axis.

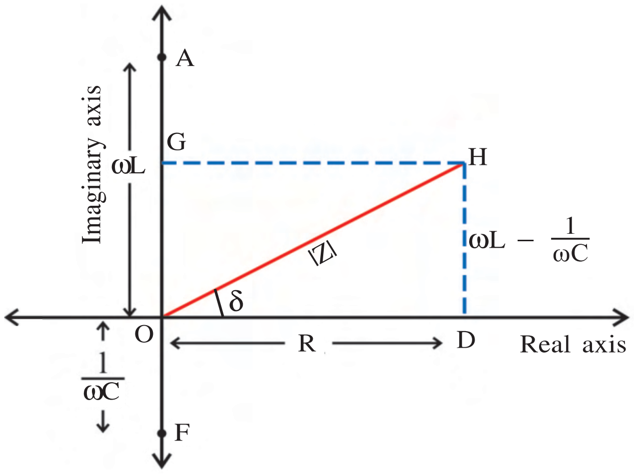


Figure 2.3 Geometrical Representation of Z

From this the point H showing the complex number Z is obtained. The impedance Z, given by equation (2.3.13) is represented in the complex plane in figure 2.3 as under,

In figure $OD = R$, $OA = \omega L$ and $OF = \frac{1}{\omega C}$

$\therefore OG = \omega L - \frac{1}{\omega C}$ = imaginary part of Z.

$OH = Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$

From figure (2.3),

$$|Z| = \sqrt{OD^2 + DH^2} \tag{2.3.21}$$

$$= \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} \tag{2.3.22}$$

The phase difference δ is obtained from,

$$\therefore \tan\delta = \frac{HD}{OD} = \frac{\left(\omega L - \frac{1}{\omega C}\right)}{R} \tag{2.3.23}$$

Thus showing the impedance in the complex plane, values of δ and $|Z|$ can be easily found out geometrically. Moreover, since the values of ω, L, C and R are known using equations (2.3.22) and (2.3.23) respectively, values of $|Z|$ and δ can be obtained. Hence, the equation showing the relation between the current and the voltage can be written.

To find the impedance of the given circuit, we can use the same laws for $j\omega L$ and $-\frac{j}{\omega C}$ as those used for series and parallel combination of resistances R.

For different circuits the relations between the current and the voltage can be obtained with the help of above geometrical arrangements.

2.4 Different cases of A.C. Circuits

(1) A.C. Circuit with only Resistance : In LCR circuit when inductor (L) and capacitor (C) are absent, it will be a circuit with only resistance. In the equation

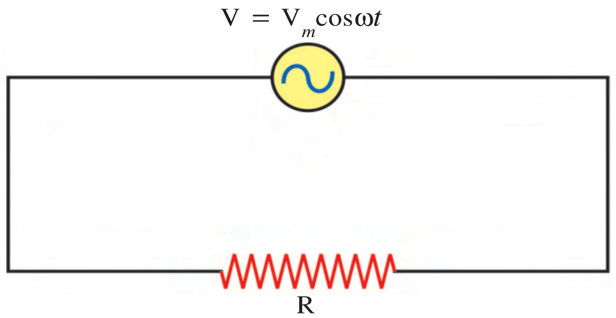


Figure 2.4 A.C. Circuit with only R

$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$ for LCR circuit, by substituting $\omega L = 0$ and $\frac{1}{\omega C} = 0$, we get $|Z| = R$ for this circuit and the value of δ from equation (2.3.23) would be zero. Thus, the equation (2.3.19) showing the relation between the current and voltage would be of the form.

$$I = \frac{V_m \cos\omega t}{R} \quad (2.4.1)$$

Thus, we can see that in the A.C. circuit with only resistance the phases of current and voltage are equal.

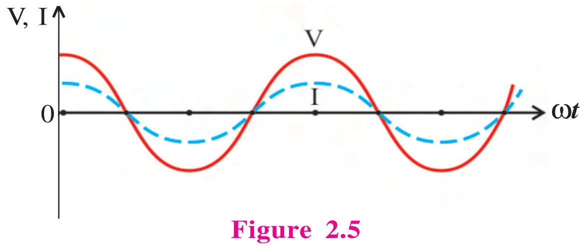


Figure 2.5

(2) A.C. Circuit with only Inductor : As seen earlier, a circuit with only inductor means in LCR circuit capacitor C and resistance R are absent. For this circuit $Z = j\omega L$ and $|Z| = \omega L = X_L$ (because $\frac{1}{\omega C} = 0$ and $R = 0$).

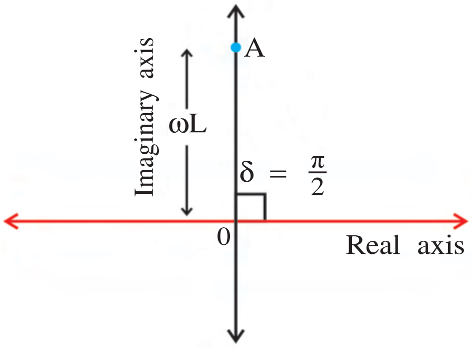


Figure 2.6

Z is shown by point A in the complex plane in figure 2.6.

Here OA makes an angle of $\frac{\pi}{2}$ with the real axis, which shows that $\delta = \frac{\pi}{2}$ and $OA = \omega L = |Z|$. Substituting values of $|Z|$ and δ in equation (2.3.19).

$$I = \frac{V_m \cos\left(\omega t - \frac{\pi}{2}\right)}{\omega L} = \frac{V_m \cos\left(\omega t - \frac{\pi}{2}\right)}{X_L} \quad (2.4.2)$$

This shows that current is lagging behind the voltage in phase by $\frac{\pi}{2}$.

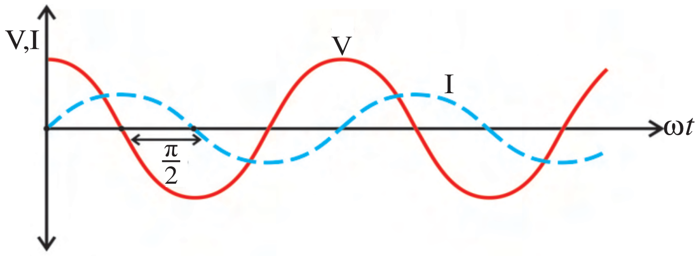


Figure 2.7

(3) A.C. Circuit with only Capacitor : In this case only capacitor is present, hence $Z = -\frac{j}{\omega C}$ and $|Z| = \frac{1}{\omega C} = X_C$ is shown by point F in the complex plane in figure 2.8. It is clear from the figure 2.8. that $\delta = -\frac{\pi}{2}$. Thus, the relation between the current and the voltage would be

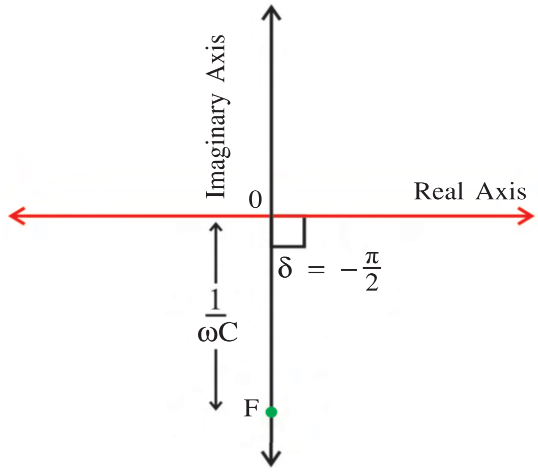


Figure 2.8

$$I = \frac{V_m \cos\left(\omega t + \frac{\pi}{2}\right)}{\left(\frac{1}{\omega C}\right)} = \frac{V_m \cos\left(\omega t + \frac{\pi}{2}\right)}{\frac{1}{\omega C}} \qquad (2.4.3)$$

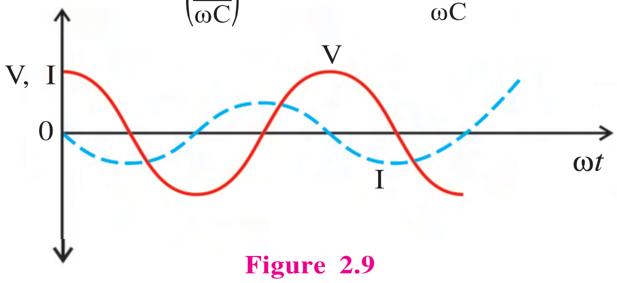


Figure 2.9

Thus, in A.C. circuit with capacitor, only current leads the voltage in phase by $\frac{\pi}{2}$. This fact is shown in the figure 2.9.

(4) A.C. Circuit with R and L Joined in Series : For this circuit, $Z = R + jX_L = R + j\omega L$ and $|Z| = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + (\omega L)^2}$. In the figure 2.10, Z is shown by point H in the complex plane. From the figure, it is clear that

$$\tan\delta = \frac{\omega L}{R}$$

$$\therefore \delta = \tan^{-1}\left(\frac{\omega L}{R}\right) = \tan^{-1}\left(\frac{X_L}{R}\right) \qquad (2.4.4)$$

In this circuit current lags behind the voltage in phase by δ.

Here, the electric current $I = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + (\omega L)^2}} \qquad (2.4.5)$

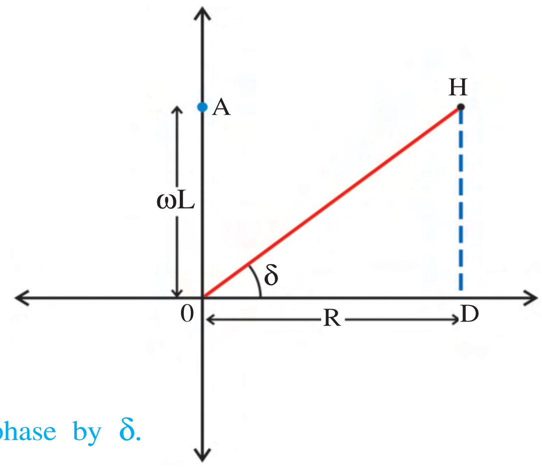


Figure 2.10

(5) A.C. Circuit with R and C Joined Series : For this circuit $Z = R - \frac{j}{\omega C} = R - jX_C$

$$\therefore |Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + X_C^2}$$

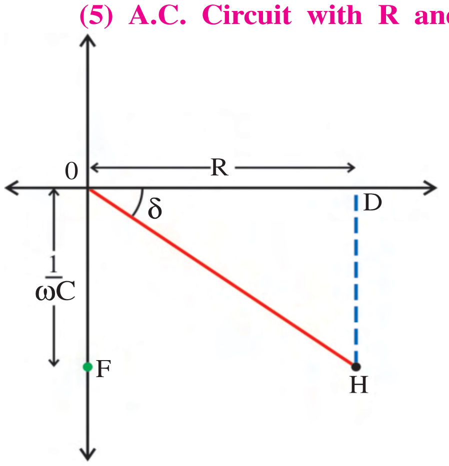


Figure 2.11

This Z is shown in figure 2.11 by point H. Here, as shown in the figure, δ is negative and has magnitude given by

$$\delta = \tan^{-1}\left(\frac{1}{\omega CR}\right) = \tan^{-1}\left(\frac{X_C}{R}\right) \qquad (2.4.6)$$

In this case the electric current leads the voltage in phase by δ. Here electric current

$$I = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + (X_C)^2}} \qquad (2.4.7)$$

(6) A.C. Circuit with L and C in Series : For this circuit $Z = j\omega L - \frac{j}{\omega C} = jX_L - jX_C$

$$\therefore |Z| = \omega L - \frac{1}{\omega C} = X_L - X_C$$

Assuming $\omega L > \frac{1}{\omega C}$, Z obtained here is shown in the complex plane by point G. Here, $\delta = \frac{\pi}{2}$. Thus, in L–C series circuit if $\omega L > \frac{1}{\omega C}$, the current lags behind the voltage in phase by $\frac{\pi}{2}$. (If $\omega L < \frac{1}{\omega C}$ then ? Think yourself)

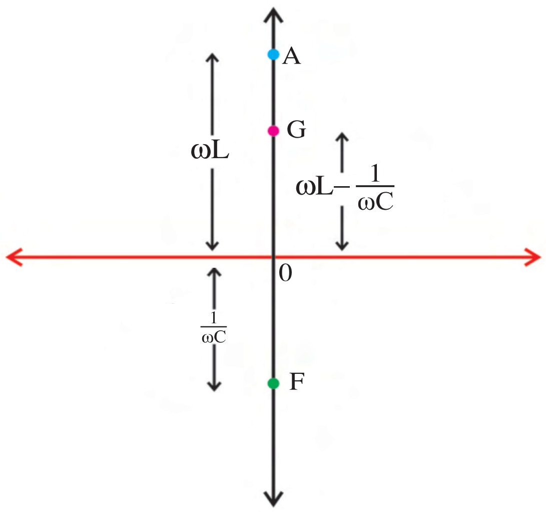


Figure 2.12

(7) A.C. Circuit with Parallel Combination of L and C, and R in Series with this Combination : A circuit with L and C joined in parallel and R in series with this combination is as shown in the figure 2.13.

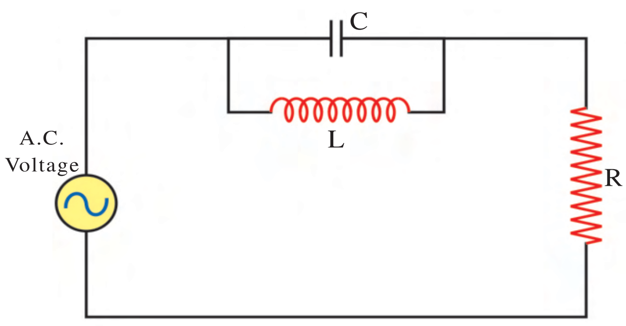


Figure 2.13

The effective impedance Z of this circuit can be obtained as follows using the laws of series-parallel combinations. If the impedance of the parallel combination of L and C is Z_1,

$$\frac{1}{Z_1} = \frac{1}{Z_C} + \frac{1}{Z_L} = \frac{1}{\frac{-j}{(\omega C)}} + \frac{1}{j\omega L} = j\left(\omega C - \frac{1}{\omega L}\right) \qquad (2.4.8)$$

$$\therefore Z = \frac{1}{j\left(\omega C - \frac{1}{\omega L}\right)} = -\frac{j}{\left(\omega C - \frac{1}{\omega L}\right)} \qquad (2.4.9)$$

Moreover, R and Z_1 are in series

$$\therefore Z = R + Z_1$$

$$\therefore Z = R - \frac{j}{\left(\omega C - \frac{1}{\omega L}\right)} \qquad (2.4.10)$$

Assuming $\omega L > \frac{1}{\omega C}$, Z can be represented as shown in the figure 2.14.

From equation (2.4.10),

$$|Z| = \sqrt{R^2 + \frac{1}{\left(\omega C - \frac{1}{\omega L}\right)^2}} \qquad (2.4.11)$$

From equations (2.3.19) and (2.4.12), we get electric current,

$$I = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + \frac{1}{\left(\omega C - \frac{1}{\omega L}\right)^2}}} \qquad (2.4.12)$$

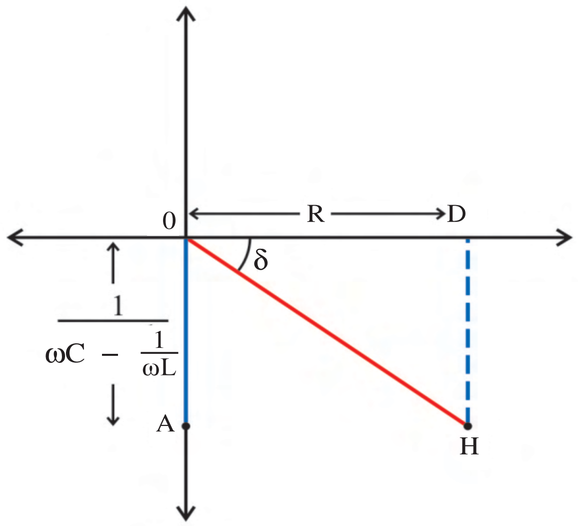


Figure 2.14

This equation shows the relation between the current and the voltage in the present circuit.

$$\text{Here, } \tan\delta = \frac{HD}{OD} = \frac{1}{R\left(\omega C - \frac{1}{\omega L}\right)} \qquad (2.4.13)$$

2.5 r.m.s. Values of Voltage and Current

Till now we have seen equations like, $V = V_m \cos\omega t$ and $I = I_m \cos(\omega t \pm \delta)$ for voltage and current respectively. Here, V and I continuously vary with time periodically. In this condition by joining a simple voltmeter or ammeter properly in the circuit, it is not possible to measure voltage or current. If we try to find the average values of A.C. voltage or A.C. current, we get zero, because sine or cosine function appears in their formulae. You know that the average value of sine or cosine function over an interval of one period is zero. That is,

$$< V > = V_m \left[\frac{1}{T}\int_0^T \cos\omega t \, dt\right] = 0$$

In practice, specially designed A.C. voltmeter and A.C. ammeter are used to measure A.C. voltage and A.C. current. These meters give r.m.s. (root mean square) value of A.C. voltage and A.C. current.

Root means square (r.m.s) of a quantity means the square root of the mean (average) of the squares of that quantity. In the present case the average of the square is taken over an interval of one periodic time*. To obtain r.m.s. value of $V = V_m \cos\omega t$, we should get the average of V^2 over one periodic time and then find the square root of it.

$$\text{Average } V^2 = < V^2 > = < V_m^2 \cos^2\omega t > \qquad (2.5.1)$$

$$= V_m^2 \left\langle \frac{1+\cos 2\omega t}{2} \right\rangle = V_m^2 \left\langle \frac{1}{2} + \frac{\cos 2\omega t}{2} \right\rangle$$

Foot note : *If $f(t)$ is a function of time (t), the average value of this function over an interval of time T, is given by $\langle f(t) \rangle = \frac{1}{T}\int_0^T f(t)\, dt$.

$$= V_m^2 \left\langle \frac{1}{2} \right\rangle + \frac{V_m^2}{2}\left(\frac{1}{T}\int_0^T \cos 2\omega t.dt\right)$$

But, $\left\langle \frac{1}{2} \right\rangle = \frac{1}{2}$ and $\frac{1}{T}\int_0^T \cos 2\omega t.dt = 0$

$$\therefore \; <V_m^2> = \frac{V_m^2}{2} \qquad (2.5.2)$$

$$\therefore \; V_{rms} = \sqrt{<V^2>} = \frac{V_m}{\sqrt{2}} \qquad (2.5.3)$$

Similarly, $I_{rms} = \dfrac{I_m}{\sqrt{2}}$ (2.5.4)

2.6 Series Resonance

In order to understand the phenomenon of resonance in L-C-R series circuit, consider equation (2.3.19).

$$I = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\therefore \; I = I_m \cos(\omega t - \delta)$$

where, $I_m = \dfrac{V_m}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

From equation (2.5.4)

$$I_{rms} = \frac{I_m}{\sqrt{2}} = \frac{\frac{V_m}{\sqrt{2}}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$$

$$\therefore \; I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \frac{V_{rms}}{|Z|} \qquad (2.6.1)$$

Equation (2.6.1) shows that if we go on changing the values of angular frequency ω of the voltage, then the values of I_{rms} will also go on changing and at one definite value $\omega = \omega_0$, we will get

$$\omega_0 L = \frac{1}{\omega_0 C} \qquad (2.6.2)$$

and in this condition $|Z|$ becomes minimum and I_{rms} becomes maximum.

$$I_{rms} = \frac{V_{rms}}{R} = I_{rms}(\text{max}) \qquad (2.6.3)$$

Thus, for a definite angular frequency (ω_0) of the voltage, value of r.m.s. current becomes maximum. This is called the series resonance in L-C-R A.C. series circuit.

From equation (2.6.2)

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (2.6.4)$$

From this $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

Here ω_0 is called the natural angular frequency or the resonant angular frequency and f_0 is called resonant frequency of L-C-R A.C. series circuit.

Here, note that resonance is produced when the reactive component of impedence $\left(\omega L - \frac{1}{\omega C}\right)$ becomes zero; that is the imaginary part of impedence becomes zero.

In the figure 2.15, graphs of I_{rms} against ω for L-C-R series circuit are shown for two values of R ($R_1 < R_2$), which are called resonance curves. From the figure, it is clear that resonance curve is sharper for smaller value of R.

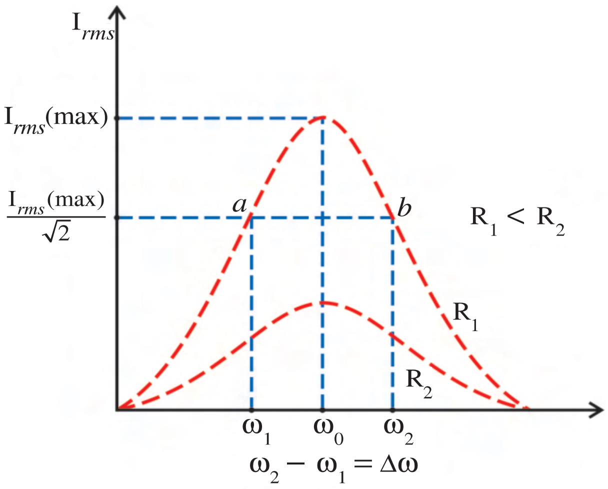


Figure 2.15 Resonance Curves L-C-R Series Circuit

Q-factor : The sharpness of the L-C-R resonance curve is measured by a quantity called the Q-factor.

In the circuit, the maximum power is proportional to the square of maximum value of rms current $[I_{rms}\ (max)]^2$. When I_{rms} becomes $\frac{I_{rms}(max)}{\sqrt{2}}$, the value of power becomes half of its maximum value. The value of $\frac{I_{rms}(max)}{\sqrt{2}}$ corresponding to this power is shown in the figure 2.15. From the figure, it is clear that for this value of current two angular frequencies ω_1 and ω_2 are found.

$(\omega_2 - \omega_1)$ is called half-power bandwidhth $(\Delta\omega)$.

From this discussion, it is clear that as the half-power band-width is smaller when the sharpness of resonance curve is more. To understand this fact, Q-factor is defined by the following formula :

$$Q = \frac{\omega_0}{\Delta\omega} = \frac{f_0}{\Delta f} \qquad (2.6.5)$$

Here, it is clear that as the Q-factor is larger the sharpness of the curve is more.

Moreover,

$$\Delta\omega = \frac{R}{L} \qquad (2.6.6)$$

(Derivation of this formula is given in Appendix-B at the end of this chapter, only for information). Substituting this value of $\Delta\omega$, in equation (2.6.5) we get,

$$Q = \frac{\omega_0 L}{R} \qquad (2.6.7)$$

But, $\omega_0 = \frac{1}{\sqrt{LC}}$

$$\therefore \; Q = \frac{1}{R}\sqrt{\frac{L}{C}} \qquad (2.6.8)$$

From this formula it can be seen that, Q-factor depends on the values of the all circuit-components.

From the value of the Q-factor, we can now infer how is the tuning of the circuit and also its selectivity.

Resonance circuit is used to select (or to tune) the desired frequency, out of many frequencies incident on the antenna of radio or TV. in order to change desired frequency, the arrangement is made to change either L or C or both. Here, note that resonance cannot be obtained in RL and RC circuit.

Illustration 1 : An A.C. Source of 230 V is connected in series with a 8.0 mH inductor, 80 μF capacitor and a 400 Ω resistor. Calculate (1) The resonant frequency (2) The impedance of the circuit and the value of the current at the resonant frequency (3) The *rms* value of the voltage across the components of the above circuit.

Solution :

(1) The resonant frequency $f = \frac{1}{2\pi\sqrt{LC}}$

$$\therefore f = \frac{1}{(2)\,(3.14)\sqrt{8\times10^{-3}\times 80\times10^{-6}}} = \frac{1}{6.28\times8\times10^{-4}} = 199 \text{ Hz}$$

(2) $$|Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + (X_L - X_C)^2}$$

$$X_L = \omega L = 2\pi f L = (2)\ (3.14)\ (199)\ (8 \times 10^{-3}) = 10\ \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C} = \frac{1}{(2)(3.14)(199)(80\times10^{-6})} = 10\ \Omega$$

At resonance $X_L = X_C$

$\therefore |Z| = R = 400\ \Omega$

At resonance current in the circuit $I = \frac{V}{R} = \frac{230}{400} = 0.575$ A

(3) Potential difference between two ends of the inductor

$V_L = I_{rms} X_L = (0.575)\ (10) = 5.75$ volt

Similarly potential difference across the capacitor.

$V_C = I_{rms}X_C = (0.575)\ (10) = 5.75$ volt

and potential difference across the resitance

$V_R = I_{rms}R = (0.575)\ (400) = 230$ volt

Illustration 2 : For which value of ω will the impedance of figure 2.13 be maximum ? What will be the value of I_{rms} in the above case ? What will be the maximum value of impedance ?

Solution : As per the equation (2.4.12)

$$|Z| = \left[R^2 + \frac{1}{\left(\omega C - \frac{1}{\omega L}\right)^2}\right]^{\frac{1}{2}}$$

When the term $\left(\omega C - \frac{1}{\omega L}\right)^2$ becomes minimum, the |Z| term becomes maximum.

$$\therefore \ \omega C - \frac{1}{\omega L} = 0$$

$$\therefore \ \omega = \frac{1}{\sqrt{LC}}$$

$\therefore$ |Z| = infinite

$$\therefore \ I_{rms} = \frac{V}{|Z|} = 0$$

Illustration 3 : The A.C. voltage and the current in an L-C-R A.C. series circuit are given by the following expression. $V = 200\sqrt{2}\cos(3000t - 55°)$ V, $I = 10\sqrt{2}\cos(3000t - 10°)$ A. Calculate the impedance and the resistance of the above circuit.

Solution : Phase difference between current in the circuit and voltage is 45°.

$$\therefore \ \tan\delta = \tan 45° = 1$$

Now for L-C-R series circuit $\tan\delta = \dfrac{\omega L - \frac{1}{\omega C}}{R}$

$$\therefore \ \frac{\omega L - \frac{1}{\omega C}}{R} = 1$$

$$\therefore \ R = \omega L - \frac{1}{\omega C}$$

$$\therefore \ \text{Impedance } |Z| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{R^2 + R^2} = R\sqrt{2}$$

$$\therefore \ |Z| = \frac{V_m}{I_m} = \frac{200\sqrt{2}}{10\sqrt{2}} = 20\ \Omega$$

$$\therefore \ R\sqrt{2} = 20$$

$$\therefore \ R = 14.14\ \Omega$$

Illustration 4 : An electric current has both A.C. and D.C. components. The value of the D.C. component is equal to 12 A while the A.C. component is given as I = 9 $\sin\omega t$ A. Determine the formula for the resultant current and also calculate the value of I_{rms}.

Solution : Resultant current (at any instant of time) will be I = 12 + 9 $\sin\omega t$ (1)

Now, $I_{rms} = \sqrt{\langle I^2 \rangle} = \sqrt{\langle 12 + 9\sin\omega t \rangle^2} = \sqrt{\langle 144 + 216\sin\omega t + 81\sin^2\omega t \rangle}$

Here, the average is taken over a time interval equal to the periodic time.

$\therefore I_{rms} = \sqrt{\langle 144 \rangle + 216\langle \sin\omega t \rangle + 81\langle \sin^2\omega t \rangle}$

Now $\langle 144 \rangle = 144$, $216\langle \sin\omega t \rangle = 0$ and $81\langle \sin^2\omega t \rangle = 81 \times \frac{1}{2} = 40.5$

$\therefore I_{rms} = \sqrt{144 + 40.5} = 13.58$ A

Illustration 5 : Calculate the resultant inductance of two inductors L_1 and L_2 when they are connected in parallel in A.C. circuit.

Solution : Let Z_{L_1} and Z_{L_2} be the inductive reactance of the two coils. Since they are connected in parallel, the resultant reactance will be,

$$Z = \frac{Z_{L_1} Z_{L_2}}{Z_{L_1} + Z_{L_2}} = \frac{(j\omega L_1) \times (j\omega L_2)}{j\omega L_1 + j\omega L_2} \quad (1)$$

If the resultant inductance is equal to L, then Z = $j\omega L$.

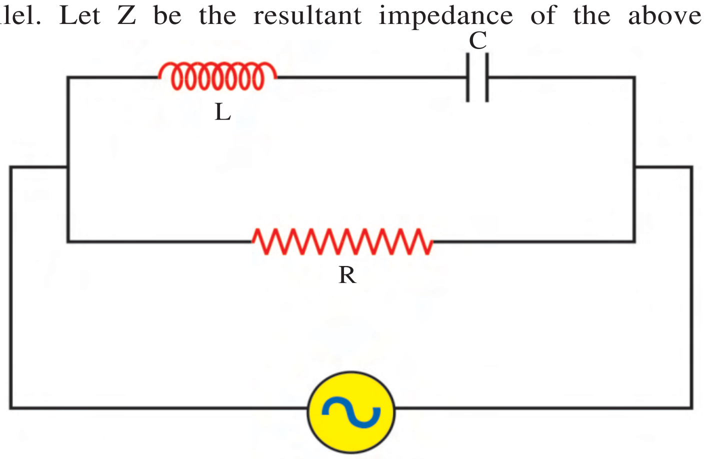
$$j\omega L = \frac{j\omega^2 L_1 L_2}{\omega L_1 + \omega L_2}$$

$$\therefore L = \frac{L_1 L_2}{L_1 + L_2}$$

Illustration 6 : Calculate the impedance Z of the given circuit.

Solution : Let Z_1 be the effective impedance of L and C in series, then $Z_1 = Z_L + Z_C$.

Now Z_1 and R are connected in parallel. Let Z be the resultant impedance of the above parallel connection, then



$$Z = \frac{Z_1 R}{Z_1 + R} = \frac{(Z_L + Z_C)R}{Z_L + Z_C + R}$$

$$= \frac{j\left(\omega L - \frac{1}{\omega C}\right)R}{j\left(\omega L - \frac{1}{\omega C}\right) + R}$$

$$= \frac{j(X_L - X_C)R}{j(X_L - X_C) + R}$$

Multiplying the complex numbers in numerator and the denominator with their respective complex conjugate,

$$\therefore \quad |Z| = \left\{\frac{-Rj(X_L - X_C) \times Rj(X_L - X_C)}{\{j(X_L - X_C) + R\}\{R - j(X_L - X_C)\}}\right\}^{\frac{1}{2}}$$

$$\therefore \quad |Z| = \left[\frac{R^2(X_L - X_C)^2}{R^2 + (X_L - X_C)^2}\right]^{\frac{1}{2}}$$

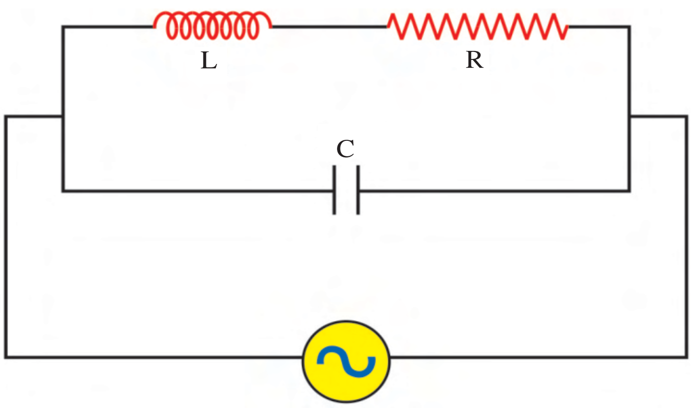
In the above expression when $X_L = X_C$, $|Z| = 0$ (Resonance can be obtained.)

Illustration 7 : Obtain the resonance angular frequency for the circuit shown in the figure.

Solution : Let Z_1 be the resultant impedance of the inductor L and resistor R.

$Z_1 = R + jX_L = R + j\omega L$

Let Z be the resultant impedance of the above circuit, hence



$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_C}$$

$$\frac{1}{Z} = \frac{1}{R + j\omega L} + j\omega C$$

$$\left[\because \quad \frac{1}{-\frac{1}{\omega C}j} = -\frac{\omega C}{j} = j\omega C\right]$$

(Multiplying and dividing the first term on the right hand side by $R - j\omega L$.)

$$\frac{1}{Z} = \frac{R - j\omega L}{R^2 + \omega^2 L^2} + j\omega C$$

$$= \frac{R + j(\omega C R^2 + \omega^3 L^2 C - \omega L)}{R^2 + \omega^2 L^2}$$

$$\therefore \quad Z = \frac{R^2 + \omega^2 L^2}{R + j(\omega C R^2 + \omega^3 L^2 C - \omega L)}$$

The imaginary part (coefficient of j) in the denominator of the above equation should be zero, for Z to be maximum.

$\therefore \quad \omega C R^2 + \omega^3 L^2 C - \omega L = 0$

$\therefore \quad \omega^2 L^2 C = L - CR^2$

$$\therefore \quad \omega^2 = \frac{1}{LC} - \frac{R^2}{L^2}$$

$$\therefore \quad \omega = \sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$$

Illustration 8 : The series combination of R(Ω) and capacitor C(F) is connected to an A.C. source of V volts and angular frequency ω. If the angular frequency is reduced to $\frac{\omega}{3}$, the current is found to be reduced to one-half without changing the value of the voltage. Determine the ratio of the capacitive reactance and the resistance.

Solution : First case : (We shall indicate the r.m.s. value of I and V as I and V for the sake of convenience.)

$$I = \frac{V}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \quad \therefore \quad I^2 = \frac{V^2}{R^2 + Xc^2} \qquad (1)$$

Second case :

$$\frac{I}{2} = \frac{V}{\sqrt{R^2 + \frac{9}{\omega^2 C^2}}} \quad \therefore \quad \frac{I^2}{4} = \frac{V^2}{R^2 + 9Xc^2} \qquad (2)$$

Dividing equation (1) by (2), we have,

$$4 = \frac{R^2 + 9Xc^2}{R^2 + Xc^2}$$

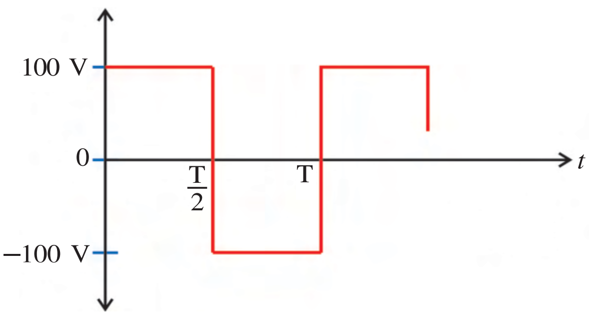
$$\therefore \quad 4R^2 + 4Xc^2 = R^2 + 9Xc^2$$

$$\therefore \quad 5Xc^2 + 4R^2 = R^2$$

$$\therefore \quad \frac{Xc}{R} = \sqrt{\frac{3}{5}}$$

Illustration 9 : The maximum value of an A.C. voltage is equal to 100 V for a square wave shown in the figure. Calculate the rms value of the voltage.

Solution : $V_{rms} = \left[\frac{1}{T}\int_0^T V^2(t)dt\right]^{\frac{1}{2}}$



$$= \left[\frac{1}{T}\left\{\int_0^{\frac{T}{2}}(100)^2 dt + \int_{\frac{T}{2}}^{T}(-100)^2 dt\right\}\right]$$

$$= \left[\frac{1}{T}\left\{10^4\left(\frac{T}{2} - 0\right) + 10^4\left(T - \frac{T}{2}\right)\right\}\right]^{\frac{1}{2}} = \left[\frac{1}{T}\left\{10^4\frac{T}{2} + 10^4\frac{T}{2}\right\}\right]^{\frac{1}{2}} = \left[10^4\right]^{\frac{1}{2}} = 100 \text{ V}$$

Illustration 10 : The medium wave broadcast signal in a radio can be tuned from 600 kHz to 1200 kHz. If the effective value of the inductance of an inductor connected in the L-C circuit is 100 mH, find the range of the variable capacitor.

Solution : L = 100 mH, f_{max} = 1200 kHz, f_{min} = 600 kHz

For Tuning (means for resonance) frequency $f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$

$$\therefore \quad 4\pi^2 f^2 = \frac{1}{LC}$$

$$\therefore \quad C = \frac{1}{4\pi^2 f^2 L} \quad \therefore \quad C_{max} = \frac{1}{4\pi^2 f^2_{min} L}, \quad \therefore \quad C_{min} = \frac{1}{4\pi^2 f^2_{max} L}$$

$$\therefore \quad C_{max} = \frac{1}{(4)(3.14)^2 (600 \times 10^3)^2 \ (100 \times 10^{-3})}$$

$$= 0.7 \times 10^{-12} \text{ F}$$

$$= 0.7 \text{ pF}$$

Similarly $C_{min} = \frac{1}{(4)(3.14)^2 (1200 \times 10^3)^2 \ (100 \times 10^{-3})}$

$$= 0.176 \times 10^{-12} \text{ F}$$

$$= 0.176 \text{ pF}$$

Thus, the range of the variable capacitor is from 0.176 pF to 0.7 pF.

2.7 Phasor Method

Addition of harmonic functions can be easily done with the method of phasor. To understand what a phasor is, consider a harmonic function.

$$I = I_m \cos(\omega t + \delta) \qquad (2.7.1)$$

A vector with magnitude I_m is drawn from the origin of coordinate system in X–Y plane, as shown in figure 2.16, which makes an angle with X-axis equal to phase $(\omega t + \delta)$. From the figure 2.16, the following points are clear.

(1) The phase $(\omega t + \delta)$ changes with time. It means the angle made by the vector I_m with X-axis in figure 2.16 changes with time. Thus the vector drawn here is not steady but rotates in X–Y plane with an angular frequency ω. Such a vector is called the rotating vector. Such a rotating vector is the phaser or the rotor.

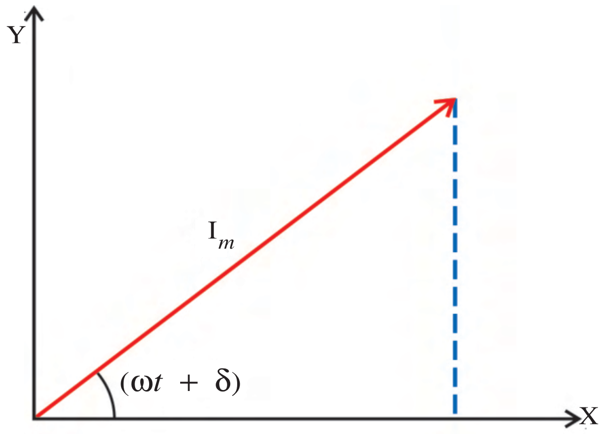


Figure 2.16

Here, note that I is scalar only. We are merely representing it as a rotating vector.

(2) At $t = t$ time, the x-component of this vector is $I_m \cos(\omega t + \delta)$, which gives the instantaneous value of I. If we want to add functions like

$I_1 \cos(\omega t + \delta_1)$, $I_2 \cos(\omega t + \delta_2)$ .... etc., now it becomes easier. We should draw phasors for all functions at time and then add their X-components, algebraically. Thus, the final algebra becomes simple.

(3) There is one more advantage of this method. If we take *y*-component of this function, it is $\cos\left[\frac{\pi}{2} - (\omega t + \delta)\right] = \sin(\omega t + \delta)$. Thus by considering Y-components of the vectors we can also deal with sine functions in the similar manner.

(4) Suppose we want to add two harmonic functions,

$$I_1 = I_{1m} \cos(\omega t + \delta_1) \qquad (2.7.2)$$

$$I_2 = I_{2m} \cos(\omega t + \delta_2) \qquad (2.7.3)$$

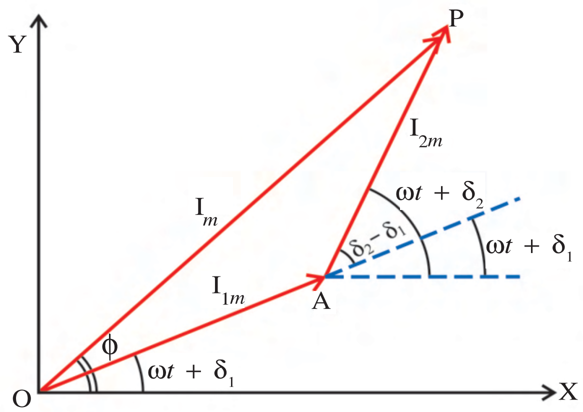


Figure 2.17

To find the resultant function, vectors representing I_{1m} and I_{2m} are drawn as shown in the figure 2.17 and then their vector addition is found. For this draw vector in the direction making angle of $(\omega t + \delta_1)$ with X-axis. Now, from the end point A of this vector draw a vector I_{2m} in the direction making an angle of $(\omega t + \delta_2)$ with X-axis. To find the resultant vector from starting point O of I_{1m} draw a vector to the end point P of I_{2m}.

From the geometry of the figure 2.17, it is clear that the vector representing I at time t, represents the resultant function obtained at that time, of the two given harmonic functions. Its amplitude is I_m (= OP) (to scale) and its phase at time t is ϕ. We can also get the functional form of I from the law of triangle of vectors.

From the figure the angle between the two vectors representing two harmonic functions. I_1 and I_2 is $(\delta_2 - \delta_1)$

Now, $I_m^2 = I_{1m}^2 + I_{2m}^2 + 2I_{1m}I_{2m}\cos(\delta_2 - \delta_1)$

Let the phase difference between the functions I_1 and I_2 be $(\delta_2 - \delta_1) = \delta$

$\therefore\ I_m^2 = I_{1m}^2 + I_{2m}^2 + 2I_{1m}I_{2m}\cos\delta$

Thus we get the resultant function also. We should remember here, that the magnitudes of the veacors representing I_1, I_2 and I are respectively I_{1m}, I_{2m} and I_m.

2.8 Use of Phasor Method in an A.C. Circuit

This method can be very easily used for obtaining the phase relation between applied voltage and current.

A.C. Circuit Containing only Resistance : For circuit containing only resistance phase difference between applied voltage V and current I is zero hence phasor for voltage and current will be in the same direction as shown in the figure 2.18. (It may be noted that the phasor of I can be taken in any arbitrary direction. The phasor of V can be drawn by knowing the phase difference between V and I).

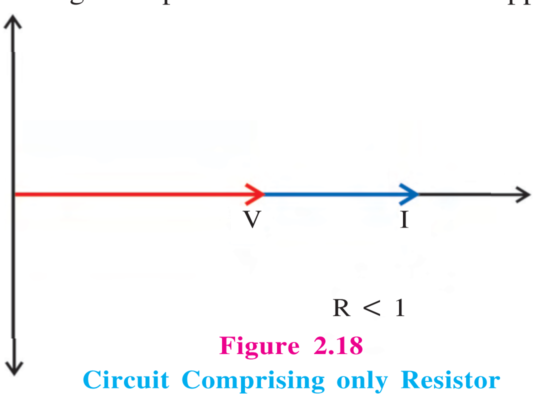


Figure 2.18
Circuit Comprising only Resistor

Circuit Containing only Inductor : We have already studied algebraically this circuit in section (2.4). In this circuit current I lags the voltage V by $\frac{\pi}{2}$ rad in phase angle or the voltage leads current by a phase angle of $\frac{\pi}{2}$. If I is represented along the X-direction then the phasor representation of V will be along the positive Y-direction. As shown in figure 2.19.

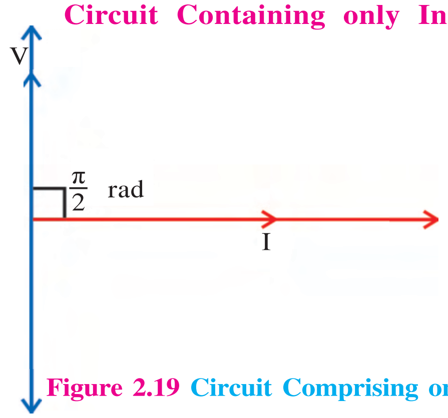


Figure 2.19 Circuit Comprising only Inducter

A.C. Circuit Containing only Capacitor : In the circuit the current I leads the voltage V by a phase of $\frac{\pi}{2}$ or the voltage lags the current by a phase of $\frac{\pi}{2}$. The phasor diagram for this circuit is as shown in figure 2.20.

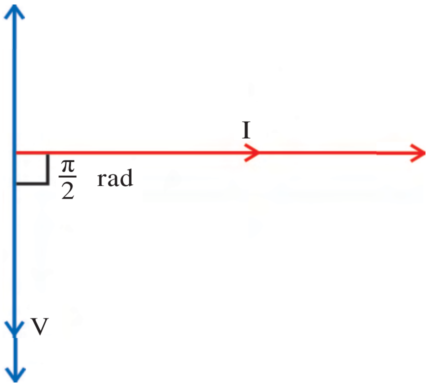


Figure 2.20 **Circuit Comprising only Capacitor**

L-C-R Series A.C. Circuit : From the above information the phasor diagram for each component of L-C-R series circuit will be as shown in the figure 2.21.

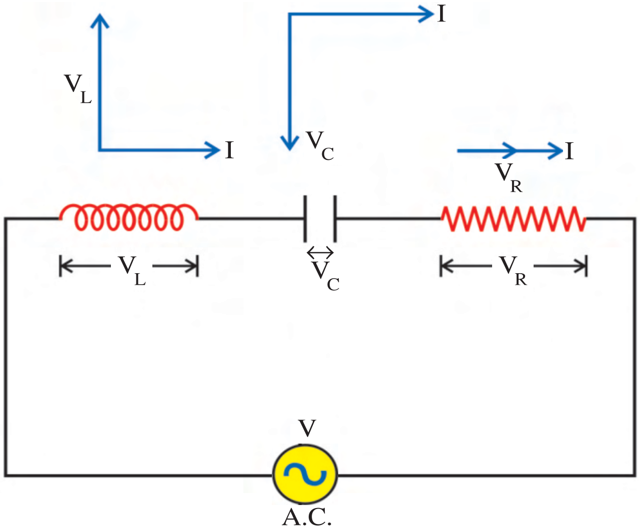


Figure 2.21 **Series Circuit**

In the present case as L, C and R are in series, the current passing through each component will be same. If applied voltage is V then,

$$\vec{V} = \vec{V_L} + \vec{V_C} + \vec{V_R} \qquad (2.8.1)$$

Where V_L, V_C and V_R are potential difference between two ends of inductor, capacitor and resistance respectively. Suppose the phasor of current I is shown in X-direction, then the phase diagrams for each components will be as shown in figure 2.22.

It is obvious from the figure 2.22 that

$$V^2 = (V_L - V_C)^2 + V_R^2$$

If I_m is the maximum value of the current

$$V_R = I_m R,\ V_L = I_m X_L \text{ and } V_C = I_m X_C$$

$$\therefore\ V^2 = I_m^2 (X_L - X_C)^2 + I_m^2 R^2$$

$$\therefore\ V = I_m \sqrt{(X_L - X_C)^2 + R^2}$$

As we have taken the maximum current

$$V = V_m$$

$$\therefore\ I_m = \frac{V_m}{\sqrt{(X_L - X_C)^2 + R^2}} = \frac{V_m}{|Z|} \qquad (2.8.2)$$

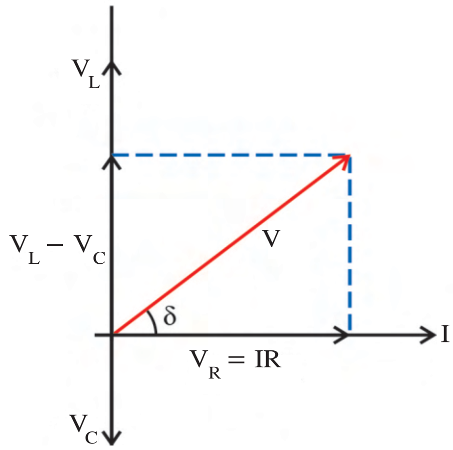


Figure 2.22

Now from figure the angle between voltage phasor and current phasor is δ. In the present case (when $V_L > V_C$) current lags the voltage by phase angle δ. If $V_L < V_C$ then current leads the voltage by phase angle δ.

From figure,

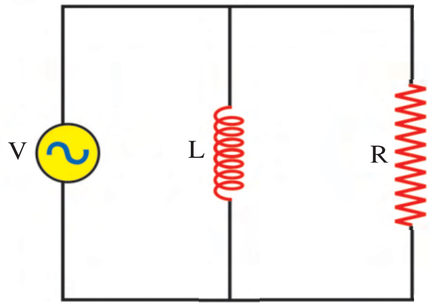
$$\tan\delta = \frac{V_L - V_C}{V_R}$$

$$= \frac{I_m X_L - I_m X_C}{I_m R}$$

$$\therefore \tan\delta = \frac{X_L - X_C}{R} \qquad (2.8.3)$$

$$\delta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) \qquad (2.8.4)$$

Illustration 11 : An inductor L and resistor R are connected in parallel with an A.C. source of V volt. Determine the total current I in terms of X_L and R. Also determine the phase difference between the current and the voltage. Use the method of phasor diagram.

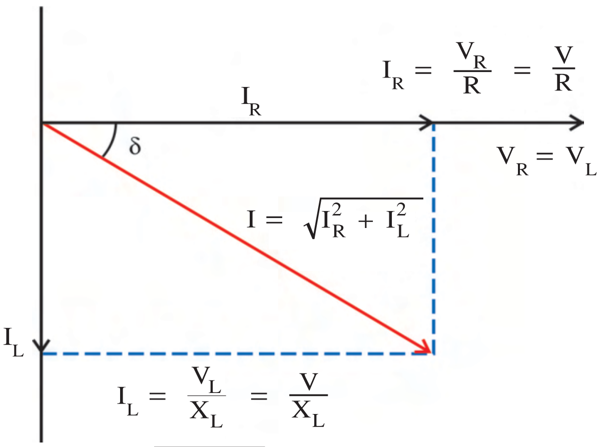


Solution : Here R and L are parallel. Hence the potential difference across them is equal. On representing the phasor of this voltage on X-axis and with reference to it representing the current phasor, the situation will be as shown in the figure.

(1) Here phasor I_R and phasor V_R are in same phase.

$\therefore I_R = \frac{V_R}{R}$ and it is in the X-direction as shown in the figure.

(2) Moreover the current in the inductor is lagging behind the voltage V_L in phase by $\frac{\pi}{2}$. Hence I_L will be on the negative Y-axis as shown in the figure.



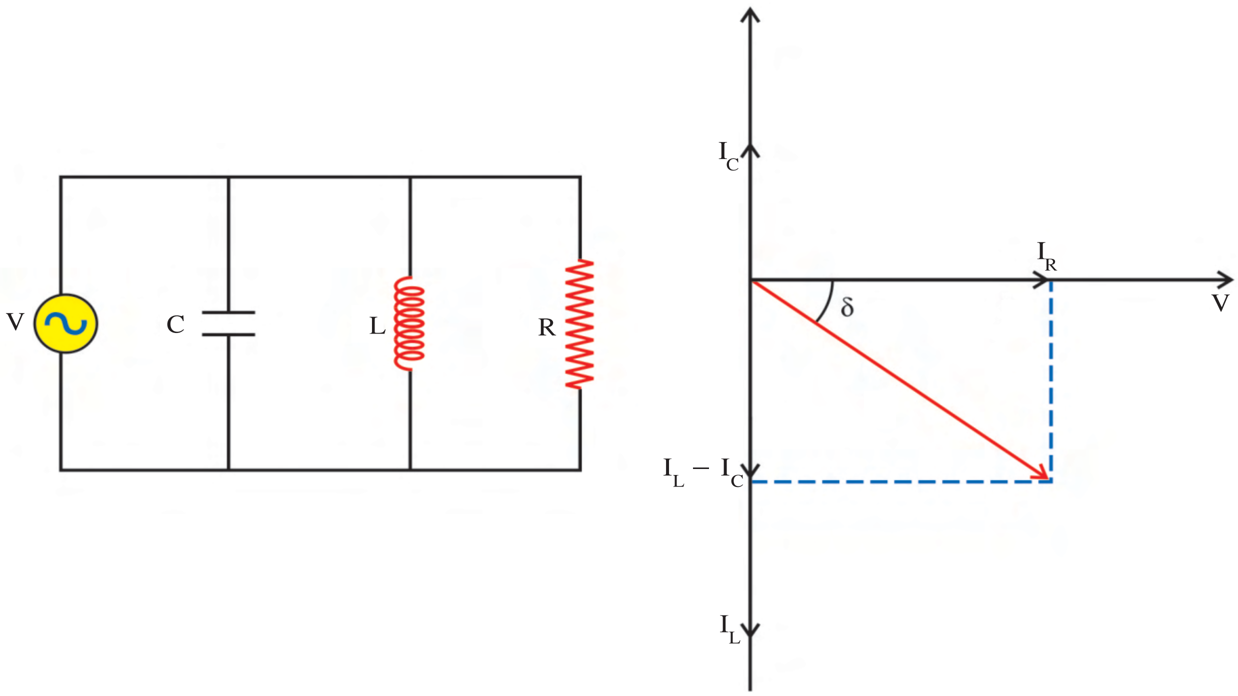
$\therefore I_L = \frac{V_L}{X_L}$. From the figure $I = \sqrt{I_R^2 + I_L^2} = V = \sqrt{\frac{1}{R^2} + \frac{1}{X_L^2}}$

$$\tan\delta = \frac{I_L}{I_R} = \frac{V}{X_L}\frac{R}{V} = \frac{R}{X_L}$$

$$\therefore \delta = \tan^{-1}\frac{R}{X_L}$$

Illustration 12 : Derive the expression for the total current flowing in the circuit using the phasor diagram.

Solution : The phasor diagram of the voltage and current is as shown in figure. In order to obtain the total current, we shall have to consider the addition of the currents. From the diagram we have,



$$I = \sqrt{I_R^2 + (I_L - I_C)^2}$$

But, $I_R = \frac{V}{R}$; $I_L = \frac{V}{X_L}$ and $I_C = \frac{V}{X_C}$

$$\therefore \quad I = V\sqrt{\frac{1}{R^2} + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

From the figure, we have, $\tan\delta = \frac{I_L - I_C}{I_R} = \frac{\frac{1}{X_L} - \frac{1}{X_C}}{\frac{1}{R}}$

$$\therefore \quad \tan\delta = R\left(\frac{1}{X_L} - \frac{1}{X_C}\right)$$

2.9 L-C Oscillations

If the two ends of a charged capacitor (C) are connected by conductor or a resistor, the capacitor gets discharged and energy stored in the capacitor (i.e. the energy stored in the electric field between the two plates) is dissipated in the form of joule heat.

Now let us think what happens when the two plates of charged capacitor are connected with the inductor (L) having very low resistance which can be neglected (ideally zero). Such a circuit is shown in the figure 2.23 which is known as L-C circuit. Here, initially (that $t = 0$) the capacitor is in charged condition, hence we can think of the situation as follows :

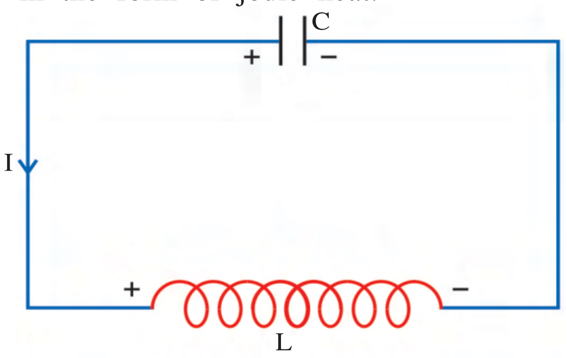


Figure 2.23

Suppose at $t = 0$ the charge on the capacitor is Q_0 and current in the circuit is zero. Here, it is assumed that the capacitor is brought in the circuit at time $t = 0$. The moment at which inductor is joined in the circuit the charge on the capacitor starts decreasing (i.e. capacitor starts discharging) and current starts in the circuit.

Due to the discharging of the capacitor suppose at time $t = t$ the charge on the capacitor = Q and curent in the circuit = I.

Hence applying Kirchoff's second law to this circuit at time $t = t$.

$$-L\frac{dI}{dt} + \frac{Q}{C} = 0$$

But, $I = -\frac{dQ}{dt}$ ($\therefore$ Charge on the capacitor dereases.)

$$\therefore \ L\frac{d^2Q}{dt^2} + \frac{Q}{C} = 0$$

$$\therefore \ \frac{d^2Q}{dt^2} = -\frac{Q}{LC} \qquad (2.9.1)$$

This equation is analogus to the differential equation $\frac{d^2y}{dt^2} = -\omega_0^2 y$ of the simple harmonic motion. Here, the role of charge Q is similar to the displacement y and in place of ω_0^2 the term $\frac{1}{LC}$ is appearing.

Thus, the solution of the equation in our case is $Q = Q_m \sin(\omega_0 t + \phi)$ (2.9.2)

Here Q_m and ϕ are the constants of the solution and can be determined from the initial conditions as follows.

When $t = 0$, $Q = Q_0$ on substituting these values in equation (2.9.2).

$$Q_0 = Q_m \sin\phi \qquad (2.9.3)$$

Differentiating equation (2.9.2) with respect to time

$$I = \frac{dQ}{dt} = Q_m \omega_0 \cos(\omega_0 t + \phi)$$

But at $t = 0$, $I = 0$

$$\therefore \ 0 = Q_m \omega_0 \cos\phi \qquad (2.9.4)$$

Here Q_m and ω_0 are non zero.

$$\therefore \cos\phi = 0$$

$$\therefore \ \phi = \frac{\pi}{2} \qquad (2.9.5)$$

substituting this value of ϕ in equation (2.9.3)

$$Q_m = Q_0 \qquad (2.9.6)$$

Using equation (2.9.5) and (2.9.6) in equation (2.9.2)

$$Q = Q_0 \sin(\omega_0 t + \frac{\pi}{2})$$

$\therefore \quad Q = Q_0 \cos\omega_0 t$ (2.9.7)

This equation shows that the charge on the capacitor changes periodically. Moreover, from this equation

$$I = \frac{dQ}{dt} = -Q_0\omega_0 \sin\omega_0 t \quad (2.9.8)$$

It can be seen from the above equation that the current I in the circuit (i.e. current in the inductor) is also changing periodically.

At time $t = 0$ charge on the capacitor is maximum and current in the inductor is zero. In this situation the intensity of the electric field produced between the plates of the capacitor is maximum and energy stored (associated with electricfield) $\left(U_E = \frac{1}{2}\frac{Q^2}{C}\right)$ is also maximum. At this time ($t = 0$) the current in the inductor is zero, there is no magnetic field associated with it. Hence no energy is associated with inductor.

As time passes, the charge on the capacitor decreases and as a result of this energy (U_E) associated with electric field also decreases. As this charge is passing through the inductor, the current (I) in the inductor increased and as a result of this the magnetic field associated with it and the energy ($U_B = \frac{1}{2}LI^2$) associated with magnetic field also increases.

Thus, the energy stored in electric field of capacitor is transformed in the energy stored in the magnetic field associated with inductor.

When the charge on the capacitor becomes zero (ie. $Q = 0$), the current (I) in the inductor becomes maximum. At this time the total energy stored in the electric field is transferred in to magnetic field.

After this time the charge on the capacitor increase but the polarity of the two plates is reversed. With this polarity charge on the capacitor becomes maximum and this process continues periodically and the initial situation (the situation at time $t = 0$) is established. This process is repeated continuously. In short the electric charge oscillates between the two plates of the capacitor via inductor. This phenomenon is called oscillations in L-C circuit or L-C oscillations.

During this oscillation the electric field associated with capacitor and energy (U_E) associated with it and the magnetic field associated with inductor and energy (U_B) associated with it are shown in the figure 2.24. with the different time intervals of a periodic time of oscillations. ($t = 0$, $t < \frac{T}{4}$, $t = \frac{T}{4}$, $\frac{T}{4} < t < \frac{T}{2}$, $t = \frac{T}{2}$, $\frac{T}{2} < t < \frac{3}{4}T$, $t = \frac{3}{4}T < t < T$ and $t = T$.)

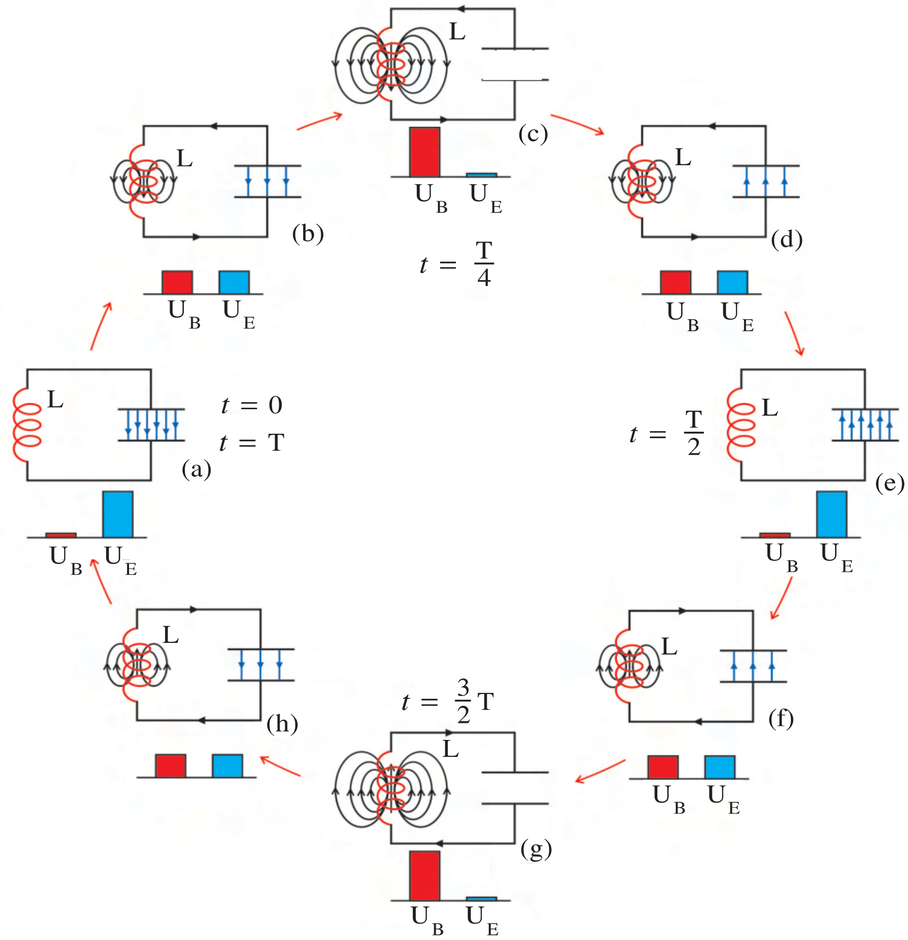


Figure 2.24 **L-C Oscillations (For Information Only)**

Here the electric field associated with capacitor and magnetic field associated with inductor changes with time. These changing electric and magnetic fields radiates electromagnetic radiation. Due to continuous emission of electromagnetic radiation, energy of circuit decreases gradually. Thus oscillating charge emits electromagnetic waves. This L-C circuit also called tank circuit. If the energy is provided in the L-C circuit which equals energy emitted then the continuous emission of the electromagnetic wave is obtained.

Illustration 13 : Show that for free L-C oscillations, the sum of energy stored in capacitor and energy stored in inductor is constant at any instant of time.

Solution : Let Q_0 be the initial (at time $t = 0$) on the capacitor (C). When this capacitor is connected with inductor (L), free oscillations start and its natural angular frequency $\omega_0 = \frac{1}{\sqrt{LC}}$.

Here, instantaneous charge

$$Q = Q_0 \cos\omega_0 t$$

$$\therefore \ I = \frac{dQ}{dt} = -Q_0\omega_0 \sin\omega_0 t$$

At some instant of time t energy stored in the capacitor

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\frac{Q^2}{C} = \frac{Q_0^2}{2C}\cos^2\omega_0 t \quad (\because \ V = \frac{Q}{C})$$

At the same time t energy stored in the inductor

$$U_M = \frac{1}{2}LI^2 = \frac{1}{2}LQ_0^2\omega_0^2 \sin^2(\omega_0 t) = \frac{Q_0{}^2}{2C} \sin^2\omega_0 t \qquad (\because \omega_0 = \frac{1}{\sqrt{LC}})$$

Summation of these two energies

$$U = U_E + U_B = \frac{Q_0^2}{2C}(\cos^2\omega_0 t + \sin^2\omega_0 t)$$

$$= \frac{Q_0^2}{2C}$$ Here, Q_0 and C are not dependent on time

$\therefore$ U = Constant.

2.10 Power and Energy Associated with L, C and R in an A.C. Circuit

According to the definition of power (P)

$$P = VI \tag{2.10.1}$$

In an A.C. circuit voltage and current both changes with time. Hence power represented according to equation (2.10.1) can be called instantaneous power for A.C. circuit. But in practice we cannot measure instantaneous power. In practice real power is defined and it is measured.

Real power = Average power for the entire period of the cycle.

For L-C-R circuit instantaneous power

$$P = VI$$

$$= V_m \cos(\omega t)\ I_m \cos(\omega t - \delta) \tag{2.10.2}$$

$$= V_m I_m \cos\omega t\ \cos(\omega t - \delta)$$

$$\text{But } \cos\omega t\ \cos(\omega t - \delta) = \frac{1}{2}\cos\delta + \frac{1}{2}\cos(2\omega t - \delta) \tag{2.10.3}$$

$$P = \frac{V_m I_m}{2}(\cos\delta + \cos(2\omega t - \delta)) \tag{2.10.4}$$

$\therefore$ According to the definition of real power (now onwards we will consider power P as real power unless specifically it is mentioned.)

$$P = \frac{V_m I_m}{2}\left[\frac{1}{T}\int_0^T \cos\delta\, dt + \frac{1}{T}\int_0^T \cos(2\omega t - \delta)dt\right]$$

But, $\int_0^T \cos(2\omega t - \delta)dt = 0$ and $\int_0^T \cos\delta\, dt = T\cos\delta$

$$\therefore P = \frac{V_m I_m}{2}\frac{T}{T}\cos\delta$$

$$\therefore P = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\delta \tag{2.10.5}$$

Here, $\cos\delta$ is called power factor.

Equation (2.10.5) can also be written in the form of r.m.s. value as follows.

$$\therefore P = V_{rms} I_{rms} \cos\delta \tag{2.10.6}$$

Special Cases :

(1) Circuit with only Resistor : For this circuit phase difference $\delta = 0$.

$\therefore P = V_{rms} I_{rms}$

(2) A.C. Circuit with only Inductor : Phase difference between voltage and current

$\delta = \frac{\pi}{2}$ $\therefore \cos\frac{\pi}{2} = 0$

$\therefore P = 0$

Thus, power in the A.C. circuit with only inductor is zero.

When current in the inductor increases, the energy drawn from the source is stored in the form of magnetic field associated with inductor and when current decreases this stored energy is given back to the source as a result power consumed is zero. Thus with the help of the inductor current can be controlled without wasting the energy in A.C. circuit. (The choke used in tubelight which is an inductor which does this work.).

(3) Circuit with Capacitor Only : The phase difference between voltage and current for this circuit $\delta = -\frac{\pi}{2}$ $\therefore \cos\left(-\frac{\pi}{2}\right) = 0$.

Thus, also in this case power $P = 0$

In this case when charge is accumulated on the two plates of the capacitor, the energy obtained from the source is stored in the electric field produced between two plates of capacitor and when capacitor discharges, the energy stored is given back to the source. As result, power consumed is zero.

(4) For L-C-R A.C. Circuit : From the figure 2.3

$$\cos\delta = \frac{R}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \frac{R}{|Z|} \qquad (2.10.8)$$

On substituting this value of $\cos\delta$ in equation (2.10.6) and calculating power, it can be seen that this power is less than the power obtained in the circuit with only resistor.

When only inductor or only capacitor is there in the A.C. circuit, power consumed is zero. In this situation, the current flowing in the circuit is called watt less current.

Illustration 14 : In an L-C-R A.C. series circuit $L = 5$ H, $\omega = 100$ rads^{-1}, $R = 100\ \Omega$ and power factor is 0.5. Calculate the value of capacitance of the capacitor.

Solution : Power factor $\cos\delta = \frac{R}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}}$

taking square on both sides $\cos^2\delta = \frac{R^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$

But, $\cos\delta = 0.5 = \frac{1}{2}$

$$\therefore \frac{1}{4} = \frac{R^2}{R^2 + (\omega L - \frac{1}{\omega C})^2}$$

$$\therefore R^2 + (\omega L - \frac{1}{\omega C})^2 = 4R^2$$

$$\therefore (\omega L - \frac{1}{\omega C})^2 = 3R^2$$

$$\therefore \omega L - \frac{1}{\omega C} = \sqrt{3} R$$

$$\therefore \omega L - \sqrt{3} R = \frac{1}{\omega C}$$

$$\therefore C = \frac{1}{\omega}\left(\frac{1}{\omega L - \sqrt{3} R}\right)$$

$$= \frac{1}{100}\left(\frac{1}{100 \times 5 - \sqrt{3} \times 100}\right)$$

$$= \frac{10^{-2}}{500 - 173.2}$$

$$= \frac{10^{-2}}{326.8} = 306 \times 10^{-7}$$

$$= 30.6 \times 10^{-6} \text{ F}$$

$$= 30.6 \ \mu\text{F}$$

2.11 Transformer

Power (P = VI) generated in the power station is to be sent to residences and industries which are very very far from the power station through hundreds of kilometers long cable network.

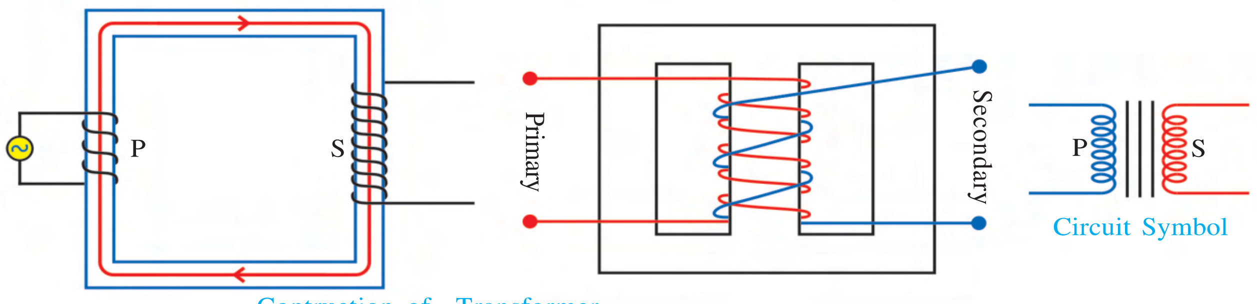
The cables have their own resistance (R). In practice the cable having zero resistance is not possible. Hence, when current (I) passes through the cable the power equals I^2R is transformed in the form of joule heat and wasted. Hence to save energy it is utmost necessary to decrease this wastage of energy. For this the current (I) should be decreased without changing the value of power (P = VI) before sending the current to cable network. Here, it is obvious that for the given value of power (P = VI), if we decrease I, the value of the voltage V has to be increased. For the safety reasons and electrical devices used in practice requires low voltage (generally 230 V or 240 V), it becomes necessary to reduce the voltage before this power is delivered to the residences or industries.

According to the above discussion, we should use a device in which without wasting power (ideally), A.C. voltage can be increased or decreased. Such a device is the transformer. The transformer with which output voltage can be increased is called step-up transformer and with which output voltage can be decreased is called step-down transformer.

It may be noted that in an ideal transformer power is not wasted. Only voltage can be increased or decreased, and correspondingly current is decreased or increased.

Principle : Transformer works on the principle of electro-magnetic induction.

Construction : Figure 2.25 shows the construction of a transformer and symbolic circuit diagram. Here, the two coils of conducting wires are wound very close to each other on a rectangular (or constituting closed loop) iron core having very high value of permeability as shown in figure 2.25. These copper coils are isolated from each other and also from core. One of these coils is called primary coil P and the other coil is called secondary coil S. Primary coil is connected with A.C. source.



(a) (b) (c)

Figure 2.25

In step-up transformer, number of turns are less in primary and copper wire is thick. Whereas in secondary number of turns are more and copper wire is thin. In step-down transformer the situation is reversed. In practice the two coils are wound on the core having shape as shown in the figure 2.25 (*b*), on one another, such that the secondary coil remains on the primary. The iron core is constituted of several layers of strips of two pieces having the shape of English alphabets I and E placing them side by side such that the final shape of the core becomes as shown in the figure 2.25 (*b*). The position of two pieces of the layer I and E are interchanged in the layers coming one after another in the core to obtain the shape of the core as shown in the figure 2.25 (*b*). These layers or strips are insulated.

Due to core constructed as discussed above and winding the secondary coil on the primary coil almost all magnetic field lines due to current in the primary coil are associated with secondary coil and eddy currents can be reduced.

The magnetic flux Φ_S and Φ_P linked with secondary coil (S) and primary coil (P) are respectively proportional to their number of turns N_S and N_P.

$$\therefore \frac{\text{Magnetic flux linked with secondary coil } \Phi_S}{\text{Magnetic flux linked with primary coil } \Phi_P} = \frac{\text{Number of turns in secondary coil } N_S}{\text{Number of turns in primary coil } N_P} \quad (2.11.1)$$

As primary is connected with A.C. source, the current passing through it is changing continuously with time (periodically) and hence magnetic flux linked with primary coil and as a result of that the magnetic flux linked with the secondary coil are changing continuously with time (periodically).

The frequency of the A.C. Voltage induced in the secondary has the frequency as that of the voltage in primary.

According to the Faraday's law

Induced emf in the primary, $\varepsilon_P = -\frac{d\Phi_P}{dt}$ and

induced emf in secondary, $\varepsilon_S = -\frac{d\Phi_S}{dt}$

Now from equation (2.11.1)

$$\Phi_S = \frac{N_S}{N_P}\Phi_P$$

$$\therefore \frac{d\Phi_S}{dt} = \frac{N_S}{N_P}\frac{d\Phi_P}{dt}$$

$$\therefore \varepsilon_S = \frac{N_S}{N_P}\varepsilon_P$$

$$\therefore \frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P} = r \qquad (2.11.2)$$

Here, r is called transformation ratio.

For step-up transformer $r > 1$ and for step-down transformer $r < 1$.

It is obvious that by selecting appropriate transformation ratio, step-up transformer or step-down transformer can be prepared.

We have assumed that in transformer there is no loss of power. Hence

Instantaneous output power (i.e. instantaneous power in the secondary coil) = Instantaneous input power (i.e. instantaneous power in the primary coil)

$$\therefore \varepsilon_S I_S = \varepsilon_P I_P$$

$$\therefore \frac{\varepsilon_S}{\varepsilon_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = r \qquad (2.11.3)$$

The assumption studied above is ideal. Thus, this type of transformer is called ideal transformer. In practice some power is lost in the magnetization and demagnetization of the core as well as in the formation of eddy currents on surface of the core. As a result the output power is less than the input power.

Illustration 15 : In an ideal step-up transformer input voltage is 110 V and current flowing in the secondary is 10 A. If transformation ratio is 10, calculate output voltage, current in primary and input and output power.

Solution : Transformation ratio $r = \frac{N_S}{N_P} = 10$

(1) $\frac{\varepsilon_S}{\varepsilon_P} = \frac{N_S}{N_P} \quad \therefore \varepsilon_S = \varepsilon_P \frac{N_S}{N_P} = 110(10)$

$\therefore$ Output voltage $\varepsilon_S = 1100$ V

(2) $\varepsilon_P I_P = \varepsilon_S I_S \Rightarrow I_P = \frac{\varepsilon_S}{\varepsilon_P} I_S$

$\therefore I_P = \frac{N_S}{N_P} I_S = (10)(10) = 100$ A

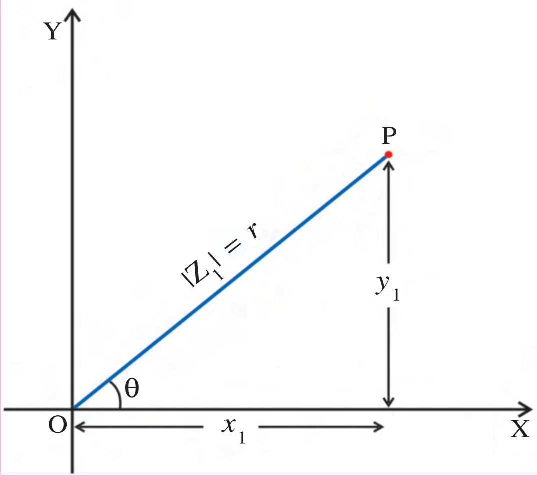
(3) Input power = Output power

$\therefore \varepsilon_P I_P = \varepsilon_S I_S = (1100)(10) = 11000$ W

Appendix A

Complex numbers (For Information Only) : Complex number Z is represented as $x + jy$, where $j = \sqrt{-1}$. Here x and y are real and imaginary parts of the complex number. Similarly a complex function $f(Z)$ is represented as $f(Z) = f_1(x, y) + jf_2(x, y)$. The important points related to our discussion are as follows :

(1) Any complex number can be represented by an appropriate point in a complex plane formed by the x and y variables. The number $Z_1 = x_1 + jy_1$ is represented as a point p in figure. The x co-ordinate of P gives us the real part of Z_1 and its y co-ordinate gives the imaginary part of z_1. The magnitude of the complex number is equal to r. i.e. $|Z_1| = r$.



Here, from figure,

$x_1 = r\cos\theta$ and $y_1 = r\sin\theta$

$\therefore\ Z_1 = r\cos\theta + jr\sin\theta$

$\therefore\ Z_1 = r(\cos\theta + j\sin\theta)$

since $e^{j\theta} = \cos\theta + j\sin\theta$

Hence, the complex number can also be represented as $\therefore Z = |Z|\ e^{j\theta} = re^{j\theta}$ where, $r = \sqrt{x^2+y^2}$

(2) Z* is known as the complex conjugate of the complex number Z. It is obtained by replacing j with $-j$.

$\therefore\ Z^* = x - jy$ moreover $ZZ^* = (x + jy)(x - jy) = (x^2 + y^2) = |Z|^2$

(3) We shall employ the following method to calculate the real and imaginary part of $\frac{1}{Z}$.

$$\frac{1}{Z} = \frac{Z^*}{ZZ^*} = \frac{Z^*}{|Z|^2} = \frac{(x - jy)}{(x^2 + y^2)} = \frac{x}{x^2 + y^2} - j\frac{y}{x^2 + y^2}$$

The real part of $\frac{1}{Z} = \frac{x}{x^2 + y^2}$ and the imaginary part of $\frac{1}{Z} = \frac{y}{x^2 + y^2}$.

The real part of the complex number will be represented as Re(Z) and the imaginary part as Im(Z).

Appendix B

Derivation of formula for Δω (For Information Only) : When the angular frequency is equal to ω_2 we have

$$I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \left(\omega_2 L - \frac{1}{\omega_2 C}\right)^2}}$$

But, $\omega_2 = \omega_0 + \frac{\Delta\omega}{2}$ (From figure 2.15)

$$\therefore\ I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \left\{\left(\omega_0 + \frac{\Delta\omega}{2}\right)L - \frac{1}{\left(\omega_0 + \frac{\Delta\omega}{2}\right)C}\right\}^2}} \qquad \text{(A–1)}$$

Now, substituting the value $C = \frac{1}{\omega_0^2 L}$ in the expression, $\left(\omega_0 + \frac{\Delta\omega}{2}\right)L - \frac{1}{\left(\omega_0 + \frac{\Delta\omega}{2}\right)C}$, we have,

$$\omega_0 L + \frac{(\Delta\omega)L}{2} - \frac{\omega_0^2 L}{\omega_0 + \frac{\Delta\omega}{2}}$$

$$= \frac{\omega_0^2 L + \frac{(\Delta\omega)L\omega_0}{2} + \frac{(\Delta\omega)L\omega_0}{2} + \frac{(\Delta\omega)^2 L}{4} - \omega_0^2 L}{\omega_0 + \frac{\Delta\omega}{2}}$$

$$= \frac{(\Delta\omega)L\omega_0}{\omega_0 + \frac{\Delta\omega}{2}}$$ [Here, we have ignored the term containing $(\Delta\omega)^2$]

Now, $$\frac{(\Delta\omega)L\omega_0}{\omega_0 + \frac{\Delta\omega}{2}} = (\Delta\omega)\ L\omega_0\left(\omega_0 + \frac{\Delta\omega}{2}\right)^{-1}$$

$$= (\Delta\omega)L\left(1 + \frac{\Delta\omega}{2\omega_0}\right)^{-1}$$

$$= (\Delta\omega)L\left(1 - \frac{\Delta\omega}{2\omega_0}\right)$$

$$= (\Delta\omega)L$$

(Here too, we have ignored the second and higher order terms of $(\Delta\omega)$. Substituting the above result in equation (A–1) we have,

$$\therefore\ I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + (L\Delta\omega)^2}} \qquad \text{(A–2)}$$

At angular frequency, ω_2 we have,

$$I_{rms} = \frac{I_{rms}\ (\max)}{\sqrt{2}} = \frac{V_{rms}}{\sqrt{2}R}$$

Substituting the above value of I_{rms} in equation (8.7.6) we have,

$$\frac{V_{rms}}{\sqrt{2}R} = \frac{V_{rms}}{\sqrt{R^2 + (L\Delta\omega)^2}}$$

$$\therefore\ 2R^2 = R^2 + (L\Delta\omega)^2$$

$$\therefore\ R^2 = (L\Delta\omega)^2$$

$$\therefore\ R = L\Delta\omega \Rightarrow \Delta\omega = \frac{R}{L}$$

SUMMARY

1. In the present chapter we have studied different A.C. circuits. We have observed equivalence between mechanical quantities and electrical quantities from the similarity between differential equation for electrical charge.

$$\frac{d^2Q}{dt^2} + \frac{L}{R}\frac{dQ}{dt} + \frac{Q}{LC} = \frac{V_m}{L}\cos\omega t$$

and the differential equation for forced escillations in mechanics

$$\frac{d^2y}{dt^2} + \frac{b}{m}\frac{dy}{dt} + \frac{k}{m}y = \frac{F_0}{m}\sin\omega t.$$

For L-C-R AC. circuit the expression for complex current $i = \dfrac{V_m e^{j\omega t}}{R + j\left(\omega L - \frac{1}{\omega C}\right)}$ is obtained by using complex number. Above expression can be compared with Ohm's law for instantaneous values of voltage and current.

From this we note that effect of inductor and capacitor on current is similar to the effect of R on the current and can be given by $j\omega L$ and $\frac{-j}{\omega C}$ respectively.

$j\omega L$ and $\frac{-j}{\omega C}$ are known as inductive reactance of an inductor and capacitive reactance of a capacitor respectively. Their symbols are Z_L and Z_C and values are $|Z_L|$ and $|Z_C|$ and they are denoted by X_L and X_C respectively.

$\therefore\ |Z_L| = X_L = \omega L$ and $|Z_C| = X_C = \frac{1}{\omega C}$

Summation of Z_L, Z_C and R is called impedance (Z) of series circuit.

$\therefore\ Z = R + Z_L + Z_C = R + j\left(\omega L - \frac{1}{\omega C}\right).$

And its magnitude (value) $|Z| = \sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}$

We have obtained expression for complex current by solving differential equation for charge and from the real part of solution expression for current I.

$I = \dfrac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + (\omega L - \frac{1}{\omega C})^2}} = \dfrac{V_m \cos(\omega t - \delta)}{|Z|}$ is obtained where δ is the phase difference between current and voltage which can be obtained from formula $\tan\delta = \dfrac{\left(\omega L - \frac{1}{\omega C}\right)}{R}$.

2. (1) For A.C. circuit containing only inductor (L)

$Z = j\omega L = jX_L$ and $|Z| = \omega L = X_L,\ \delta = \frac{\pi}{2}$

Current $I = \dfrac{V_m \cos(\omega t - \frac{\pi}{2})}{\omega L} = \dfrac{V_m \cos(\omega t - \frac{\pi}{2})}{X_L}$

(2) For A.C. circuit containing only capacitor (C)

$$Z = \frac{-j}{\omega C} \text{ and } |Z| = \frac{1}{\omega C} = X_C,\ \delta = -\frac{\pi}{2}$$

$$\text{Current } I = \frac{V_m \cos\left(\omega t - \frac{\pi}{2}\right)}{\left(\frac{1}{\omega C}\right)} = \frac{V_m \cos\left(\omega t - \frac{\pi}{2}\right)}{X_C}$$

(3) For A.C. containing R and L in series

$$Z = R + j\omega L \ \therefore\ Z\sqrt{R^2 + (\omega L)^2} = \sqrt{R^2 + X_L^2}$$

$$\delta = \tan^{-1}\left(\frac{\omega L}{R}\right) \text{ and}$$

$$\text{Current } I = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + (\omega L)^2}} = \frac{V_m \cos(\omega t - \delta)}{\sqrt{R^2 + (X_L)^2}}$$

(4) For A.C. circuit containing R and C in Series

$$Z = R - \frac{j}{\omega C} = R - jX_C$$

$$\therefore\ |Z| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2} = \sqrt{R^2 + X_C^2}$$

$$\text{Here, } \delta = \tan^{-1}\left(\frac{1}{\omega CR}\right) \text{ and } \delta \text{ is negative}$$

$$\text{Current } I = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + (\frac{1}{\omega C})^2}} = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + X_C{}^2}}$$

(4) For A.C. circuit containing L and C in series

$$Z = j\omega L - \frac{j}{\omega C} = jX_L - jX_C$$

$$\therefore\ |Z| = \omega L - \frac{1}{\omega C} = X_L - X_C$$

$$\text{Considering } \omega L > \frac{1}{\omega C};\ \delta = \frac{\pi}{2}$$

$$\text{and Current } I = \frac{V_m \cos(\omega t - \frac{\pi}{2})}{\omega L - \frac{1}{\omega C}} = \frac{V_m \cos(\omega t - \frac{\pi}{2})}{X_L - X_C}$$

(5) For A.C. circuit where R is in series with the parallel combination of L and C

$$Z = R - \frac{j}{\omega C - \frac{1}{\omega L}} = R - \left(\frac{j}{X_C - X_L}\right)$$

For $\omega C > \frac{1}{\omega L}$, $\delta = \tan^{-1}\left(\frac{1}{R\,(\omega C - \frac{1}{\omega L})}\right)$

$$|Z| = \sqrt{R^2 + \frac{1}{(\omega C - \frac{1}{\omega L})}} \quad \text{and}$$

$$\text{Current } I = \frac{V_m \cos(\omega t + \delta)}{\sqrt{R^2 + \frac{1}{(\omega C - \frac{1}{\omega L})^2}}}$$

3. Formula for r.m.s. values of voltage and current are

$$V_{rms} = \frac{V_m}{\sqrt{2}} \quad \text{and} \quad I_{rms} = \frac{I_m}{\sqrt{2}}$$

where V_m and I_m are the maximum values of voltage and current respectively.

4. At resonance in L-C-R circuit $\omega_0 L = \frac{1}{\omega_0 C}$ where, ω_0 is the angular frequence and current will be $I_{rms} = \frac{V_{rms}}{R}$

As $\omega_0 L = \frac{1}{\omega_0 C} \Rightarrow \omega_0 = \frac{1}{\sqrt{LC}}$

Q-factor (quality factor) $= \frac{\omega_0}{\Delta\omega}$

Here $\Delta\omega$ is known as Half Power Bandwidth and $\Delta\omega = \frac{R}{L}$

$$\therefore \; Q = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Q factor determines sharpness of the $I_{rms} \rightarrow \omega$ curve.

5. We saw that in different cases of A.C. circuit the phase difference between voltage and current can be obtained easily using phasor.

6. It is also observed from the oscillations of the charge in L–C tank circuit that when maximum charge is on the plates of capacitor the total energy is stored in the electric field produced in the capacitor and when maximum current is flowing in the inductor total energy is stored in the magnetic field produced in the inductor. Moreover, the angular frequency of the oscillations,

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

7. Real power in an A.C. circuit is given by $P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\delta$ where, V_m and I_m are the maximum voltage and current respectively and δ is the phase difference between voltage and current. Here, $\cos\delta$ is known as power factor.

(i) When only resistance R is there in the circuit,

$\delta = 0 \Rightarrow \cos\delta = 1$

$\therefore P = V_{rms} I_{rms}$

(ii) When only inductor is there in the circuit,

$\delta = \frac{\pi}{2} \Rightarrow \cos\delta = 0$

$\therefore P = V_{rms} I_{rms} (0) = 0$

(iii) When only capacitor is there in the circuit,

$\delta = -\frac{\pi}{2} \Rightarrow \cos\delta = 0$

$\therefore P = V_{rms} I_{rms} (0) = 0$

Thus, in an A.C. circuit containing only inductor or only capacitor power P = 0. In this situation current flowing in the circuit is called wattless current.

8. A.C. voltage can be increased or decreased with the help of the transformer. The transformer which increases A.C. voltage is called step-up transformer and the transformer which decreases A.C. voltage is called step-down transformer.

In an ideal transformer,

Instantaneous input power $(I_P \varepsilon_P)$ = Instantaneous output power $(I_S \varepsilon_S)$

Transformer works on the principle of electromagnetic induction.

$$\frac{\varepsilon_S}{\varepsilon_P} = \frac{I_P}{I_S} = \frac{N_S}{N_P} = r$$

Here, r is called transformation ratio.

In the transformers used in practice some part of the electrical power in the primary coil is wasted in the form of heat and some part is used in magnetzing and demagnetizing the core and producing eddy currents. Hence the output power is less than the input power.

EXERCISE

For the following statements choose the correct option from the given options :

1. In an A.C. circuit in 1 second current reduces to zero value 120 times. Hence the frequency of A.C. current is .... Hz.

(A) 50 (B) 100 (C) 60 (D) 120

2. In L–R A.C. circuit at time t current is I and time rate of change of current is $\frac{dI}{dt}$ what will be the potential difference between the two ends of the inductor ?

(A) $L\frac{dI}{dt}$ (B) $\frac{1}{L}\frac{dI}{dt}$ (C) LI (D) $\frac{L}{I}$

3. On decreasing the angular frequency of A.C. source used in L-C-R series circuit, the capacitive reactance .... and inductive reactance ....

(A) Increases, Decreases (B) Increases, Increases

(C) Decreases, Increases (D) Decreases, Decreases

4. When does the impedance of a series L-C-R AC circuit become minimum ?

(A) When the resistance is equal to zero.

(B) When the impedance is equal to zero.

(C) When the electric current is equal to zero.

(D) When the imaginary part of the impedance is equal to zero.

5. The Value of the Q factor in an L-C-R series circuit is ....

(A) dependent on the frequency of the A.C. source.

(B) dependent on the values of all the three components L, R and C.

(C) dependent only on the values of L and C.

(D) it may or may not depend on the power factor.

6. V and I are given by the following equation in an A.C. circuit :

$V = 100 \sin(100t)$ V, $I = 100 \sin\left(100t + \frac{\pi}{3}\right)$ mA. The power in the circuit is equal to .... W.

(A) 10^4 (B) 10 (C) 2.5 (C) 5.0

7. Current of $\frac{50}{\pi}$ Hz frequency is passing through an A.C. circuit having series combination of resistance $R = 100\ \Omega$ and inductor $L = 1$ H, then phase difference between voltage and current is ....

(A) 60° (B) 45° (C) 30° (D) 90°

8. If in an A.C. L–C series circuit $X_L > X_C$. Hence current ....

(A) lags behind the voltage by $\frac{\pi}{2}$ in phase (B) leads the voltage by $\frac{\pi}{2}$ in phase

(C) leads the voltage by π in phase (D) lags behind the voltage by π in phase

9. What is the r.m.s. value of the current for A.C. current $I = 100\cos(200t + 45^\circ)$ A.

(A) $50\sqrt{2}$ A (B) 100 A (C) $100\sqrt{2}$ A (D) Zero

10. Resonance frequency for L-C-R A.C. series circuit is f_0 = ...

(A) $\frac{1}{2\pi\sqrt{LC}}$ (B) $\frac{2\pi}{\sqrt{LC}}$ (C) $\frac{\sqrt{LC}}{2\pi}$ (D) $\frac{2\pi}{LC}$

11. A coil of inductance L and resistance R is connected to an A.C. source of V volt. If the angular frequency of the A.C. source is equal to ω rads^{-1}, then the current in the circuit will be ....

(A) $\frac{V}{R}$ (B) $\frac{V}{L}$ (C) $\frac{V}{R + L}$ (D) $\frac{V}{\sqrt{R^2 + \omega^2 L^2}}$

12. One inductor (of inductance L henry) is connected to an A.C. source, then the current flowing through the inductor I = ... A.

(A) $\frac{V_0}{\omega L}\sin\left(\omega t+\frac{\pi}{2}\right)$ (B) $\frac{V_0}{\omega L}\sin\left(\omega t-\frac{\pi}{2}\right)$

(C) $V_0\,\omega L\sin\left(\omega t-\frac{\pi}{2}\right)$ (D) $\frac{\omega L}{V_0}\sin\left(\omega t+\frac{\pi}{2}\right)$

13. The potential difference between the two ends of the three components of L-C-R series A.C. circuit are V_L, V_C and V_R respectively. Then voltage of A.C. source is ....

(A) $V_L + V_C + V_R$ (B) $V_R + V_L - V_C$

(C) $\sqrt{V_R^2 + (V_L + V_C)^2}$ (D) $\sqrt{V_R^2 + (V_L - V_C)^2}$

14. In R–C circuit when charge on the plates of the capacitor is increasing, the energy obtained from the source is stored in ....

(A) electric field (B) magnetic field

(C) gravitational field (D) both magnetic field and gravitational field

15. In an oscillating L-C circuit the maximum charge on the capacitor is Q. What will be the charge on the plate of the capacitor, when energy stored in magnetic field and electric field are equal ?

(A) $\frac{Q}{3}$ (B) $\frac{Q}{\sqrt{2}}$ (C) Q (D) $\frac{Q}{2}$

16. For L–C–R A.C. circuit resonance frequency is 600 Hz and frequencies at half power points are 550 Hz and 650 Hz. What will be the Q-factor ?

(A) $\frac{1}{6}$ (B) $\frac{1}{3}$ (C) 6 (D) 3

17. An alternating voltage given as $V = 200\sqrt{2}\sin 100t$ V is applied to a capacitor of 1μF. The current reading of the ammeter will be equal to .... mA.

(A) 100 (B) 20 (C) 40 (D) 80

18. The power in an A.C. circuit is given as $P = V_{rms} I_{rms}\cos\delta$. The power factor at the resonance frequency of a series L–C–R circuit will be ....

(A) zero (B) 1 (C) $\frac{1}{2}$ (D) $\frac{1}{\sqrt{2}}$

19. The output power in a step-up transformer is ....

(A) greater than the input power (B) equal to the input power

(C) maintained even during the power cut (D) less than the input power

20. In an L-C oscillator circuit having a completely charged capacitor, with the passage of time ....

(A) The electric current increases gradually.

(B) The energy of the circuit continuously increases.

(C) The energy of the circuit continuously decreases.

(D) There is a continuous absorption of the electromagnetic wave.

21. The equivalent inductance of two inductors is 2.4 H when connected in paralled and 10 H when connected in series, then the individual inductance is

(A) 6 H, 4 H (B) 5 H, 5 H (C) 7 H, 3 H (D) 8 H, 2 H

22. Which device is used to increase or decrease A.C. voltage ?

(A) Oscillator (B) Voltmeter (C) Transformer (D) Rectifier

23. For step-down transformer value of transformation ratio is ....

(A) $r > 1$ (B) $r < 1$ (C) $r = 1$ (D) $r = 0$

24. If for an ideal step-up transformer current in primary is I_P and current in secondary is I_S, their respective voltages are V_P and V_S, then

(A) $I_S V_S = I_P V_P$ (B) $I_S V_S > I_P V_P$ (C) $I_S V_S < I_P V_P$ (D) $I_S V_P = I_P V_S$

25. In an A.C. circuit current is 2 A and voltage is 220 V and power is 40 W power factor is ....

(A) 0.9 (B) 0.09 (C) 1.8 (D) 0.18

ANSWERS

1. (C)	2. (A)	3. (A)	4. (D)	5. (B)	6. (C)
7. (B)	8. (A)	9. (A)	10. (A)	11. (D)	12. (B)
13. (A)	14. (A)	15. (B)	16. (C)	17. (B)	18. (B)
19. (D)	20. (C)	21. (A)	22. (C)	23. (B)	24. (A)
25. (B)					

EXERCISE

Answer the following questions in brief :

1. Which value of the A.C. voltage can be measured by an A.C. voltmeter ?
2. Give units of $\frac{1}{\omega L}$ and ωC.
3. At resonance what is the phase difference between voltage and current in an A.C. L-C-R series circuit ?
4. State the condition for resonance in an A.C., L-C-R series circuit.
5. Which quantity enables us to know sharpness of resonance ?
6. Define half power band width.
7. On which factors does Q-factor depend ?
8. Define real power.
9. Give relation between output power and input power for an ideal transformer.
10. Give relation between real power and maximum power.
11. What does Q-factor give measure of ?
12. What is emitted by an oscillating charge ?
13. On which principle does a transformer work ?
14. What is done to reduce the effects of eddy currents in a transformer ?
15. What is meant by transformation ratio ?
16. What is meant by an ideal transformer ?

17. Give maximum value of the energy associated with an inductor in L-C oscillator ?
18. Give maximum value of the energy associated with a capacitor in an L-C oscillator ?
19. What is meant by r.m.s. value ?

Answer the following questions :

1. Explain alternating current using circuit containing only resistance R and drawing graph of current and time ($I \rightarrow t$).
2. L, C and R are connected in series to an A.C. voltage $V = V_m \cos\omega t$. Obtain the differential equation for the charge.
3. Write the differential equation for current in an A.C. L–C–R series circuit in complex form and derive expression for complex current.
4. Write the formula for impedance of A.C., L–C–R series circuit and represent it in complex plane. Hence obtain the equation of magnitude of Z and phase difference.
5. Obtain the expression for current in an A.C. circuit containing only inductor. (Draw necessary figure and graph.)
6. Obtain the expression for current in an A.C. circuit containing only capacitor (Draw necessary figure and graph.)
7. Obtain the expression for the current in an A.C. circuit containing resistor and an inductor in series (Draw necessary figure and graph.)
8. Obtain the expression for the current in an A.C. circuit containing resistor and the capacitor in series (Draw necessary figure and graph.)
9. Obtain the expression for the current in an A.C. circuit containing an inductor and a capacitor in series (Draw necessary figure and graph.)
10. Obtain the expression for resonance frequency and rms current at resonance in an A.C. L–C–R series circuit using expression for rms current I_{rms}.
11. Draw the graph of $I_{rms} \rightarrow \omega$ for an A.C., L–C–R series circuit and hence explain Q-factor.
12. Using the expressions for charge and current for L–C oscillator, explain L–C oscillations.
13. Derive expression $P = V_{rms} I_{rms} \cos\delta$ for an A.C. circuit.
14. Using $P = V_{rms} I_{rms} \cos\delta$ discuss the special cases for power consumed in an A.C. circuit.
15. Explain necessity of transformer for power transmission and distribution.

Solve the following examples :

1. Find the necessary inductance, if 110 V, 10 W ratting bulb is to be used with 220 V A.C. source having frequency 50 Hz. [**Ans. :** L = 6.67 H]
2. L = 8.1 mH, C = 12.5 μF and R = 100 Ω are connected in series with A.C. source of 230V and frequency 500 Hz. Calculate voltage across the two ends of resistance. [**Ans. :** 230 V]
3. In medium wave broadcast a radio can be tuned in the frequency range 800 kHz to 1200 kHz. In L–C circuit of this radio effective inductance is 200 μH, what should be the range of the variable capacitor ? [**Ans. :** 88 pF to 198 pF]
4. An inductor of 0.5 H and 200 Ω resistor are connected in series with A.C. source of 230 V and frequency 50 Hz, then calculate : (1) Maximum current in the inductor (2) Phase difference between current and voltage and time difference (time lag) [**Ans. :** I_{max} = 1.28 A, 38°, 8' and 2.1 ms]

5. In an ideal transformer input A.C. voltage is 220 V. Current in secondary coil is 2.5 A. If the ratio of number of turns in primary coil and secondary coil is 1 : 10, find

(1) Output voltage

(2) Current in primary coil

(3) Input and output power. [**Ans. :** 2200 V, 25 A, 5500 W]

6. In an A.C. circuit L and R connected in series. Maximum A.C. voltage is 220 volts. If reactance of inductor is 60 Ω and resistance is 80 Ω, find power and power factor. [**Ans. :** 193.6 W, 0.8]

7. Prove that the average value of an A.C. voltage source is given by $V = V_m \sin\omega t$ is equal to $\frac{2V_m}{\pi}$ for half period of its cycle.

8. The value of the A.C. voltage of a generator is V = 0 volts at time $t = 0$. At time $t = \frac{1}{100\pi}$ seconds, the voltage V = 2 volts. The voltage keeps on increasing upto 100 volts. After that it starts decreasing. Determine the frequency of the voltage source. [**Ans. :** 1 Hz]

9. An A.C. circuit contains only an inductor. The frequency of voltage source is 159.2 Hz. And V_m = 100 V. The inductance of the inductor is L = 1 H. Obtain the expression for the current flowing in the circuit. Consider voltage to be $V = V_m \cos\omega t$.

[**Ans. :** $I = 0.1\cos\left(1000\,t - \frac{\pi}{2}\right)$ A]

10. In an A.C. circuit maximum voltage and maximum currents are 220 V and 4.4 A respectively. Calculate power and power factor in the circuit. (Here X_C = 30 Ω and R = 40 Ω) [**Ans. :** 387.2 W, 0.8]

11. In an A.C. circuit R and L are connected in series with source of maximum 220 V and frequency 50 Hz. If L = 1 H and R = 100 Ω. Find the maximum current in inductor and phase difference between voltage and current.

[**Ans. :** 0.668 A, 72°, 20']

12. A.C. Current is given by following formula $I = I_1 \sin\omega t + I_2 \cos\omega t$. Show that rms value of this current given by $I_{rms} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$.

13. The tank circuit of L-C oscillator contains a capacitor of 30 μF and an inductor of 27 mH. Find the natural angular frequency of oscillations. [**Ans. :** 1.111×10^3 rads^{-1}]

•

3 Electromagnetic Waves

3.1 Introduction

Dear friends, when you wake up in the morning, you would have seen the golden (yellow – orange) sun rising from the East. Further you could have observed colourful flowers on green plants–trees and colourful birds in the sky. How can we see such things ? You would say that we can see by our eyes and our mind analyzes that picture. But how do our eyes see ? This is dependent upon the electromagnetic waves. Our eyes can see the picture in terms of electromagnetic waves. The visible radiations (and other radiations) produced from the sun reach the earth. Animals–birds, fruits–flowers etc. reflect electromagnetic waves of different wavelengths (or frequencies) corresponding to their colours, due to which we can see the respective colours of the objects.

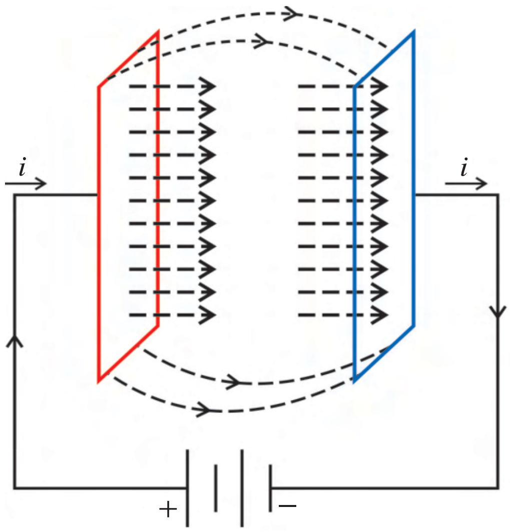
In nineteenth century, the great scientist Maxwell presented the laws of electricity and magnetism; like Gauss's law, Ampere's law, Faraday's law, and the formation of closed loops by the magnetic field, in the form of differential equations. This lead to the concept of the electromagnetic waves. While examining the symmetry between (consistency among) the differential equations for the electric and magnetic fields, it was observed that some term was missing in equation of Ampere's law. Maxwell postulated the missing term in terms of the displacement current. Now the resulting differential equations for electric field $\vec{E}$ and magnetic field $\vec{B}$ were identical with the wave equations. Not only this, but from these wave equations it was established that the velocity of these waves is equal to the velocity of light in vacuum. This established the fact that light waves are nothing but the waves associated with electric field $\vec{E}$ and magnetic field $\vec{B}$ i.e., electromagnetic waves.

Displacement Current : Apart from curing the defect in Ampere's law, Maxwell's term has a certain aesthetic appeal. Just as a changing magnetic field induces electric field (Faraday's law) similarly a changing electric field induces a magnetic field. The real confirmation of Maxwell's theory came in 1887 with Hertz's experiments on electromagnetic radiation.

In Ampere's law, according to Maxwell's opinion,

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i + ?$$

Maxwell called this missing term as displacement current.



What exactly is implied (meant) by the displacement current ? This displacement current does not have the significance of a current in the sense of being the motion of charges. It can be clearly demonstrated by considering a simple capacitor circuit as shown in figure 3.1.

A current i enters the positive plate and leaves the negative plate of a parallel plate capacitor.

This current cannot continue for longer time. When the capacitor becomes fully charged the current becomes zero. If the capacitor plates are very close together then the field between them is,

$$E = \frac{1}{\varepsilon_0}\sigma = \frac{1}{\varepsilon_0}\frac{Q}{A}$$

Figure 3.1 A Simple Capacitor Circuit

Where, Q is the charge on the plate, and A is its area. Thus, when the capacitor is getting charged between the plates

$$\frac{\partial E}{\partial t} = \frac{1}{\varepsilon_0 A}\frac{dQ}{dt} = \frac{1}{\varepsilon_0 A} i$$

$$\therefore \ \varepsilon_0 \frac{\partial E}{\partial t} = \frac{i}{A} = J_d$$

$\therefore \ \varepsilon_0 A \frac{\partial E}{\partial t} = i_d$ which is called the displacement current.

In integral form,

$$\varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot \vec{da} = i_d$$

Adding integral form of displacement current in Ampere's law

$$\oint \vec{B} \cdot \vec{dl} = \mu_0 i_c + \mu_0 \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot \vec{da}$$

or $\oint \vec{B} \cdot \vec{dl} = \mu_0 i_c + \mu_0 i_d = \mu_0 (i_c + i_d)$

This equation is known as Ampere-Maxwell law, which shows that the total current passing through any surface of which the closed loop is the perimeter is the sum of the conduction current and the displacement current. Out side the capacitor plates, there is only conduction current and inside the capacitor, there is only displacement current.

Maxwell's Equations (For Information Only)

(1) $\oint \vec{E} \cdot \vec{da} = \frac{q}{\varepsilon_0}$ (Gauss's law for electricity)

(2) $\oint \vec{B} \cdot \vec{da} = 0$ (Gauss's law for magnetisam)

(3) $\oint \vec{E} \cdot \vec{dl} = \frac{-d\phi_B}{dt}$ (Faraday's law)

(4) $\oint \vec{B} \cdot \vec{dl} = \mu_0 (i_c + i_d)$ (Ampere–Maxwell law)

Where, i_c = conduction current, and

$i_d = \varepsilon_0 \int \frac{\partial \vec{E}}{\partial t} \cdot \vec{da}$ = displacement current

3.2 Transverse nature of Electromagnetic Waves

After the mathematical representation of electromagnetic waves by Maxwell, it took time before the experimental confirmation was established. After about 32 years Hertz gave the proof for the existence of electromagnetic waves in the laboratory.

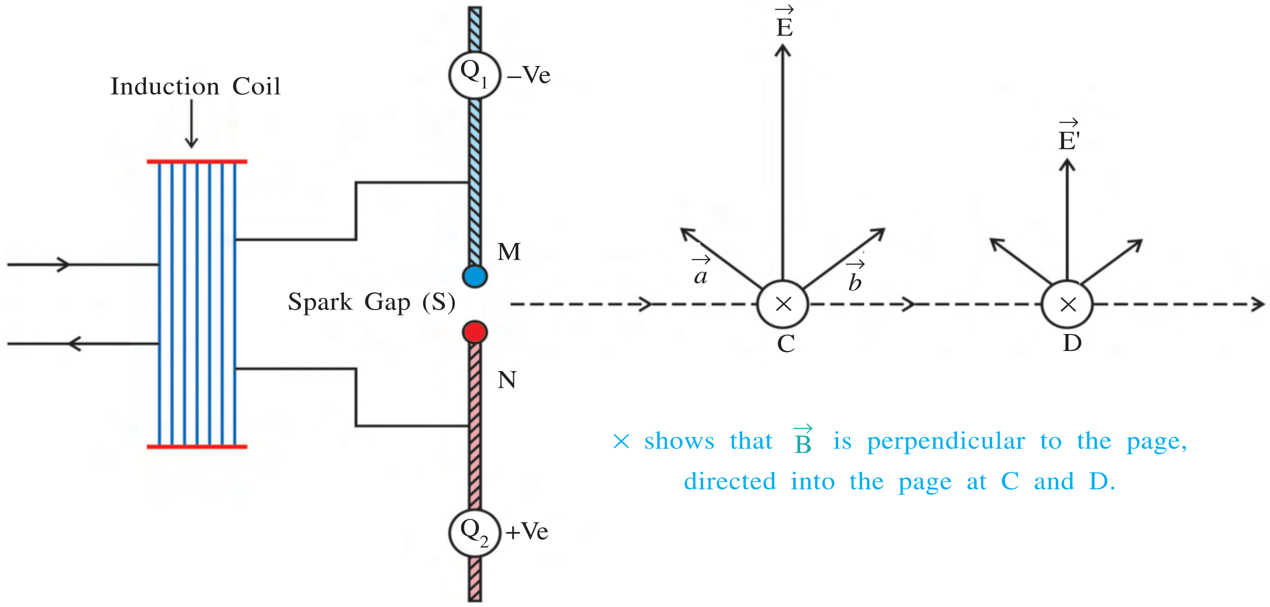


Figure 3.2 A Simple Arrangement of Hertz's Experiment

As shown in figure 3.2, two metallic spheres Q_1 and Q_2 are connected by means of metallic rods M and N, and a spark gap S is formed between the two rods. Spark can be produced in the spark gap by applying a large potential difference between the rods using an induction coil. The spheres Q_1 and Q_2 constitute a capacitor, while the rods behave as an inductor. Such an arrangement can be considered equivalent to L–C oscillator circuit, and known as Hertzian Dipole. At a moment, when one sphere Q_1 is negatively charged and Q_2 is positively charged, the electric field produced at points C and D is shown in the figure. When the spark is produced, the electrons pass from sphere Q_1 towards Q_2 through the spark gap S. This electron current induces magnetic field at points C and D as shown in the figure 3.2. When spark passes, the sphere Q_1 becomes less negative and sphere Q_2 becomes less positive with time, and then after some time the polarity on the spheres Q_1 and Q_2 is reversed, and so on. This process keeps on repeating in a fixed time interval.

Thus, oscillating charges are responsible for the generation of periodically varying electric field in the space. Further, the oscillating charges generate varying electric current which in turn is responsible for the generation of periodically varying magnetic field. (It can be found from the Ampere's right hand rule that the induced magnetic field is perpendicular to the electric field). This way the electromagnetic waves are generated. The frequency of the generated electromagnetic waves is equal to the frequency of oscillation of the electric charges. The frequency of these waves can be varied by changing the distance between the spheres. In the case of electromagnetic waves,

c (velocity) $= \lambda$ (wavelength) $\times f$ (frequency)

Seven years after the Hertz's experiment Jagdishchandra Bose generated electromagnetic waves having the wavelength in the range of 5 mm to 25 mm. Almost during the same time, an Italian scientist Marconi succeeded in the transmission of electromagnetic waves upto a distance of several miles.

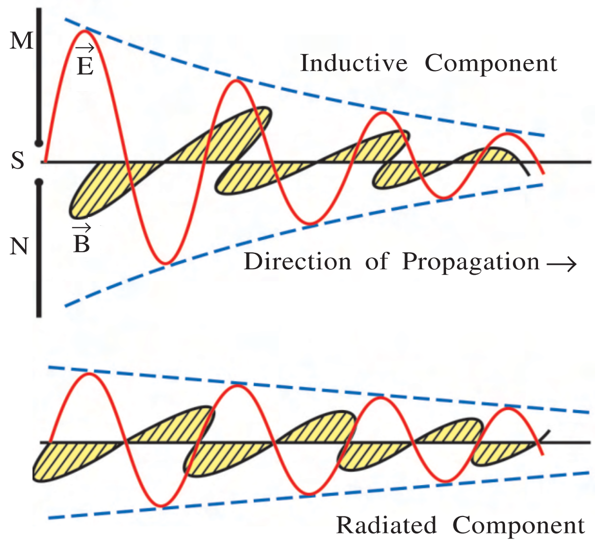


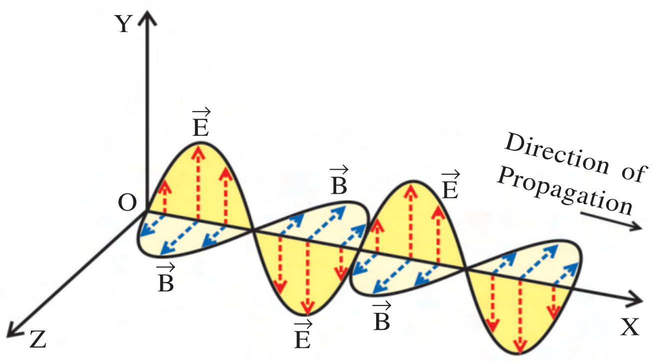
Figure 3.3 Electric and Magnetic Fields at any Instant

According to Maxwell's theory, these electric and magnetic fields do not come into existence instantaneously. In the region closer to the oscillating charges, the phase difference between $\vec{E}$ and $\vec{B}$ fields is $\frac{\pi}{2}$ and their magnitude quickly decreases as $\frac{1}{r^3}$ (where r = the distance from the source). These components of the transmitted waves near their origin (or fields) are called Inductive Components (See figure 3.3).

At large distances $\vec{E}$ and $\vec{B}$ are in phase and the decrease in their magnitude is comparatively slower with distance, as per $\frac{1}{r}$. These components of electromagnetic radiation are called Radiated Components.

When an elctromagnetic wave passes through any point in the space, the electric field and magnetic field vectors oscillate in mutually perpendicular planes, perpendicular to the direction of propagation of the wave at that point (See figure 3.4)

Suppose $\vec{E}$ and $\vec{B}$ are zero at some point away from the origin at any instant of time. As the time passes, the values of $\vec{E}$ and $\vec{B}$ increase with time, reach a maximum value and then start decreasing again to become zero. After that the direction of the fields are reversed. Now the fields start to increase in this reversed direction, reach maximum value and then again decrease to become zero. This process continues periodically as long as the electromagnetic waves pass through that point. This is the meaning of 'oscillating electric and magnetic fields'. The energy of the electromagnetic waves is equal to the kinetic energy of the charges oscillating between the two spheres.



Note : direction of propagation is that of $\vec{E} \times \vec{B}$

Figure 3.4

3.3 Characteristics of Electromagnetic Waves

In figure 3.4 we saw an electromagnetic wave propagating along the X–direction. Here electric field $\vec{E}$ lies in X–Y plane and is parallel to Y–axis, whereas magnetic field $\vec{B}$ lies in X–Z plane and is parallel to Z–axis. The characteristics of electromagnetic waves are like this :

(1) Representation in the Form of Equations : For the electromagnetic wave shown in figure 3.4 at time t, the y component (E_y) of electric field varies according to sine function, whereas its E_x and E_z components are zero. Hence the equation for the E_y component is

$$E_y = E_0 \sin(\omega t - kx) \qquad (3.3.1)$$

which in vector form is

$$\vec{E} = E_y \hat{j} = [E_0 \sin(\omega t - kx)]\hat{j} \qquad (3.3.2)$$

where ω = angular frequency, and $k = \frac{2\pi}{\lambda}$ = magnitude of wave vector. $\vec{k}$ is in the direction of propagation of the wave.

The speed of propagating wave is $c = \frac{\omega}{k}$

Similarly, as $B_x = B_y = 0$, the component B_z of magnetic field is

$$\vec{B} = B_z \hat{k} = [B_0 \sin(\omega t - kx)]\hat{k} \qquad (3.3.3)$$

(2) In electromagnetic wave the relation between the magnitudes of $\vec{E}$ and $\vec{B}$ is $\frac{E}{B} = c$.

Here, remember that the electromagnetic waves are self sustaining oscillations of electric and magnetic fields in free space, or vacuum.

In the region far away from the source, electric and magnetic fields oscillate in phase.

The constituent particles of material medium are not oscillating with the vibrations of electric and magnetic fields. It means that they are non-mechanical waves.

(3) The velocity of the electromagnetic waves in vacuum (free space) is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \qquad (3.3.4)$$

This fact was derived for the first time by Maxwell, using the equations for the electric and magnetic fields. Here,

$\mu_0 = 4\pi \times 10^{-7}$ N A^{-2} = permeability of free space,

$\varepsilon_0 = 8.85419 \times 10^{-12}$ C^2 N^{-1} m^{-2} = permittivity of free space, and

$c = 2.99792 \times 10^8$ ms^{-1}. This value is equal to the magnitude of velocity of light in vacuum.

The velocity of the electromagnetic waves propagating through any medium is given by

$$v = \frac{1}{\sqrt{\mu\varepsilon}} \qquad (3.3.5)$$

where, μ = permeability of the medium, and ε = permittivity of the medium.

Thus, the velocity of light depends on electric and magnetic properties of the medium. Now, for any medium, relative permeability $\mu_r = \frac{\mu}{\mu_0}$ and, relative permittivity $\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = K$

where, K = dielectric constant of the medium.

Hence, from equation (3.3.5)

$$v = \frac{1}{\sqrt{\mu_0 \mu_r \varepsilon_0 \varepsilon_r}} = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \frac{1}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r \varepsilon_r}} = \frac{c}{\sqrt{\mu_r K}} \qquad (3.3.6)$$

Therefore, the refractive index of the medium is

$$n = \frac{c}{v} = \sqrt{\mu_r \varepsilon_r} = \sqrt{\mu_r K} \qquad (3.3.7)$$

The velocity of electromagnetic waves in free space or vacuum is an important fundamental constant.

(4) Electromagnetic waves are transverse waves.

(5) Electromagnetic waves possess energy, and they can carry energy from one place to the other. Electromagnetic waves carry energy from the Sun to the earth, thus making the life possible on the earth.

(6) Electromagnetic waves exert pressure on a surface when they are incident on it, called radiation pressure. This imparts linear momentum to the surface.

If ΔU is the energy of electromagnetic waves incident on a surface of area A in Δt time, normal to the direction of flow of energy, then assuming that the energy is completely absorbed, the momentum delivered to this surface is

$$\Delta p = \frac{\Delta U}{c} \text{ (for complete absorption)}$$

$$\text{and } \Delta p = 2\frac{\Delta U}{c} \text{ (for complete reflection)} \quad (3.3.8)$$

which also exerts the radiation pressure (P_S) on the surface.

(7) Electromagnetic fields are prevalent in region where the electromagnetic waves propagate. The electromagnetic energy per unit volume (energy density) in a region is given by

$$\rho = \rho_E + \rho_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} \quad (3.3.9)$$

where ρ_E = energy density associated with electric field, and ρ_B = energy density associated with magnetic field.

We obtained these relations for capacitor and inductor, in case of stationary fields. In the case of electromagnetic waves, the $\vec{E}$ and $\vec{B}$ fields are oscillating as per sine or cosine functions. Hence, we have to replace E and B in equation (3.3.9) by E_{rms} and B_{rms} to calculate the energy density for electromagnetic waves.

$$\therefore \rho = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{B_{rms}^2}{2\mu_0} \quad (3.3.10)$$

$$\text{Now, } c^2 = \frac{1}{\varepsilon_0\mu_0} \Rightarrow \mu_0 = \frac{1}{\varepsilon_0 c^2}$$

$$\text{Also, } B_{rms} = \frac{E_{rms}}{c}$$

$$\therefore \rho = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{\frac{E_{rms}^2}{c^2}}{\frac{2}{\varepsilon_0 c^2}} = \frac{1}{2}\varepsilon_0 E_{rms}^2 + \frac{1}{2}\varepsilon_0 E_{rms}^2$$

$$\therefore \rho = \varepsilon_0 E_{rms}^2 \quad (3.3.11)$$

Similarly we can obtain $\rho = \dfrac{B_{rms}^2}{\mu_0}$

(8) The intensity of radiation (I) is defined as the radiant energy passing through unit area normal to the direction of propagation in one second.

$$I = \frac{\text{Energy/time}}{\text{Area}} = \frac{\text{Power}}{\text{Area}}$$

Figure 3.5 shows the radiation energy passing through a unit cross sectional area in one second confined within a volume of length equal to c. If ρ is the energy density, then the energy in the above volume is $\rho.c$.

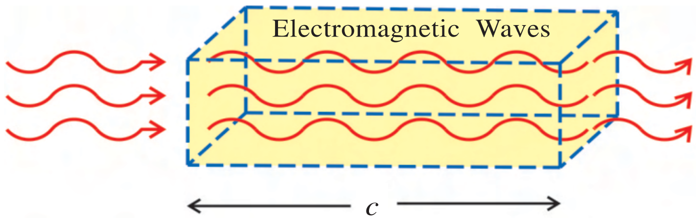


Figure 3.5 Radiation Passing through a Unit Cross Sectional Area Per Second

$$\therefore \; I = \rho.c. = \varepsilon_0 c E^2_{rms} \qquad (3.3.12)$$

Similarly, we can obtain

$$I = \frac{cB_{rms}^2}{\mu_0}$$

(9) $\vec{E} \times \vec{B}$ gives the direction of propagation of the electromagnetic wave.

Illustration 1 : Prove that the unit of $\dfrac{1}{\sqrt{\mu_0 \varepsilon_0}}$ is that of velocity, using unit of μ_0 and ε_0.

Solution : The unit of $\mu_0 = \dfrac{N}{A^2}$, and the unit of $\varepsilon_0 = \dfrac{C^2}{Nm^2}$

$$\therefore \left[\frac{1}{\sqrt{\mu_0 \varepsilon_0}}\right] = \frac{1}{\sqrt{\dfrac{N}{A^2}\,\dfrac{C^2}{Nm^2}}} = \frac{1}{\sqrt{\dfrac{A^2 s^2}{A^2 m^2}}} = ms^{-1}$$

Illustration 2 : A 1000 W bulb is kept at the centre of a spherical surface and is at a distance of 10 m from the surface. Calculate the force acting on the surface of the sphere by the electromagnetic waves, along with E_0, B_0, and intensity I. Take the working efficiency of the bulb to be 2.5 % and consider it as a point source. $\varepsilon_0 = 8.85 \times 10^{-12}$ SI and $c = 3.0 \times 10^8$ ms^{-1}. Also calculate the energy density on the surface.

Solution : The energy consumed every second by a 1000 W bulb = 1000 J.

As the working efficiency of the bulb is equal to 2.5 %, the energy radiated by the bulb per second

$$\Delta U = 1000 \times \frac{2.5}{100}$$

$$\therefore \; \Delta U = 25 \; Js^{-1}$$

Considering, the bulb at the centre of the sphere, surface area of the sphere.

$A = 4\pi R^2 = (4)\ (3.14)\ (10^2) = 1256\ m^2$

Intensity, $I = \dfrac{\text{The energy of the incident radiation per second}}{\text{Area}} = \dfrac{25}{1256} = 0.02\ Wm^{-2}$

Now, $I = \varepsilon_0 c E_{rms}^2 = 0.02$

$$\therefore\ E_{rms} = \left[\frac{0.02}{8.85 \times 10^{-12} \times 3.0 \times 10^8}\right]^{\frac{1}{2}} = 2.74\ Vm^{-1}$$

$$\text{Now, } B_{rms} = \frac{E_{rms}}{c} = \frac{2.74}{3.0 \times 10^8} = 9.13 \times 10^{-9}\ T$$

$E_0 = \sqrt{2}\,E_{rms} = 3.87\ Vm^{-1}$ and $B_0 = \sqrt{2}\,B_{rms} = 1.29 \times 10^{-8}\ T$

The total energy incident on the surface = 25 J

$\therefore$ The momentum (Δp) imparted to the surface in one second (= force),

$$\Delta p = \frac{\Delta U}{c} = F = \frac{25}{3 \times 10^8} = 8.33 \times 10^{-8}\ N$$

From $I = \rho c$, energy density

$$\rho = \frac{I}{c} = \frac{0.02}{3 \times 10^8} = 6.67 \times 10^{-11}\ Jm^{-3}$$

3.4 Electromagnetic Spectrum and Primary Facts of its Applications

After the Maxwell's theory of electromagnetic wave and its successful demonstration by Hertz, scientists started producing electromagnetic waves of different wavelengths.

It was established in the year 1906 that the X-rays discovered by Rontgen in 1895 were also electromagnetic waves. From that time onwards till now electromagnetic waves of wavelengths ranging from approximately 10^{-15} m to 10^8 m have been studied. Electromagnetic waves are continuously spreaded in this wavelength range. Out of the entire range of wavelengths, our eye is sensitive to only small region of wavelengths from about 4000 Å to about 7000 Å. We are blind as far as the other wavelengths are concerned (But this is actually the God given gift to us, otherwise the infrared radiation emitted by the surroundings during night, as well as other radiations of different wavelengths present during night could not allow us to slip, and there could not be any night for us). The sensitivity and the maximum response of the eyes of the various species is different to the electromagnetic spectrum. Many of the species are sensitive to the infrared or ultraviolet region along with visible region. Electromagnetic waves have been classified as per their wavelengths or frequencies, called electromagnetic spectrum (see figure 3.6). There are no sharp boundaries dividing the various sections of the electromagnetic spectrum.

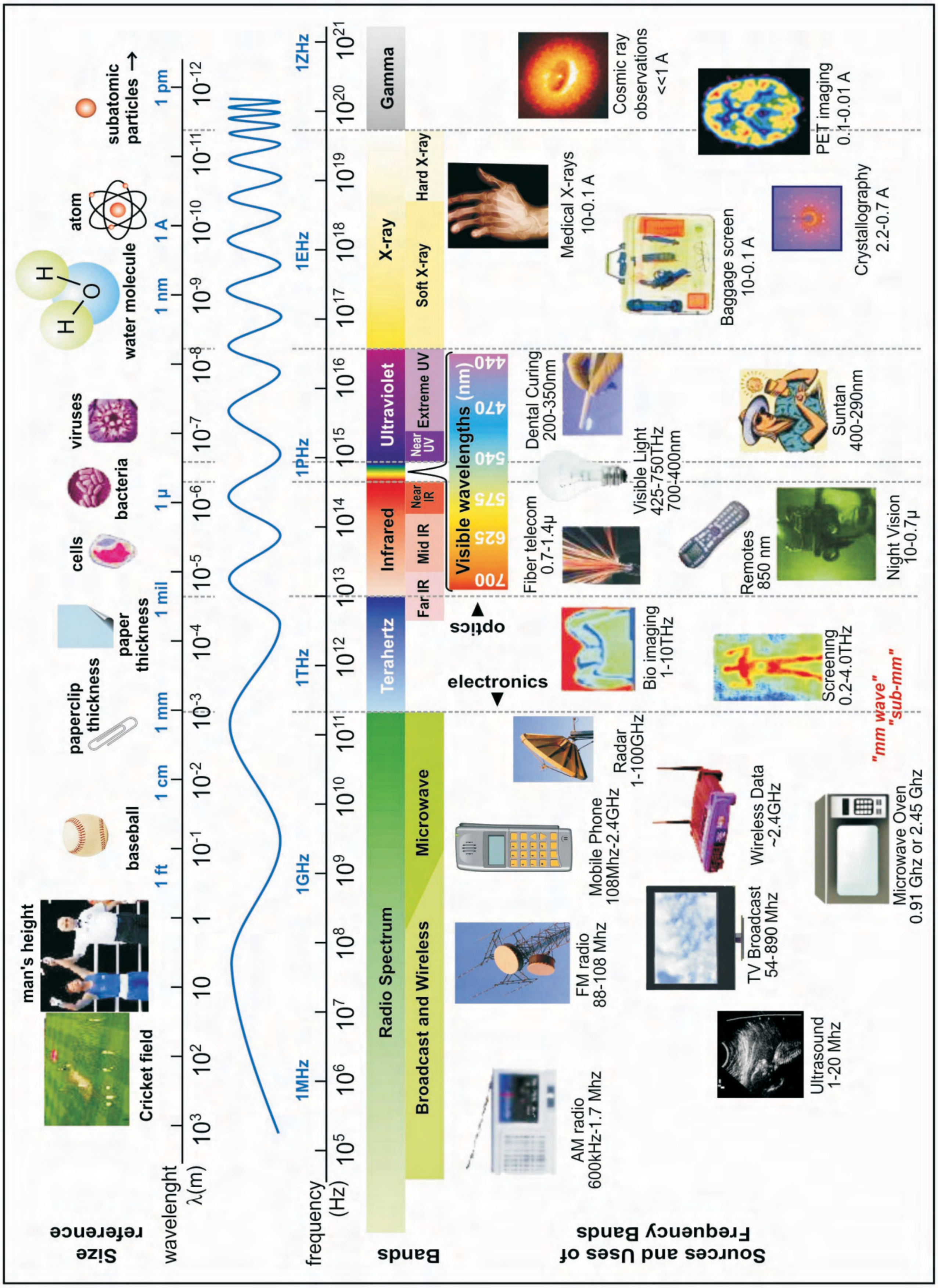


Figure 3.6 Electromagnetic Spectrum (Only For Information)

The brief discussion and applications of different regions of electromagnetic spectrum is as follows :

(1) Radio Waves : Radio waves are produced by the accelerated motion of charges in conducting wires. They are used in radio and television communication systems. The AM (amplitude modulated) band ranges from 530 kHz to 1710 kHz. Higher frequencies upto 54 MHz are used for short wave band. TV signals range from 54 MHz to 890 MHz. The FM (frequency modulated) radio band extends from 88 MHz to 108 MHz. Cellular phones use UHF (Ultra High Frequency) band for voice communication.

(2) Microwaves : Microwaves of about 0.3 GHz to 300 GHz range of frequency are produced by klystrons, magnetrons or Gunn diodes. They are suitable for radar systems used in aircraft, navigation and satellite communication. Radar also provides the basis for the speed guns used to time fast balls, tennis – serves, and intercepter vans by traffic police. Domestic microwave ovens operate at 0.915 GHz or 2.45 GHz to cook food or warm it up. When any food containing water molecules is placed in microwave oven, the water molecules rotate with this frequency. Thus, the microwave energy is transferred efficiently to the kinetic energy of water molecules, which raises the temperature of any food containing water.

(3) Infrared Waves : Infrared waves are produced by hot bodies and molecules. This band of waves lies between microwave band and the visible spectrum. The water molecules present in most materials readily absorb infrared waves (CO_2, NH_3 etc., also absorb infrared waves), due to which their thermal motion (thermal vibration) increases. This vibration increases the internal energy and hence the temperature of the substance increases. This is why the infrared waves are often called Heat Waves. Infrared lamps are used in physiotherapy.

The visible light from the sun is absorbed by the earth's surface, which in turn is re-radiated in terms of infrared radiations. This radiation is absorbed by green house gases like CO_2 and water vapour. This way, the infrared radiation plays an important role in maintaining the earth's warmth or average temperature through the greenhouse effect.

Infrared detectors are used in remote sensing satellites, for military purposes and in agriculture. The remote controller of TV, video players and hi–fi systems also use infrared (IR) LED (Light Emitting Diodes) for their operation.

(4) Visible Rays : Visible rays are a part of the radiation coming from the sun. These rays are also produced by flames, bulbs, incandesncent lamps etc. The visible region has frequency range from about 4×10^{14} Hz to about 7×10^{14} Hz or a wavelength range from about 700 nm to about 400 nm, respectively. Our eyes are sensitive to this range of wavelengths. Different animals are sensitive to different range of wavelengths. For example, snakes can also detect infrared rays, which help them to grab their prey from the infrared radiation being emitted by its body even during night.

For Information Only : The frequency (wavelength) range of visible spectrum is not well defined. The relative sensitivity of human eye to visible light of various wavelengths is shown in the given figure 3.7.

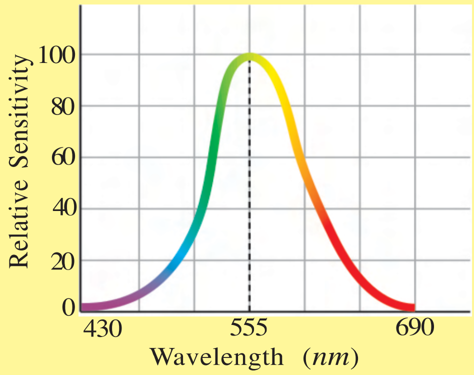


Figure 3.7

If we consider the limits as the wavelengths at which the sensitivity of eye drops to 1% of its maximum value, then these limits are about 430 nm and 690 nm. Human eye can detect electromagnetic waves some what beyond these limits if the intensity of light is high enough. The maximum sensation is produced at wavelength of about 555 nm, called yellow-green.

(5) Ultraviolet Rays : Ultraviolet (UV) radiation is produced by special types of lamps and very hot bodies. The sun is also an important source of UV radiation. Fortunately most of the UV radiation is absorbed in the ozone layer in the atmosphere which lies at an altitude of about 40-50 km from the surface of the Earth. UV radiation in large quantity is harmful to human body. UV exposure induces more production of melanin, causing tanning of the skin. Ordinary glass absorbs most of the UV radiation, so one can get less tanned or sun burn through the glass windows.

A large amount of UV radiation is produced by welding arcs. Hence, welders use face masks with dark glass windows or wear special glass goggles to protect their eyes. UV radiation has very short wavelength range from about 400 nm down to 0.6 nm. Thus, UV radiations can be focused into very narrow beams for high precision application such as LASIK (Laser-Assisted in Situ Keratomileusis) eye surgery. In some water purifiers UV lamps are used to kill germs.

The depletion of ozone from the UV protective ozone layers by the presence of CFCs (Chlorofluoro carbons) gas (such as freon) is a matter of international concern.

(6) X-rays : X-rays can be generated by bombarding a metal target by high energy electrons. The X-ray spectrum lies from about (10^{-8} m) down to about 10^{-4} nm (10^{-13} m) beyond the ultraviolet region in electromagnetic spectrum. X-rays are used in medical applications, to find the fracture in bones, as well as in a treatment of certain types of cancer. Because X-rays can damage or destroy living tissues and organisms, care must be taken to avoid unnecessary or over exposure.

(7) Gamma Rays : Gamma rays are produced during nuclear reactions and also emitted by radioactive nuclei. Gamma rays lie in the upper frequency range of the electromagnetic spectrum and have wavelength ranging from about 10^{-10} m to less than 10^{-14} m. Gamma rays are used in medicine to destroy cancer cells.

Table 3.1 summarises of different types of electromagnetic waves, their production and detection. It should be remembered that there is no sharp demarcation between any two regions.

TABLE 3.1 DIFFERENT TYPES OF ELECTROMAGNETIC WAVES

Type	Wavelength Range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials.	Receiver's aerials (conducting wire)
Microwave	0.1 m to 1 mm	Klystron, magnetron valve, Gun diode.	Point contact diodes
Infra–red	1 mm to 700 nm	Vibration of atoms and molecules.	Thermopile, Bolometer, infrared photographic film
Visible Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level.	The eye, photocells, photographic film, photo diode (LDR), light dependent resistor
Ultraviolet	400 nm to 1 nm	Inner shell electrons in atoms moving from one energy level to a lower level.	solar cell, Photocells, photographic film
X-rays	1 nm to 10^{-3} nm	X–ray tubes or inner shell electrons of atom	Photographic film, Geiger tubes, Ionization chamber,
Gamma rays	< 10^{-3} nm	Radioactive decay of the nucleus.	– do –

SUMMARY

1. The oscillating charges are responsible for the generation of periodically varying electric field in the space. Further, the oscillating charges generate varying electric current which in turn is responsible for the generation of periodically varying magnetic field. This way the electromagnetic waves are generated.

2. The frequency of generated electromagnetic waves is equal to the frequency of oscillation of the electric charges. In case of electromagnetic waves

c (Velocity) = λ (Wavelength) $\times$ f (Frequency)

3. In the region closer to the oscillating charges, the phase difference between $\vec{E}$ and $\vec{B}$ fields is $\frac{\pi}{2}$, and their magnitude quickly decreases as $\frac{1}{r^3}$ (where r = distance from the source). These components of the transmitted waves (or fields) are called inductive components.

4. At large distances from the source, $\vec{E}$ and $\vec{B}$ are in phase and the decrease in their magnitude is comparatively slower with distance, as per $\frac{1}{r}$. These components of electromagnetic radiation are called radiated components.

5. Electromagnetic waves are self sustaining oscillations of electric and magnetic fields in free space, or vacuum. No material medium is associated with vibrations of the electric and magnetic fields.

6. The velocity of electromagnetic waves in vacuum (free space) is

$$c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} = 2.99792 \times 10^8 \text{ ms}^{-1}$$

7. The velocity of the electromagnetic waves in any medium is given by $v = \frac{1}{\sqrt{\mu\varepsilon}}$

where μ = Permeability of the medium, and

ε = Permittivity of the medium.

8. The velocity of light depends on electric and magnetic properties of the medium.

9. The refractive index of a medium is $n = \frac{c}{v} = \sqrt{\mu_r \varepsilon_r} = \sqrt{\mu_r K}$.

10. Electromagnetic waves exert pressure on a surface when they are incident on it, called radiation pressure.

11. If ΔU is the energy of electromagnetic waves incident on a surface of unit area per unit time normal to the direction of flow of energy, then assuming that the energy is completely absorbed, the momentum of the electromagnetic radiation transferred to the surface is $\Delta p = \frac{\Delta U}{c}$

which also represents radiation pressure (P_S).

12. The electromagnetic energy per unit volume (energy density) in a region is given by

$$\rho = \rho_E + \rho_B = \frac{1}{2}\varepsilon_0 E^2 + \frac{B^2}{2\mu_0} = \varepsilon_0 E^2_{rms}$$

13. The radiant energy passing through unit area normal to the direction of propagation in one second is called the intensity of radiation I.

14. The energy of the electromagnetic waves is equal to the kinetic energy of the charges oscillating between the two spheres.

EXERCISE

For the following statements choose the correct option from the given options :

1. The electromagnetic waves in the range of wavelengths from 3 mm to 100 cm are used for the purpose of satellite communication. The range of frequencies corresponding to this range of wavelengths is .... $[c = 3 \times 10^8 \text{ ms}^{-1}]$

   (A) 30 MHz to 10^4 MHz (B) 300 MHz to 10^5 MHz

   (C) 3 MHz to 3×10^8 MHz (D) 3 MHz to 10^6 MHz

2. Astronomers have found that the electromagnetic waves of wavelength 21 cm are continuously reaching the Earth's surface. The frequency of this radiation is ....

   $[c = 3 \times 10^8 \text{ m s}^{-1}]$

   (A) 1.43 GHz (B) 1.43 MHz (C) 1.43 kHz (D) 1.43 Hz

3. If v_g, v_x and v_m are the velocities of the γ-rays, X-rays and microwaves respectively in space, then

   (A) $v_g > v_x > v_m$ (B) $v_g < v_x < v_m$ (C) $v_x > v_m > v_g$ (D) $v_g = v_x = v_m$

4. If μ_r be relative permeability and K be dielectric constant of a given medium, then the refractive index of the medium is n = ...

   (A) $\sqrt{\mu_r K}$ (B) $\sqrt{\mu_0 \varepsilon_0}$ (C) $\frac{1}{\mu_r K}$ (D) $\sqrt{\frac{\mu_r}{K}}$

5. The maximum value of $\vec{E}$ in an electromagnetic wave is equal to 18 Vm^{-1}. Thus the maximum value of $\vec{B}$ is ....

   (A) 3×10^{-6} T (B) 6×10^{-8} T (C) 9×10^{-9} T (D) 2×10^{-10} T

6. An electromagnetic wave passing through the space is given by equations : $E = E_0 \sin(\omega t - kx)$ and $B = B_0 \sin(\omega t - kx)$. Which of the following is true ?

   (A) $E_0 B_0 = \omega k$ (B) $E_0 \omega = B_0 k$ (C) $E_0 k = B_0 \omega$ (D) $\frac{E_0}{B_0} = \frac{1}{\omega k}$

7. A plane electromagnetic wave is travelling along the X-direction. The electric field vector at an arbitrary point at a time is $\vec{E} = 6.3\,\hat{j}$ Vm^{-1}. The magnetic field at that point at that time is ....

   (A) $2.1 \times 10^{-8}\,\hat{k}$ T (B) $-2.1 \times 10^{-8}\,\hat{k}$ T (C) $6.3\,\hat{k}$ T (D) $-6.3\,\hat{k}$ T

8. Two opposite charged particles oscillate about their mean equilibrium position in free space, with a frequency of 10^9 Hz. The wavelength of the corresponding electromagnetic wave produced is ....

   (A) 0.3 m (B) 3×10^{17} m (C) 10^9 m (D) 3.3 m

9. The wavelengths 5890 Å and 5896 Å of sodium doublet correspond to .... region of the electromagnetic spectrum.

(A) infrared (B) visible light (C) ultraviolet (D) microwave

10. The frequency of an electromagnetic wave in free space is 2 MHz. When it passes through a region of relative permittivity $\varepsilon_r = 4.0$, then its wavelength .... and frequency ....

(A) becomes double, becomes half (B) becomes double, remains constant

(C) becomes half, becomes double (D) becomes half, remains constant

11. The rms value of electric field of the radiation coming from the Sun is 720 N/C. The average radiation density is .... Jm^{-3}.

(A) 81.35×10^{-12} (B) 3.3×10^{-3} (C) 4.58×10^{-6} (D) 6.37×10^{-9}

12. In the region closer to the oscillating charges, the phase difference between $\vec{E}$ and $\vec{B}$ fields is ...., and their magnitude quickly decreases as .... with distance r from the source.

(A) 0, r^{-1} (B) $\frac{\pi}{2}$, r^{-3} (C) $\frac{\pi}{2}$, r^{-1} (D) 0, r^{-3}

13. At large distances from the source, $\vec{E}$ and $\vec{B}$ are in phase and the decrease in their magnitude is comparatively slower with distance r as per ...., and these components are called .... components.

(A) r^{-3}, inductive (B) r^{-1}, radiated (C) r^{-3}, radiated (D) r^{-1}, inductive

14. At room temperature, if the relative permittivity of water be 80, and the relative permeability be 0.0222, then the velocity of light in water is .... m s^{-1}.

(A) 3×10^8 (B) 2.5×10^8 (C) 2.25×10^8 (D) 3.5×10^8

15. If the electric field associated with a radiation of frequency 10 MHz is $E = 10 \sin(kx - \omega t) \frac{mV}{m}$, then its energy density is .... J m^{-3}. $[\varepsilon_0 = 8.85 \times 10^{-12} C^2 N^{-1} m^{-2}]$

(A) 4.425×10^{-16} (B) 6.26×10^{-14} (C) 8.85×10^{-16} (D) 8.85×10^{-14}

16. An electromagnetic wave coming from infinity, enters a medium from the vacuum. For this wave .... is independent of the medium. [will not change in the medium]

(A) ω (B) k (C) $\frac{\omega}{k}$ (D) λ

17. For a radiation of 6 GHz passing through air, the wave number (number of waves) per 1 m length is .... (1 GHz = 10^9 Hz).

(A) 3 (B) 5 (C) 20 (D) 30

18. In Hertz's experiment the .... of the electromagnetic waves is equal to the kinetic energy of the charges oscillating between the two spheres.

(A) frequency (B) energy (C) wavelength (D) velocity.

19. The intensity of a plane electromagnetic wave with $B_0 = 1.0 \times 10^{-4}$ T is .... Wm^{-2}

$[c = 3 \times 10^8 \text{ m s}^{-1}, \mu_0 = 4\pi \times 10^{-7} \text{ NA}^{-2}]$

(A) 2.38×10^6 (B) 1.19×10^6 (C) 6×10^5 (D) 4.76×10^6

ANSWERS

1. (B)	**2.** (A)	**3.** (D)	**4.** (A)	**5.** (B)	**6.** (C)
7. (A)	**8.** (A)	**9.** (B)	**10.** (D)	**11.** (C)	**12.** (B)
13. (B)	**14.** (C)	**15.** (A)	**16.** (A)	**17.** (C)	**18.** (B)
19. (B)					

Answer the following questions in brief :

1. Who was the first scientist to demonstrate the existence of electromagnetic waves in the laboratory ?
2. Which term was missing in one of the equations relating electric and magnetic fields ?
3. What is the phase difference between $\vec{E}$ and $\vec{B}$ at large distance from the source ?
4. Who was the first scientist that generated the electromagnetic waves having wavelength ranging from 5 mm to 25 mm ?
5. Define intensity of radiations.
6. Which approximate wavelength ranges are generally not visible to human eye ?
7. Which type of waves are also called heat waves ?
8. Which type of rays are used for LASIK eye surgery ?
9. Which type of rays are produced during the nuclear reactions ?
10. Define the energy density of electromagnetic wave.

Answer the following questions :

1. Drawing the figure of Hertz's experiment, explain the generation of electromagnetic waves.
2. Drawing necessary figure, explain the inductive and radiated components of electromagnetic wave.
3. Explain any four characteristics of electromagnetic waves.
4. Give the information about the production of any two divisions of electromagnetic spectrum and their applications.

Solve the following examples :

1. The magnetic field of an electromagnetic plane wave travelling along the negative X-direction is given by $B_y = 2 \times 10^{-7} \sin (0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ T. Calculate (*a*) the wavelength and frequency of the wave. (*b*) Write the equation of the electric field.

   [**Ans. :** $\lambda = 1.26$ cm, $f = 23.9$ GHz, $E_z = 60 \sin(0.5 \times 10^3 x + 1.5 \times 10^{11} t)$ Vm^{-1}]

2. 5 % of the total energy of a 100 W bulb is converted into visible light. Calculate the average intensity at a spherical surface which is at a distance of 1 m from the bulb. Consider the bulb to be a point source and let the medium be isotropic.

   [**Ans. :** 0.4 Wm^{-2}]

3. The maximum electric field at a distance of 10 m from an isotropic point source of light is 3.0 Vm^{-1}. Calculate (*a*) the maximum value of magnetic field, (*b*) average intensity of the light at that place, and (*c*) the power of the source.

[$c = 3 \times 10^8$ ms^{-1}, $\varepsilon_0 = 8.854 \times 10^{-12}$ $C^2 N^{-2} m^{-2}$]

[**Ans. :** $B_0 = 10^{-8}$ T, I = 1.195×10^{-2} Wm^{-2}, P = 15 W]

4. An observer is at 2 m from an isotropic point source of light emitting 40 W power. What are the rms values of the electric and magnetic fields due to the source at the position of the observer ? [$c = 3 \times 10^8$ m s^{-1}, $\varepsilon_0 = 8.854 \times 10^{-12}$ $C^2 N^{-1} m^{-2}$]

[**Ans. :** $E_{rms} = 17.3$ V m^{-1}, $B_{rms} = 5.77 \times 10^{-8}$ T]

5. A plane electromagnetic wave travelling along X-direction has electric field of amplitude 300 V m^{-1}, directed along the Y-axis. (*a*) What is the intensity of the wave ? (*b*) If the wave falls on a perfectly absorbing sheet of area 3.0 m^2, at what rate is the momentum delivered to the sheet and what is the radiation pressure exerted on the sheet ?

[**Take :** $\varepsilon_0 = 8.854 \times 10^{-12}$ $C^2 N^{-1} m^{-2}$, $c = 3 \times 10^8$ ms^{-1}]

[**Ans. :** 119.529 W m^{-2}, 1.195×10^{-6} N, 3.98×10^{-7} P_a]

6. An electromagnetic wave of electric field $E = 10 \sin(\omega t - kx) \frac{N}{C}$ is incident normal to the cross-sectional area of a cylinder of 10 cm^2 and having length 100 cm, lying along X-axis. Find (*a*) the energy density, (*b*) energy contained in the cylinder, (*c*) the intensity of the wave, (*d*) momentum transferred to the cross-sectional area of the cylinder in 1 s, considering total absorption, (*e*) radiation pressure.

[**Take :** $\varepsilon_0 = 8.854 \times 10^{-12}$ $C^2 N^{-1} m^{-2}$, $c = 3 \times 10^8$ ms^{-1}]

[**Ans. :** (*a*) 4.427×10^{-10} J m^{-3}, (*b*) 4.427×10^{-13} J, (*c*) 1.3278×10^{-1} Wm^{-2} (*d*) 1.475×10^{-21} N (*e*) 1.475×10^{-18} N m^{-2}.]

●

4 Wave Optics

4.1 Introduction

We have noted in the previous semester in the chapter 'Ray Optics' that various theories have been put forward to understand the nature of light. Ray optics or geometric optics has limitations in explaining certain optical phenomena such as interference, diffraction, polarization, transmission, holography, etc. In 1678, Huygen proposed a wave theory of light. According to this theory, light energy is supposed to be transferred from one point to another in the form of waves. He, based on his wave theory, could explain the laws of reflection and refraction. Later, in 1801, Thomas Young could explain the phenomenon of interference of light. Augustin Fresnel in 1815 had developed the wave theory to explain rectilinear propagation of light. The polarization phenomenon, as discovered by Malus in 1808, remained an unsolved problem to Huygen's wave theory. Huygen's wave theory assumes light waves as longitudinal, while the polarization effect can be observed only for transverse waves. As longitudinal waves always require elastic medium for propagation, Young and Fresnel assumed presence of luminiferous ether in entire universe.

Later Young realized that light is transverse waves, though he was still believing in the presence of omnipresent ether. It was Faraday who showed that the polarization of light was affected by a strong magnetic field. This was the first hint about electromagnetic nature of light. Clerk Maxwell unified the empirical laws of electricity and magnetism into a coherent theory of electromagnetism. As studied in the previous chapter, Maxwell made the prediction that light is a high frequency electromagnetic waves. Theoretical prediction of Maxwell was confirmed by Hertz by producing and detecting electromagnetic waves. In 1887, Michelson-Morely performed the famous ether-drift experiment, and concluded that ether does not exist. Hence, light waves are high frequency non-mechanical transverse electromagnetic waves, comprising of oscillating electric and magnetic field vectors.

However, the simple wave theory capable of explaining reflection, refraction, interference, diffraction etc. is described by a single scalar function. This is known as Wave Optics or precisely Scalar Wave Optics.

In this chapter, we shall study propagation of light and related optical phenomena using the ideas of wave optics.

4.2 Wavefront and Huygen's Principle

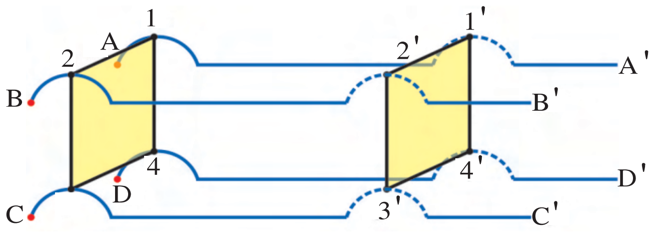
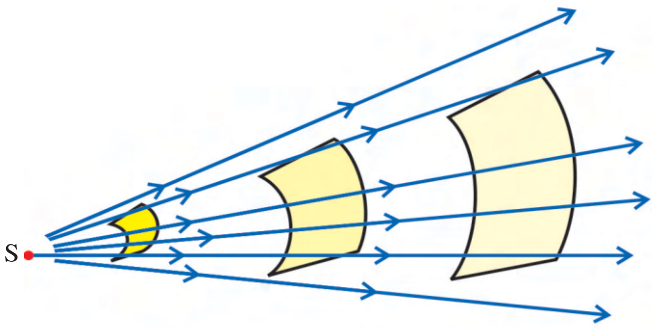


Figure 4.1 Construction of Wavefront

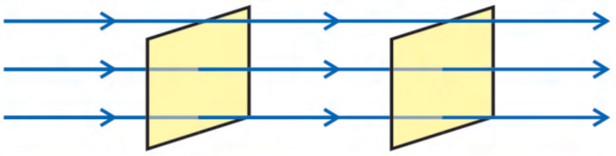
Propagation of disturbance in the medium (space) is known as wave. Thus, waves start from a source (origin) and spread out to new regions of medium (space). To understand, wave propagation, the concept of wavefront is used. As shown in figure 4.1, on identical four mutually parallel strings AA', BB', CC' and DD' four identical crests are created at points 1, 2, 3 and 4 respectively. All the particles located at the crest will begin the same state of oscillation and hence they are in the same phase. An imaginary surface passing through particles (rectangular-plane 1234 as shown in the figure 4.1) oscillating with same phase is known as a wavefront.

Since the shape of wavefront 1234 is a plane surface, it is known as a plane wavefront. Wavefront can be of various shapes.

Waves originating from a point like source and propagating in three dimensional homogeneous and isotropic medium have spherical wavefronts, while in the case of water ripples and due to linear source they are circular and cylindrical, respectively. Although, at considerably large distance (theoretically infinite) wavefronts are locally plane (See figure 4.2).



(a) Spherical Wavefront



(a) Plane Wavefront

Figure 4.2 Different Shapes of Wavefronts

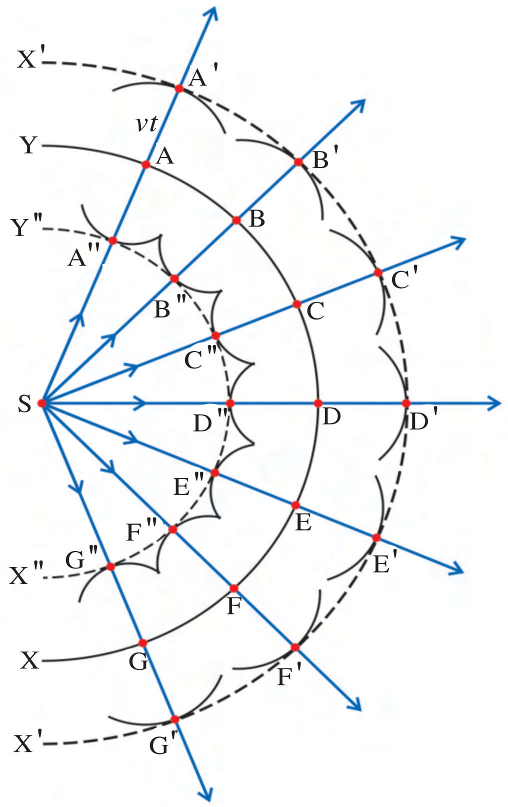
As shown in the figure 4.1, if we observe the strings after sometime crests have reached to particles 1', 2', 3' and 4'. However, their phase of oscillations remain same. Here, also we can imagine a plane wavefront 1'2'3'4'. In this way, as wave propagates ahead in the medium or space, wavefronts also move along with the wave. Thus, the propagation of wave can be visualized in the form of advancing wavefronts.

Lines perpendicular to the wavefront and indicating the direction of propagation of the wave are called rays. Remember that ray is just a geometrical concept.

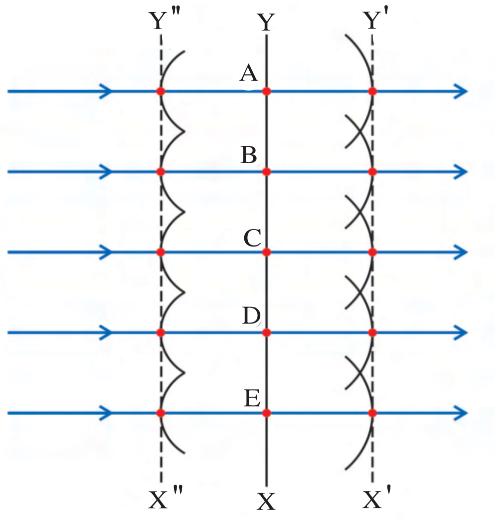
Having noted that along with the wave, wavefront also propagates, a natural question which we may ask is. How is a new wavefront formed after a very small time interval Δt ? This question can be answered by Huygen's principle.

Huygen's Principle : "Every point or particle of a wavefront behaves as an independent secondary source, and emits by itself secondary spherical waves. After a very small time interval the surface tangential to all such secondary spherical wavelets gives the position and shape of new wavefront."

As shown in figure 4.3 (a), part of corss section of spherical or cylindrical wavefront at a particular instant of time (t) is shown as XY. According to Huygen's principle, all particles of this wavefront (i.e. A,B,C ….. etc.) behave as secondary sources and emit spherical waves. If the velocity of wave is v, then we can draw spheres of radii $v\Delta t$ with these particles as centers. Now, we can imagine a surface touching these spheres as a new wavefront at later time $t + \Delta t$. In the figure 4.2 such two surfaces X'Y' and X"Y" are shown. This means that from the wavefront XY, light propagates in both forward and backward directions ! Of course, this is never experienced in day to day life. A satisfactory explanation to this apparent paradox was given by scientists named by Voigt and Kirchoff. They showed that the intensity of secondary wave, making an angle θ with the direction of propagation is proportional to a factor $\cos^2\left(\frac{\theta}{2}\right)$. For the direction of propagation of wave (i.e. forward direction) $\theta = 0$ and hence the intensity is maximum. Whereas for the backward direction ($\theta = \pi$) intensity becomes zero. Hence, the effect due to the secondary waves at X"Y" is zero or in other words, there is no back radiation of energy. Figure 4.3 (b) explains the wavefront formation for plane wave.



(a) Spherical Wavefront



(b) Plane Wavefront

Figure 4.3 Propagation of Wavefront

For the isotropic medium new wavefront maintains its original shape.

4.3 Reflection of Light through the Concept of Wavefront

To understand the phenomenon of reflection of light using the concept of wavefront, consider a plane wavefront PQ in figure 4.4. It is incident on reflecting surface AB such that point P of wavefront just touches the reflecting surface AB at $t = 0$. So, at time $t = 0$, point P starts emitting secondary spherical waves. As time passes, one by one all the points between P and Q gradually touch the surface AB, and start emitting secondary waves.

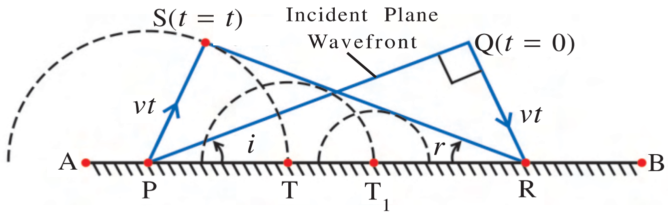


Figure 4.4 Reflection of Wavefront

Let the point Q touch the surface AB at later time t. That is, at time t, point R just starts emitting its secondary waves. During this time interval, a secondary wavefront produced by point P at time $t = 0$ has travelled a distance vt, where v is the speed of light wave in the medium. The corresponding wavefront is shown by dashed line. One such wavefront due to point T is also shown in the figure. According to Huygen's principle a common tangent drawn to such spherical wavefronts (SR in the figure) gives the new wavefront at time $t = t$.

Suppose incident and reflected wavefronts make angle i and r with reflecting surface AB, respectively. From the figure, in ΔPSR and ΔPQR, PR is common side.

$$\angle PSR = \angle PQR = \frac{\pi}{2}$$

Also, PS = vt = QR ($\because$ incident and reflected waves travel in the same medium having speed v.) 2

These facts show that ΔPSR and ΔPQR are congruent.

$\therefore \angle QPR = \angle SRP$

i.e., $i = r$

Thus, the law of reflection (Angle of incidence = Angle of reflection) can also be proved by Huygen's wave theory.

4.4 Refraction of Light Through the Concept of Wavefront

Consider a plane wavefront PQ incident from a medium with refractive index n_1 on a transparent medium having refractive index n_2 (see figure 4.5). In the present discussion, we consider only transmitted wavefronts going into the medium-2. Let at time $t = 0$, the point P just touches the surface separating two media called an interface, and starts emitting secondary waves at $t = 0$ in the medium-2.

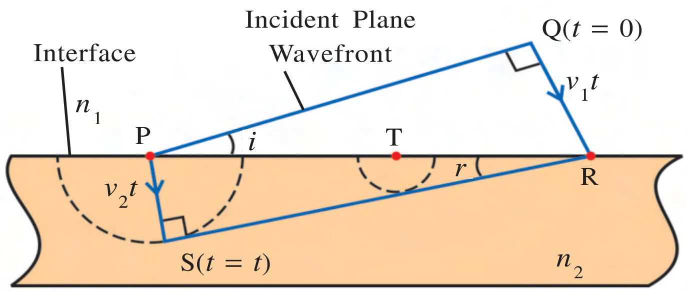


Figure 4.5 Refraction of Wavefront

Now, if speed of light wave in medium-2 is v_2, then secondary wavefront produced from point P travels a distance v_2t in the medium-2. The corresponding wavefront is shown by dashed line in the figure. Further, we assume that during this time ($t = t$), a wavefront produced from point Q has travelled a distance v_1t, and just touched the interface at point R. Here, v_1 is the speed of light in medium-1. According to Huygen's principle, a new wavefront in the medium-2 at time $t = t$ can be formed by drawing a common tangent to such spherical wavefronts (SR in the figure 4.5).

Using the geometry of the figure, angle of incidence (i.e., angle made by incident wavefront with the interface) is i and angle of refraction is r.

Also, PS = v_2t, QR = v_1t and PR is common side to ΔPQR.

$$\text{From } \Delta PQR, \sin i, \frac{QR}{PR} = \frac{v_1t}{PR}$$

and from ΔPSR, $\sin r = \frac{PS}{PR} = \frac{v_2 t}{PR}$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{v_1 t}{v_2 t} = \frac{v_1}{v_2} \qquad (4.4.1)$$

But, $\frac{v_1}{v_2} = n_{21} = \frac{n_2}{n_1}$

$$\therefore \quad \frac{\sin i}{\sin r} = \frac{n_2}{n_1} \qquad (4.4.2)$$

or

$$n_1 \sin i = n_2 \sin r \qquad (4.4.3)$$

Equation (4.4.2) or (4.4.3) is nothing but the Snell's law for refraction.

4.5 Interference

As the disturbance produced at one point in a medium (space in case of non-mechanical wave) propagates, the particles (points in case of non-mechanical wave) coming in its way oscillate according to the type of the disturbance. Now, if a particle comes under the effect of more than one wave, what will be its displacement ? What kind of situation arise? To answer such questions, we should first study principle of superposition.

Principle of Superposition : "When a particle of the medium oscillates under the effect of two or more then two waves superposing at the given particle, according to the principle of superposition the resultant displacement of the particle is equal to the vector sum of the independent displacements due to each wave."

For example, if the displacement due to one wave superposing at a point is 1 cm in upward direction, and that due to other wave is 3 cm in the same direction, the resultant displacement due to both waves will be 1 + 3 = 4 cm in upward direction. But if the displacement due to second wave is 2 cm in downward direction, the resultant displacement at a point will be 1 + (−2) = −1 cm in downward direction.

Thus, superposition principle describes a situation when more than one waves superpose (i.e., interfere) at a point.

"The effect produced by superposition of two or more waves is called interference."

4.5 (a) Interference Due to Two Waves : Suppose two harmonic waves having initial phases ϕ_1 and ϕ_2 are emitted from two point like sources S_1 and S_2 respectively. They superimpose simultaneously (i.e. at the same time t) at a point P, as shown in the figure 4.6.

We have studied in the previous chapter that electromagnetic wave is represented by oscillating electric and magnetic field vectors. However, the effect of light (i.e. visible perception) is produced only by electric field, and therefore, in the present case we write light waves produced by source S_1 and S_2 interms of electric fields ($\vec{e}$) only.

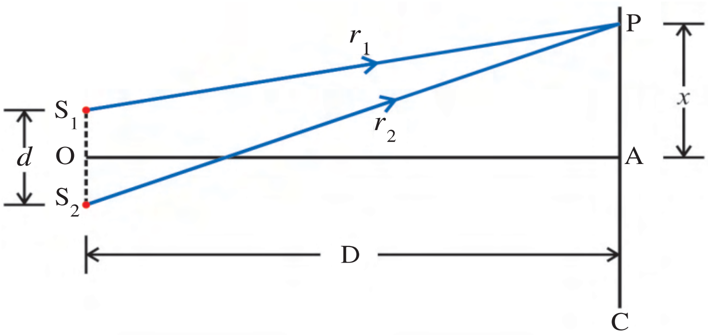


Figure 4.6 Superposition of Waves

Due to S_1 source,

$$\vec{e}_1 = \vec{E}_1 \sin(\omega_1 t - k_1 r_1 + \phi_1) \quad (4.5.1)$$

and that due to source S_2 source,

$$\vec{e}_2 = \vec{E}_2 \sin(\omega_2 t - k_2 r_2 + \phi_2) \quad (4.5.2)$$

Here, $\vec{E}_1$ and $\vec{E}_2$ represent amplitudes of electric fields, ω_1 and ω_2 denote angular frequencies of waves, and k_1 and k_2 are wave vectors. Arguments of sine function is known as phase of two waves.

Let, $\omega_1 t - k_1 r_1 + \phi_1 = \delta_1$ (4.5.3)

and $\omega_2 t - k_2 r_2 + \phi_2 = \delta_2$ (4.5.4)

Then, $\vec{e}_1 = \vec{E}_1 \sin\delta_1$ (4.5.5)

and $\vec{e}_2 = \vec{E}_2 \sin\delta_2$ (4.5.6)

Now, according to the principle of superposition, the resultant displacement at point P is,

$$\vec{e} = \vec{e}_1 + \vec{e}_2 \quad (4.5.7)$$

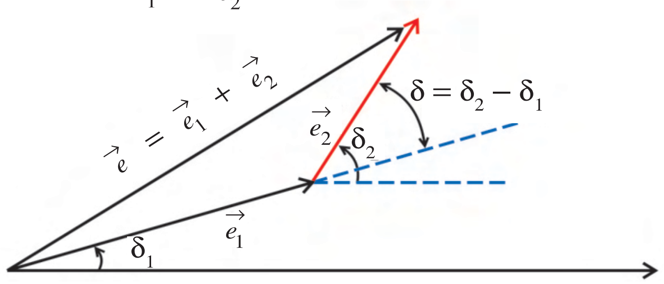


Figure 4.7 Phasor Diagram

To obtain the sum in equation (4.5.7), we use the method of phasor. (See figure 4.7)

$$\therefore\ e^2 = e_1^2 + e_2^2 + 2\vec{e}_1 \cdot \vec{e}_2$$

$$\therefore\ E^2 = E_1^2 + E_2^2 + 2E_1E_2 \cos(\delta_2 - \delta_1) \quad (4.5.8)$$

Where $\delta_2 - \delta_1 = \delta$ = angle between two vectors $\vec{e}_1$ and $\vec{e}_2$, and E is the resultant amplitude.

But, the average intensity of light is proportional to the square of amplitude, i.e. $I \propto E^2$.

Thus, equation (4.5.8) becomes.

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\, \langle \cos(\delta_2 - \delta_1) \rangle \quad (4.5.9)$$

In equation (4.5.9), I_1 and I_2 are the average intensities due to each wave. They are independent of time. The last term in above equation is known as the interference term which depends on time.

Now, $\langle \cos(\delta_2 - \delta_1) \rangle = \frac{1}{T} \int_{t=0}^{t=T} \cos(\delta_2 - \delta_1)\, dt$

$$= \frac{1}{T} \int_0^T \cos\{(\omega_2 t - \omega_1 t) + (k_1 r_1 - k_2 r_2) + (\phi_2 - \phi_1)\}\, dt \quad (4.5.10)$$

Here, T is the period of electric field oscillation.

Case I : Incoherent Sources : If two waves have different angular frequencies, i.e. $\omega_1 \neq \omega_2$. In this case, the phase difference, $\delta = (\delta_2 - \delta_1)$ between two waves is a function of time i.e. $\delta(t)$. Now, equation (4.5.10) becomes,

$$\langle \cos(\delta_2 - \delta_1) \rangle = \frac{1}{T}\int_0^T \cos(\delta(t))\, dt \tag{4.5.11}$$

But integration of cosine or sine function over its period is zero. Thus, in this situation last term in equation (4.5.9) is zero, and superposed two waves produce the average intensity $I_1 + I_2$ at point P.

The sources producing light waves with different frequencies (i.e., $\omega_1 \neq \omega_2$) are known as Incoherent Sources.

Case II : Coherent Sources : If two waves have same angular frequencies, i.e. $\omega_1 = \omega_2$.

Since two waves have same frequencies, they vibrate in such a way that the initial phase difference $\phi_2 - \phi_1$ remains same (or it can also be set to zero.) Light sources having same angular frequences and having constant initial phase difference are called Coherant Source. Here, we take $\phi_2 = \phi_1$. Also, since both waves are travelling in the same medium, their speed will be equal. Therefore, using the relation, $v = f\lambda = \frac{\omega}{k}$, we have $k_1 = k_2 = k$ ($\because \omega_1 = \omega_2$).

Thus, equation (4.5.10)

$$\langle \cos(\delta_2 - \delta_1) \rangle = \frac{1}{T}\int_0^T \cos\{k(r_1 - r_2)\}\, dt$$

$$= \frac{1}{T}\cos\{k(r_2 - r_1)\}\int_0^T dt \qquad (\because \cos(-\theta) = \cos\theta)$$

$$= \cos\{k(r_2 - r_1)\} \tag{4.5.12}$$

Putting the value of equation (4.5.12) in equation (4.5.9), and also by assuming that amplitude of both waves is equal, i.e. $I_1 = I_2 = I'$ (say) then,

$$I = I' + I' + 2\sqrt{I'I'}\ \cos k(r_2 - r_1)$$

$$= 2I'\{1 + \cos k(r_2 - r_1)\}$$

$$= 4I'\cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\} \qquad \left[\because (1 + \cos\theta) = 2\cos^2\left(\frac{\theta}{2}\right)\right]$$

$$I = I_0\cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\} \qquad \text{Where, } 4I' = I_0 = \text{maximum intensity.} \tag{4.5.13}$$

Here, $k(r_2 - r_1)$ is known as the phase difference between superposing waves.

Special Cases :

Case I : When $\frac{k(r_2 - r_1)}{2} = n\pi$ or $k(r_2 - r_1) = 2n\pi$ (4.5.14)

Where $= n = 0, 1, 2, \ldots\ldots$

Then intensity, $I = I_0 =$ maximum ($\because \cos^2 n\pi = 1$)

"If the phase difference between the superposing waves is $2n\pi$ ($n = 0, 1, 2, \ldots$), intensity at a superposing point is maximum. This interference is called constructive interference."

Substituting $k = \frac{2\pi}{\lambda}$ in equation (4.5.14)

$$\frac{2\pi}{\lambda}(r_2 - r_1) = 2n\pi$$

$\therefore$ The difference, $(r_2 - r_1) = n\lambda$ with $n = 0, 1, 2, 3, \ldots\ldots$ (4.5.15)

"If the path difference between superposing waves is $n\lambda$ ($n = 0, 1, 2, \ldots\ldots$) intensity at a superposing point is maximum. Such interference is called constructive interference."

Case II : When $\frac{k(r_2 - r_1)}{2} = (2n - 1)\frac{\pi}{2}$ or $k(r_2 - r_1) = (2n - 1)\pi$ (4.5.16)

where, $n = 1, 2, 3, \ldots\ldots$

Then intensity, $I = 0 =$ minimum ($\because \cos\left(\frac{(2n - 1)\pi}{2}\right) = 0$)

"If the phase difference between superposing waves is $(2n - 1)\pi$, ($n = 1, 2, 3, \ldots\ldots$), intensity at a superposing point is minimum. This interference is called destructive interference."

Corresponding path difference, $(r_2 - r_1) = (2n - 1)\frac{\lambda}{2}$ where $n = 1, 2, \ldots\ldots$ (4.5.17)

"If path difference between superposing waves is $(2n - 1)\frac{\lambda}{2}$ (where, $n = 1, 2, \ldots$), intensity at superposed point is minimum. Such interference is known as destructive interference."

4.5 (b) Intensity Distribution : In principle, using equation (4.5.13), intensity distribution at different points P, P_m, P_{m+1} etc., can be found (see figure 4.8).

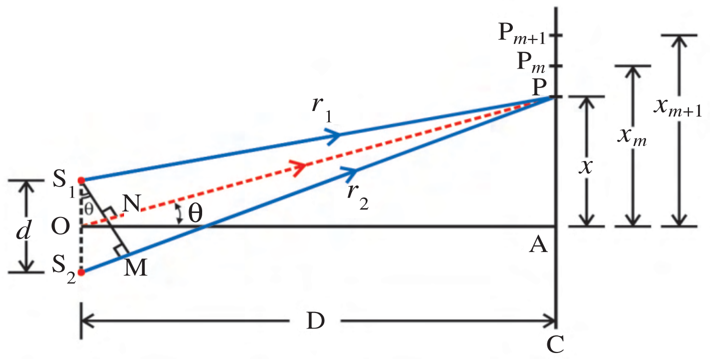


Figure 4.8 Interference of Waves

However, practically it is difficult to find path difference $(r_2 - r_1)$ directly. Therefore, we first convert equation (4.5.13) into such a form that experimentally path difference can be found. As shown in the figure 4.8, let point A on a screen is lying on perpendicular bisector of S_1S_2.

Also, suppose $S_1S_2 = d$, OA = D and position of point P from point A, AP = x and $\angle AOP = \theta$. To measure path difference, draw a perpendicular S_1M on S_2P from S_1. From the geometry of the figure,

path difference $r_2 - r_1 = S_2P - S_1P = S_2M$ (4.5.18)

In actual experiment S_1S_2 is of the order of 0.1 mm and distance D is of the order of meter. Hence, near S_1S_2, segment S_2M and ON may be considered parallel. Also, $\angle S_1NO = 90^\circ$.

$\therefore\ \angle POA = \angle S_2S_1M = \theta$ and $\sin\theta = \dfrac{S_2M}{S_1S_2}$

$\therefore\ S_2M = S_1S_2\sin\theta = d\sin\theta$

Using equation (4.5.18),

path difference $r_2 - r_1 = d\sin\theta$ (4.5.19)

Since S_1 and S_2 are very close to each other, θ (in rad) is very small.

$\therefore\ \sin\theta \approx \theta \approx \tan\theta$

$\therefore\ (r_2 - r_1) = d\tan\theta$ (4.5.20)

From ΔPOA, $\tan\theta = \dfrac{PA}{OA} = \dfrac{x}{D}$

$\therefore\ (r_2 - r_1) = \dfrac{xd}{D}$ (4.5.21)

Using equations (4.5.20) and (4.5.21) in (4.5.13), respectively, we get equation for intensity at point P.

$$I_P = I_0\cos^2\left\{\frac{k\,d\tan\theta}{2}\right\} \quad (4.5.22)$$

and

$$I_P = I_0\cos^2\left\{\frac{k\,xd}{2D}\right\} \quad (4.5.23)$$

Using this equation intensity at any point at a distance x or at an angle θ from point A can be found, which is shown in the figure 4.9. It is evident from equation (4.5.22) or (4.5.23) that intensity at any point does not change with time. This type of interference is known as stationary interference.

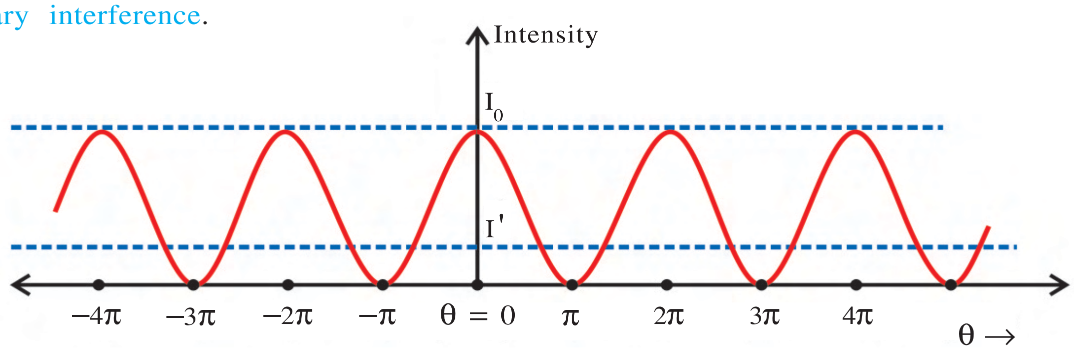


Figure 4.9 Intensity Distribution on the Screen

For the case of $\omega_1 \neq \omega_2$, waves oscillate with different frequencies. Therefore, their phase difference changes continuously. Thus, interference intensity at a point is no longer constant and it will be equal to the sum of average intensity due to both waves. For example, in the case of ordinary electric bulb, electrons transit randomly in the filament, producing waves of various frequencies. Hence, with an ordinary bulb, stationary interference pattern cannot be obtained. Thus, special techniques are required to obtain coherent sources for stationary interference pattern. They are classified into two categories : (i) division of wavefront and (ii) division of amplitude. In first type of method only narrow source is required, while for the latter, an extended source is necessary. We shall study only one method due to Young for obtaining coherent sources by using the method of division of wavefront.

For constructive interference, maximum intensity due to interference of two waves is written as,

$$I = I_0 = 4I'$$
$$= 2^2 I'$$

where $I' = I_1 = I_2$ is the intensity due to individual waves. This equation is a special case of N-source (wave) experiment as $I = N^2 I'$.

Distance Between Two Consecutive Bright Fringes : As shown in the figure 4.8, at point P_m and P_{m+1}, m^{th} and $(m + 1)^{th}$ bright fringes are produced. Using the expression for path difference, $r_2 - r_1 = \frac{xd}{D}$,

Path difference at point P_m is

$$\frac{x_m d}{D} = m\lambda \quad (4.5.24)$$

Similarly at P_{m+1} path difference is

$$\frac{x_{m+1} d}{D} = (m + 1)\lambda \quad (4.5.25)$$

∴ distance between these consecutive bright fringes is,

$$(x_{m+1} - x_m)\frac{d}{D} = \{(m + 1) - m\}\lambda = \lambda \quad (4.5.26)$$

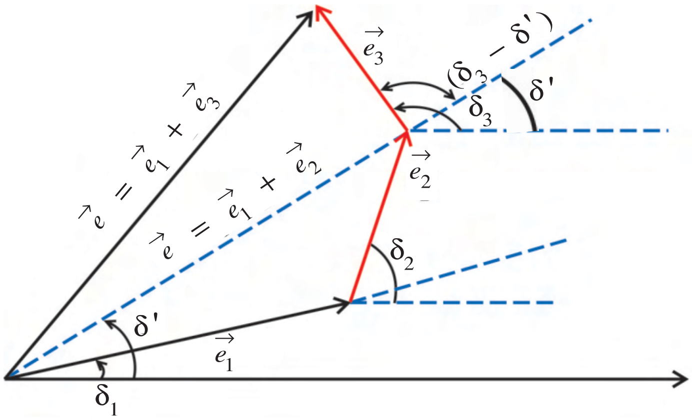
Denoting $x_{m+1} - x_m = \bar{x}$,

$$\bar{x} = \frac{\lambda D}{d} \quad (4.5.27)$$

We can similarly prove that even for two consecutive dark fringes also the distance remains same, i.e. $\bar{x}$.

Further, it can be seen from equation (4.5.27) that the distance between two consecutive bright or dark fringes does not depend on the order of the fringes. That is, all the fringes are of equal width. It is also evident from equation (4.5.27) or (4.5.23) that all bright fringes are equally bright.

Illustration 1 : Using the method of phasor diagram, prove that for constructive interference due to equally intense three waves from coherent sources, the maximum intensity is given by, $I = 3^2 I'$. Here, I' is the maximum intensity of individual waves.



Solution : As shown in the figure, first we add two vectors $\vec{e_1}$ and $\vec{e_2}$, and then $\vec{e_3}$ to their sum, using an equation (4.5.12) for coherent sources resultant intensity due to $\vec{e_1}$ and $\vec{e_2}$ is given by,

$$I_1' = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos(\delta_1 - \delta_2) \quad (1)$$

Resultant intensity due to all waves is,

$$I = I'_1 + I_3 + 2\sqrt{I'_1 I_3}\cos(\delta' - \delta_3) \qquad (2)$$

But for constructive interference, phase difference will be in multiple of $2n\pi$. Therefore, all $\cos\delta$ terms will be unity. Using equations (1) in (2), we get,

$$I = I_1 + I_2 + I_3 + 2\sqrt{I_1 I_2} + 2\sqrt{(I_1 + I_2 + 2\sqrt{I_1 I_2})I_3}$$

But, $I_1 = I_2 = I_3 = I'$ (given),

$$\therefore\ I = I' + I' + I' + 2\sqrt{I'I'} + 2\sqrt{I'I' + I'I' + (2\sqrt{I'I'})\,I'}$$

$$= 5\,I' + 2 \times 2\,I' = 9\,I'$$

$$\therefore\ I = 3^2\,I'$$

4.5 (c) Young's Double Slit Experiment : As early as in 1665, Grimaldi attempted to produce interference using sunlight into a dark room through two pinholes in a screen. Unfortunately, he could see only an average uniform illumination. The reason is now clear, as described in the previous section.

Later in 1801, British physician Thomas Young made a special arrangement to obtain two coherent sources by the method of division of a wavefront. An experimental arrangement of Young's experiment is shown in the figure 4.10.

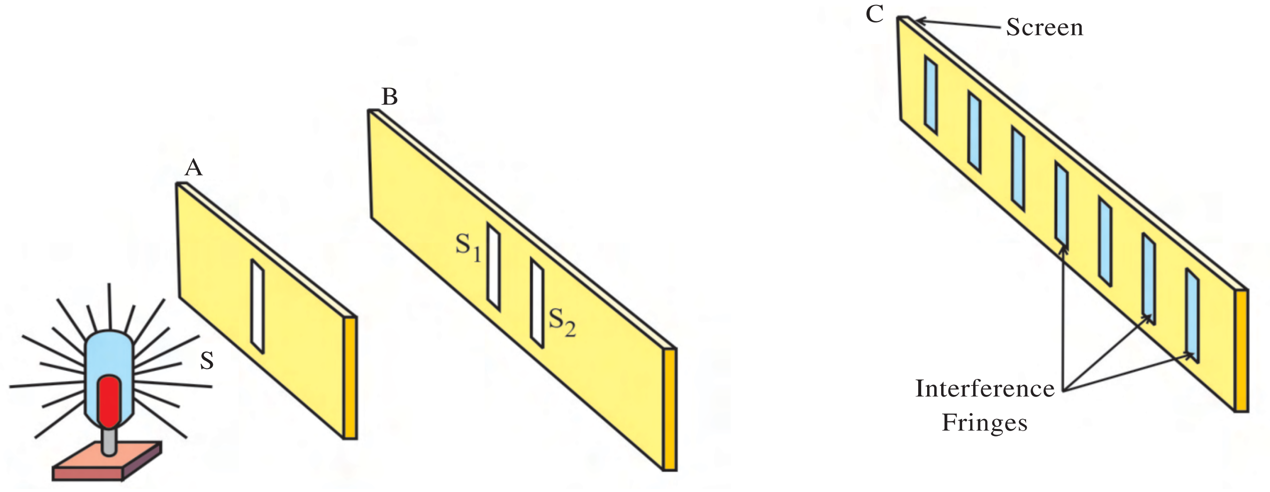


Figure 4.10 Young's Double Slit Experiment

A monochromatic light source emit cylindrical waves, which are collimated by slit S kept nearly on a screen A. Thus, slit now works as a secondary source of light and emit cylindrical waves towards the screen B. Two slits S_1 and S_2 on the screen B are kept such that $SS_1 = SS_2$. Also, distance between S_1 and S_2 is kept small, of the order of millimeter. Since S_1 and S_2 are equidistant from S, at a time only one wavefront is incident on them. According to Huygen's principle all the points on the same wave front vibrate in the same phase so that S_1 and S_2 act as coherent sources.

These cylindrical coherent waves emitted from S_1 and S_2 superpose on a screen C and produce stationary interference.

In the following figure 4.11, the cross section of the slit and the cylindrical wavefronts in a plane of the paper is shown.

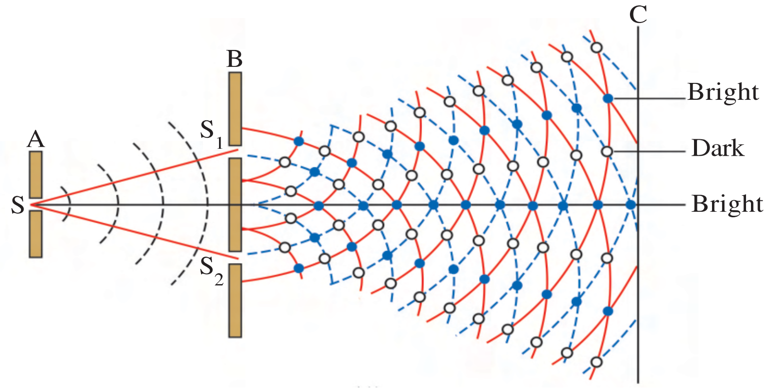


Figure 4.11 Interference Pattern Due to Cylindrical Wavefront. (Only for Information)

Here, points where constructive interference is produced are shown by solid circles, while those with destructive inference are shown by open circles.

Since in figure 4.10, secondary sources S_1 and S_2 are linear, on a screen C dark and bright fringes (bands) are seen.

It is to be noted that in his historical experiment Young had used pinholes in place of slits and white light instead of monochromatic light.

Illustration 2 : The ratio of intensities of rays emitted from two different coherent sources is α. For the interference pattern formed by them, prove that

$$\frac{I_{max}+I_{min}}{I_{max}-I_{min}} = \frac{1+\alpha}{2\sqrt{\alpha}}, \text{ where,}$$

I_{max} = Maximum of intensity in the interference fringes.

I_{min} = Minimum of intensity in the interference fringes.

Solution : For two waves, ratio of their intensities,

$$\frac{I_1}{I_2} = \alpha \text{ (given)}$$

But we know that $I \propto A^2$, where A is an amplitude.

$$\therefore \frac{I_1}{I_2} = \frac{A_1^2}{A_2^2} = \alpha$$

$$\therefore \frac{A_1}{A_2} = \frac{\sqrt{\alpha}}{1}$$

$$\therefore \frac{A_1+A_2}{A_1-A_2} = \frac{A_{max}}{A_{min}} = \frac{\sqrt{\alpha}+1}{\sqrt{\alpha}-1}$$

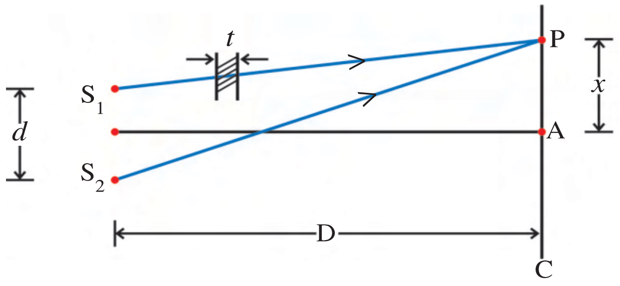
$$\therefore \frac{I_{max}}{I_{min}} = \frac{A_{max}^2}{A_{min}^2} = \frac{(1+\sqrt{\alpha})^2}{(\sqrt{\alpha}-1)^2} = \frac{(1+2\sqrt{\alpha}+\alpha)}{(1-2\sqrt{\alpha}+\alpha)}$$

$$\therefore \frac{I_{max}+I_{min}}{I_{max}-I_{min}} = \frac{(1+2\sqrt{\alpha}+\alpha)+(1-2\sqrt{\alpha}+\alpha)}{(1+2\sqrt{\alpha}+\alpha)-(1-2\sqrt{\alpha}+\alpha)}$$

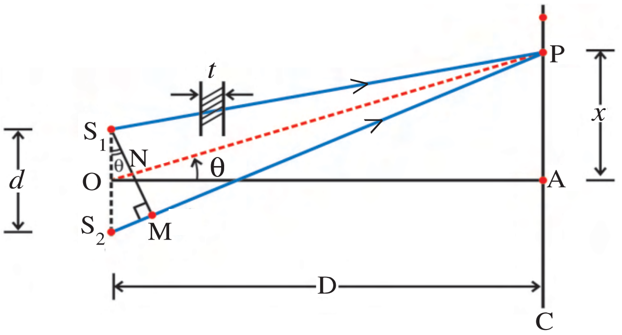
$$= \frac{\alpha+1}{2\sqrt{\alpha}}$$

Reciprocal of the above term, i.e. $\frac{I_{max}-I_{min}}{I_{max}+I_{min}}$ is known as visibility of fringes.

Illustration 3 : Young's double slit experiment is used to determine the thickness of a thin transparent sheet. An experimental arrangement to find the thickness t of transparent material having refractive index n is shown in the figure. Let the central bright fringe, which was obtained at a point A on a screen in absence of the thin sheet shifts to point P. Derive the formula for thickness of the sheet.



Solution : In absence of the sheet, path difference between S_1A and S_2A is zero. Therefore, central bright fringe is located at point A. On introducing transparent sheet in the path of beam from source S_1, the fringes get displaced towards the beam in whose path a sheet is introduced. This is called the lateral shift (x) of fringes.



Now at point P the central bright fringe is obtained. That is, path difference $S_2P - S_1P = 0$

$\therefore \{(S_2P - t) + t_{medium}\} - S_1P = 0$

where t_{medium} = pathlength in a medium (optical path) = $t\ n$

$\therefore \{S_2P - t + t\ n\} - S_1P = 0$

$\therefore$ path difference,

$S_2P - S_1P = S_2M = (n - 1)t$ (1)

From ΔS_1S_2M, $S_2M = d\sin\theta$ (2)

Since two sources S_1 and S_2 are closely placed, θ (in rad) is very small.

$\therefore \sin\theta \approx \theta \approx \tan\theta$

From ΔOAP, $\tan\theta = \frac{x}{D}$ (3)

Using equation (3) into (2),

$S_2M = \frac{xd}{D}$

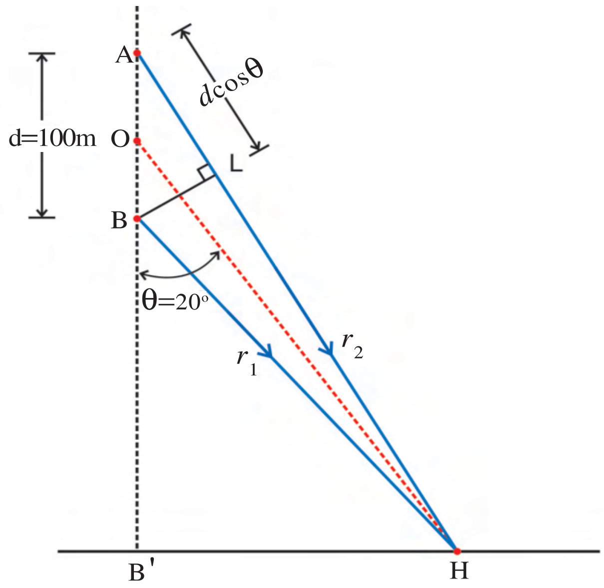
$\therefore$ from equation (4) and (1), $\frac{xd}{D} = (n - 1)t$

$\therefore$ thickness, $t = \frac{xd}{D(n - 1)}$

Illustration 4 : Two radio antennas A and B emit radio waves of frequency 1100 kHz. These waves get superposed at point H. If the distance between two antennas is 100 m and the line joining point H with the midpoint of these antennas makes an angle 20^{o} with the vertical, find resultant intensity in terms of maximum intensity (I_0) at H. Distance BH = 20 km. Take $\cos 20^{o} = 0.9397$, $\cos 62^{o} = 0.4695$.

Solution : Here, antennas A and B are two coherent sources of waves with frequency 1100×10^3 Hz.

$\therefore$ using an equation,



$$I = I_0\cos^2\left\{\frac{k(r_2 - r_1)}{2}\right\}$$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c}$$

$$= \frac{2\pi \times 1100 \times 10^3}{3 \times 10^8}$$

and $(r_2 - r_1) = AL = d\cos\theta = 100 \times \cos 20^o$

$$= 100 \times 0.9397 \times \text{m}$$

$$= 93.97 \text{ m}$$

$$\therefore I = I_0\cos^2\left\{\frac{2\pi 1100 \times 10^3 \times 93.77}{2 \times 3 \times 10^8}\right\}$$

$$\therefore I_0\cos^2\{\pi \times 0.3445\} = I_0\{\cos(62^o)\}^2$$

$$= I_0 \times (0.22)$$

$$\therefore \frac{I}{I_0} = 0.22$$

4.6 Diffraction

When waves encounter obstacles or openings like slits, they bend round the edges. This bending of waves is called diffraction. It was first discovered by Grimaldy. Since this is strictly against the idea of rectilinear propagation of light ray, we conclude that the ray optics cannot explain the phenomenon of diffraction.

To understand the phenomenon of diffraction consider the following day to day experience. We know that light and sound energy both travel in the form of waves. We have experienced that a person standing near an open door in one room may listen to a person standing on the other side of the wall but cannot see him. This implies that sound waves bend near the edge of the door showing the diffraction, but light waves do not! Then the question is why light waves do not diffract ? To explain this apparent paradox between sound waves and light waves, consider an experiment of ripple tank, as shown in the figure 4.12.

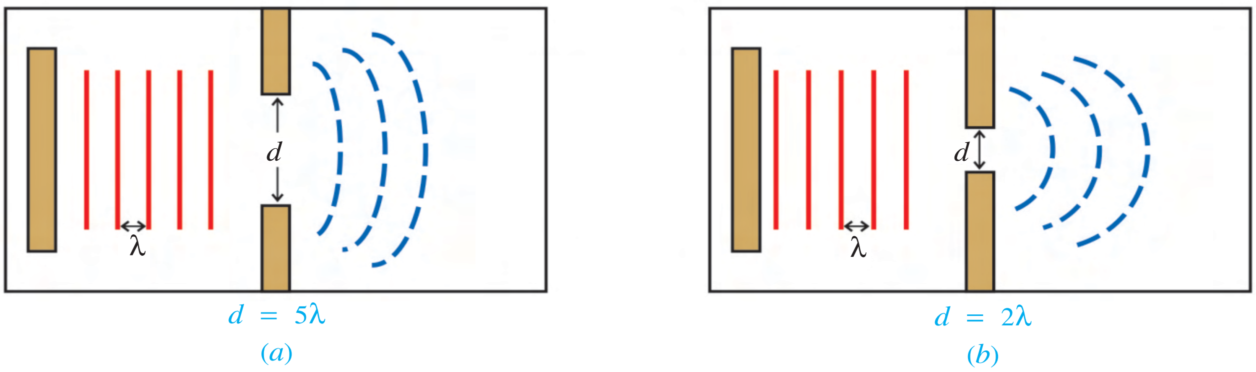


Figure 4.12 Ripple Tank Experiment for Diffraction

In this experiment, linear waves can be produced with the help of straight wooden strip by tapping periodically to the water surface. Near to this, a slit is formed by placing two blocks of wax. In this experiment the width of the slit and wave length of the waves produced can be taken as variable. Let the wave length of waves produced by the controlled oscillations of the stick be λ.

Suppose initially width (d) of the slit is kept as, $d = 5\lambda$. In this situation the waves emerging out of slit are found to be almost linear (see figure 4.12(a)). But when the width of the slit is reduced to $d = 2\lambda$, emerging waves are diffracted by considerable amount (figure 4.12(b)).

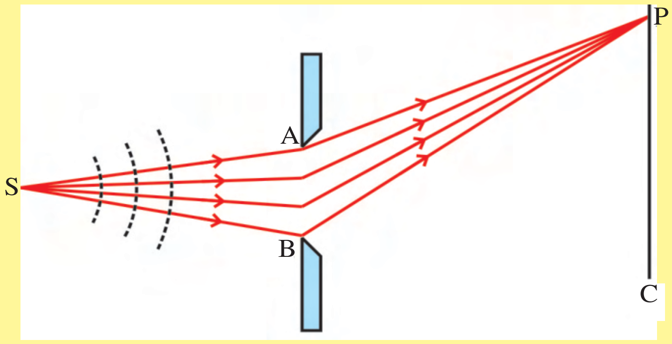
These observations show that smaller is the width of the slit, more will be the diffraction for a given wavelength. It is also found that if the wavelength and the width of the slit are so changed that the ratio $\frac{\lambda}{d}$ remains constant, amount of bending (= diffraction) does not change. Thus, we conclude that diffraction of a wave through a slit depends on the ratio $\frac{\lambda}{d}$. Also, more is the $\frac{\lambda}{d}$ ratio greater is the diffraction.

In the case of day to day life, wavelength of sound waves is typically of the order of 1m. The width of the door is also about 1 m, making the ratio, $\frac{\lambda}{d}$ nearly one. But considering average wavelength of visible portion of electromagnetic spectrum as 6000 $\mathring{A}$, i.e. 6×10^{-7} m, the ratio $\frac{\lambda}{d}$ will be of the order of 10^{-7}. This ratio is too small to produce any appreciable bending of light waves. Hence, in routine life light waves do not appear to diffract. However, if a very narrow slit is used, which increases the ratio $\frac{\lambda}{d}$, appreciable diffraction of light is also possible.

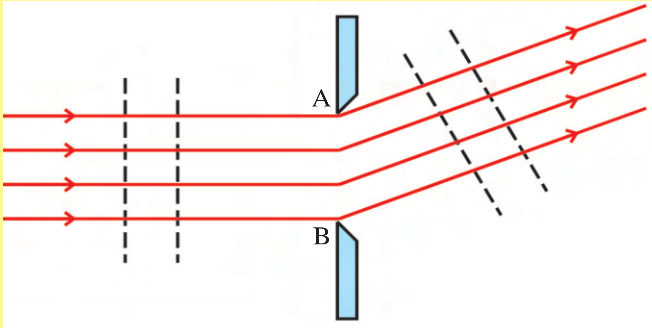
From the above discussion, we infer that in order to keep $\frac{\lambda}{d}$ ratio large for given wavelengths, width (opening) of the slit should be kept small. This requirement suggests that the complete wavefront does not pass through the slit. Slit allows only limited part of wavefront to pass through it. Thus, we say that "diffraction is the effect produced by the limited part of the wavefront."

Types of Diffraction (Only for Information) : According to the type of the wavefronts hindered by the obstacle, diffraction is classified into two types. (1) Fresnel and (2) Fraunhoffer diffraction.

When the distances between the obstacle (slit) AB and the source of light S, as well as between the obstacle AB and screen C are finite the diffraction produced is known as Fresnel diffraction, (refer the figure (a)). In Fresnel diffraction waves are spherical or cylindrical.

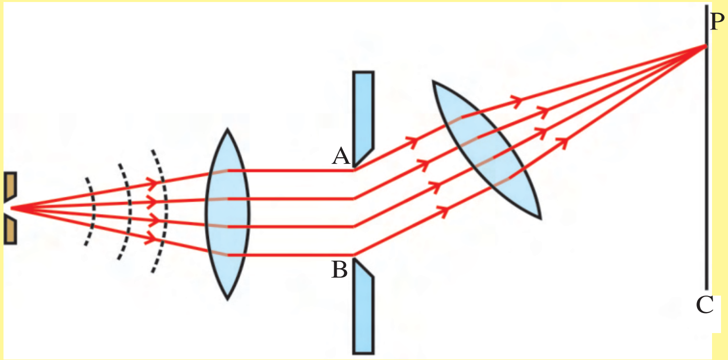


(a) Fresnel Diffraction



(*b*) Fraunhoffer Diffraction

When the light incident on slit AB is coming from infinite distance (or the incident waves are plane) and the distance between the obstacle AB and screen C is also infinite, the diffraction is called Fraunhoffer diffraction, (refer figure (*b*)).



(*c*) Laboratory Arrangement for Fraunhoffer Diffraction

Fraunhoffer diffraction can be obtained in the laboratory with an experimental arrangement as shown in figure (*c*).

Here, the source being at the focus of the convex lens, the rays incident on the slit AB are parallel. While on placing another lens in the passage of set of parallel rays diffracted in different direction, they can be focused at different points on the screen C. Thus in figure (*c*) the conditions of Fraunhoffer diffraction are fulfilled.

4.6 Diffraction Due to Single Slit

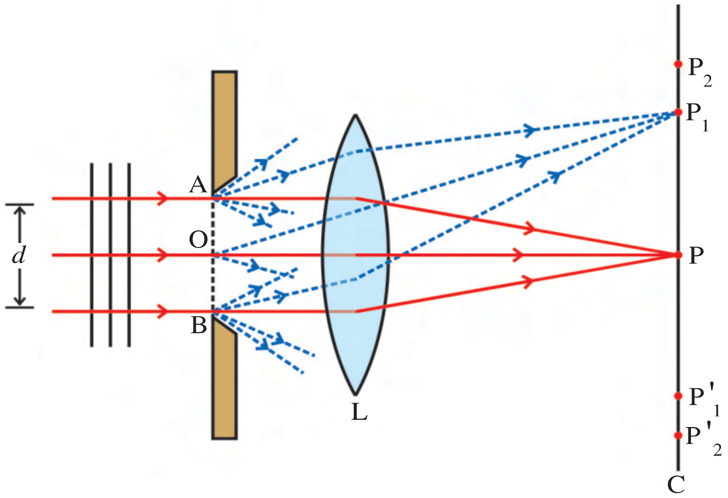


Figure 4.13 Diffraction Due to Single Slit

We now examine the diffraction pattern of plane waves (i.e. Fraunhoffer diffraction) of wavelength λ produced by a single slit of width d (See figure 4.13). When such plane wavefront arrive at a plane of slit, according to Huygen's principle, all points on the slit (like A, O, B) act as secondary sources having the same phase, and produce secondary waves. In order to produce a diffraction pattern of bright and dark fringes (i.e. interference maxima and minima) on the screen (C), converging lens (L) is often used.

Thus, now diffracted waves are converged on to the screen and produce interference pattern. Therefore, we can now use a procedure similar to the one we need to locate the fringes in Young's double slit experiment.

(1) Central Maximum : As shown in figure 4.14 (*a*), point P_0 of a screen C is lying on a perpendicular bisector of slit AB. Therefore, those waves originated from each points of a slit and diffracted normal to the plane of the slit (i.e, in the direction of incident waves, $\theta = 0$) will be all concentrated at point P_0 by a lens L. In figure 4.14 (*a*), out of many such waves only three representative rays are shown. Here, screen is at the focal plane of the lens. It is obvious from the figure that rays travelling less distance in air have to travel more distance through the lens. Since speed of waves in lens is less than their speed in air, their optical path will be equal. (Optical distance in a medium is equal to the product of refractive index of the medium to geometrical path length in air). Thus, all rays reaching to point P_0 having equal phase produce constructive interference, and point P_0 will be having maximum intensity. Point P_0 is known as Central Maximum.

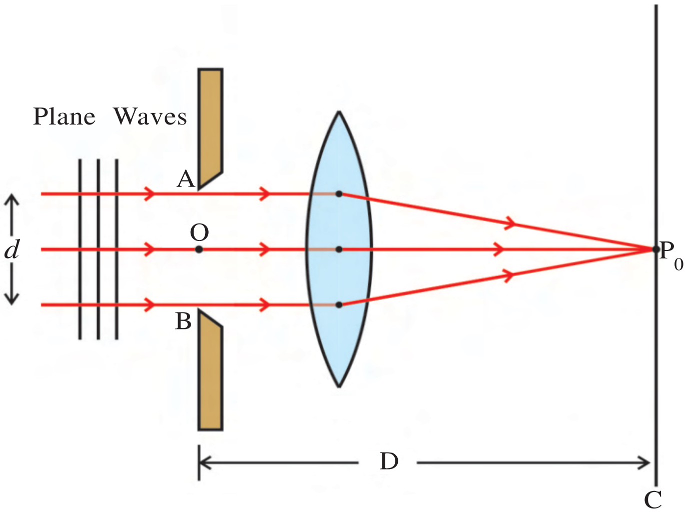


Figure 4.14 (*a*) Central Maximum

Only for Information : In laboratory experiment, lens (L) used to produce Fraunhoffer differaction decides the width of the central maximum. But for lens-less diffraction by keeping screen at infinite distance ($d \ll D$), width of central maximum is roughly equal to the width of the slit (d).

For analysis of diffraction pattern (i.e. to know the intensity distribution and location of interference fringes) mathematical treatment is so complex (which is given at the end of the chapter as an appendix for information) that we will give only logical proof.

(2) First Minimum : As shown in figure 4.14 (*b*), consider waves which are diffracted at an angle θ with respect to perpendicular bisector XP_0 of the slit. Here, point X is the midpoint of slit AB. Therefore, $AX = XB = \frac{d}{2}$. Here, secondary waves originated from all points A, X, B of slit are thought to be divided in two parts : waves from A to X and waves

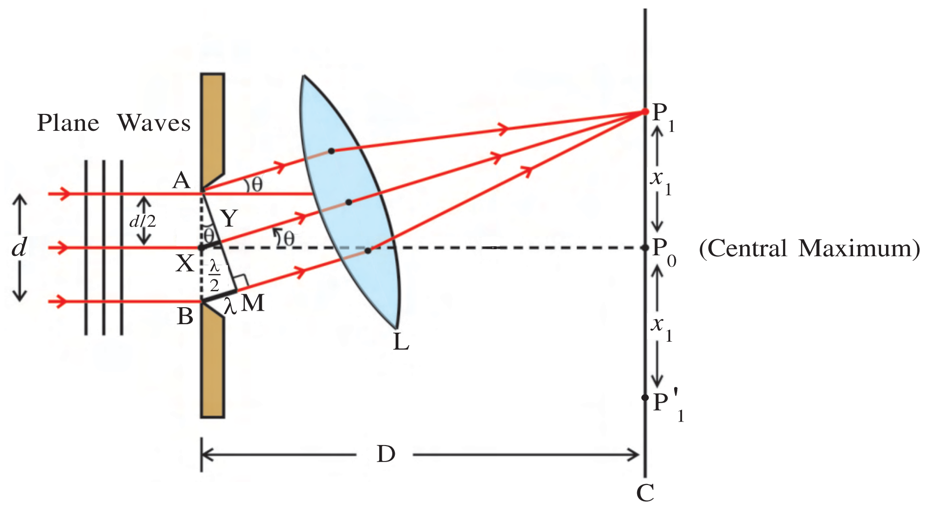


Figure 4.14 (*b*) First Minimum

from X to B. As per figure, all these waves diffracted at an angle θ are focused at point P_1 of a screen. To know whether constructive or distructive interference will take place at point P_1, we require to know phase difference between these waves. For that, draw AM $\perp$ BL. It is obvious that all the waves reaching from AM to P_1 have equal optical path.

But rays going from A and X, and reaching to P_1 have path difference of XY.

Let us assume that diffracted angle θ is such that $XY = \frac{\lambda}{2}$.

In this situation, waves from A and X will follow the condition of distructive interference at point P_1. And their resultant intensity will be zero.

Further, as corresponding to point A we have point X for which condition for distructive interference holds, like wise, corresponding to every point of part AX, we have successive points in section XB such that for every such pair, path difference at point P_1 is $\frac{\lambda}{2}$.

Thus, in totality, destructive interference will take place at point P_1 and it will be dark.

Point P_1 is known as First Minimum. From the symmetry of the figure it is obvious that at the same distance from P_0 on other side also we have first minimum (P_1').

(3) First Maximum : As shown in the figure 4.14 (*c*), suppose slit AB is assumed to be divided in three equal (odd number) parts AX_1, X_1X_2 and X_2B.

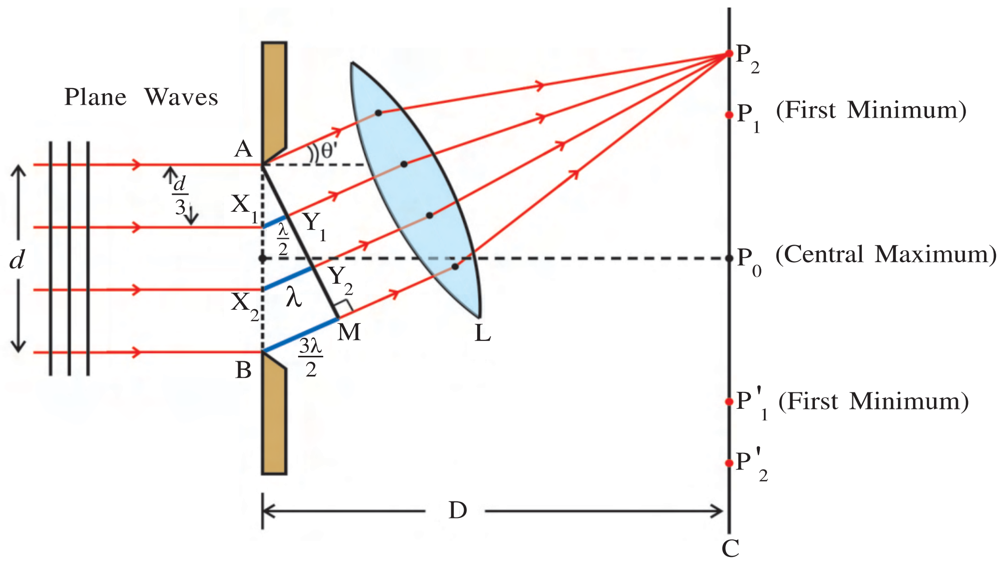


Figure 4.14 (*c*) First Maximum

Here, $AX_1 = X_1X_2 = X_2B = \frac{d}{3}$. As per figure, draw AM $\perp$ BL. Waves reaching from AM to P_2 will have equal optical path.

Waves starting from A and X_1 and imposing at point P_1 will have path difference X_1Y_1.

Let us assume that diffracted angle θ' is such that $X_1Y_1 = \frac{\lambda}{2}$, $X_2Y_2 = \lambda$, $BM = \frac{3\lambda}{2}$.

Since path difference between waves originated from A and X_1, and superimpose at point P_2 is $\frac{\lambda}{2}$, they interfere destructively. And intensity at point P_2 due to these waves will be zero.

In the same way, waves from every pair AX_1 and X_1X_2 will have path difference $\frac{\lambda}{2}$. And as explained above, resultant intensity at point P_2 due to them will be zero.

However, intensity of rays differacted at an angle θ' from section X_2B is not vanishing at point P_2. Therefore, due to this section of the slit, there remains some intensity at point P_2, and point P_2 will be bright.

Here, point P_2 is known as first maximum. It is obvious that the intensity at point P_2 is verymuch less as compared to P_0.

Of course, to know locations of higher order minima and maxima, and intensities of maxima relative to central maximum, above mentioned of logical method is not useful.

Intensity of diffracted light at any point on the screen (C) is given by the following formula (see information given in the appendix).

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \quad (4.6.1)$$

where I_0 is maximum intensity at point P_0 and

$$\alpha = \frac{\pi d \sin\theta}{\lambda} \quad (4.6.2)$$

Condition for Central Maximum : It is clear from the figure 4.13 that secondary waves from slit for which $\theta \approx 0$ (without undergoing diffraction) will meet at point P_0 on the screen, C. From equation (4.6.2), as $\theta \rightarrow 0$, $\alpha \rightarrow 0$.

Therefore, according to equation (4.6.2),

$$\text{Intensity} \quad I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 = I_0 \qquad \left(\therefore \lim_{\alpha\to 0} \frac{\sin\alpha}{\alpha} = 1\right)$$

Thus, point P_0 will be bright, which we call the central maximum. On either side of it, at equal separation, we can observe successive minima and maxima.

Condition for Minima : If $\alpha = n\pi$; $n = 1, 2, 3, ...$, according to equation (4.6.1), we get successive minima for different values of n. From equation (4.6.2),

$$\frac{\pi d \sin\theta}{\lambda} = n\pi$$

$$\therefore \ d\sin\theta = n\lambda \quad (4.6.3)$$

Equation (4.6.3) gives the condition for minima. For $n = 1$ we get first minimum (point P_1), for $n = 2$ we get second order minimum (point P_3), etc. Due to symmetry on the other side of point P_0 corresponding minima (P_1', P_3',....) are also obtained.

Condition for Maxima : If $\alpha = (2n + 1)\frac{\pi}{2}$, $n = 1, 2, 3, ...$, according to equation (4.6.1), we get successive maxima for different values of n. From equation (4.6.2),

$$\frac{\pi d \sin\theta}{\lambda} = (2n + 1)\frac{\pi}{2}$$

$$\therefore \ d\sin\theta = (2n + 1)\frac{\lambda}{2} \quad (4.6.4)$$

Equation stated above gives the condition for maxima. For $n = 1$, we get first order maxima (points P_2 and P'_2), for $n = 2$ we get second maxima (points P_4 and P'_4), etc.

(1) For first order maximum (i.e. $n = 1$)

$$\alpha = (2 \times 1 + 1)\frac{\pi}{2} = \frac{3\pi}{2}$$

$$\therefore \ I = I_0\left(\frac{\sin\left(\frac{3\pi}{2}\right)}{\frac{3\pi}{2}}\right)^2 = I_0\left(\frac{-1}{\frac{3\pi}{2}}\right)^2 = \frac{4I_0}{9\pi^2} \approx \frac{I_0}{22}$$

(2) For second order maximum (i.e. $n = 2$)

$$\alpha = \frac{5\pi}{2} \Rightarrow I = I_0\left(\frac{\sin\left(\frac{5\pi}{2}\right)}{\frac{5\pi}{2}}\right)^2 = \frac{4I_0}{25\pi^2} \approx \frac{I_0}{62}$$

Thus, the intensity of maxima decreases rapidly with the order of maxima.

Further, from equation (4.6.2), $\frac{\pi d \sin\theta}{\lambda} = \alpha$, for a given order of maxima or minima (i.e. value of α is fixed) and fixed wavelength, $\sin\theta \propto \frac{1}{d}$. This suggests that the smaller the width of slit, the larger will be θ. From figures 4.14, then points P_1, P_2, ..., etc. will be found at larger angular separation. Thus, the diffraction pattern will spread/expand on the screen. However, intensity of diffraction maxima decreases in proportion to decrease in the width of a slit. To illustrate this point, graphs of intensity versus θ for two cases, $d = 5\lambda$ and $d = 10\lambda$, are shown in the figure 4.15.

Width of Central Maximum : "The distance between two first order minimum is known as width of central maximum." As per figure 4.14(b), width of the central maximum is $2x_1$.

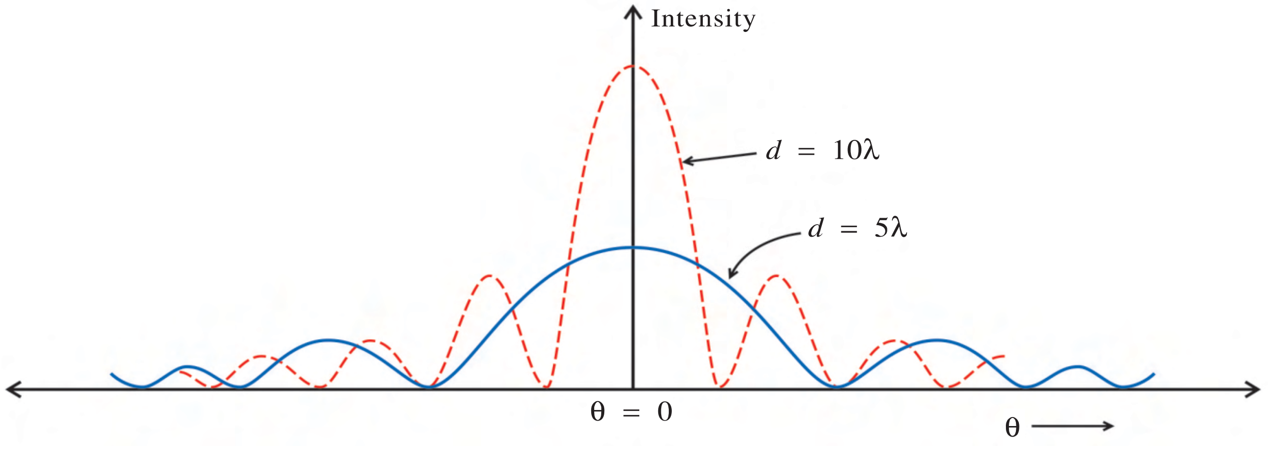


Figure 4.15 Intensity Distribution Due to Single Slit Diffraction

For first order minimum, $d\sin\theta = \lambda$ or $\sin\theta = \frac{\lambda}{d}$ (4.6.5)

Also, from the figure 4.14 (*b*), $\tan\theta = \frac{x_1}{D}$ (4.6.6)

But for small angle of diffraction θ (in rad) is small. Therefore, $\sin\theta \approx \tan\theta$. From equation (4.6.5) and (4.6.6)

$$\frac{x_1}{D} = \frac{\lambda}{d}$$

$\therefore$ Width of central maximum, $2x_1 = \frac{2\lambda D}{d}$

Angular width of central maximum is given by

$2\theta = \frac{2\lambda}{d}$ (See equation (4.6.5)).

In the case of optical instruments such as telescope or microscope, objective lens acts as a circular obstacle to the incoming wavefronts, and produces diffraction. In such diffraction pattern, due to circular aperture there is a central circular bright fringe, which is called the Airy's disc. It is surrounded by alternate dark and bright concentric rings called Airy's rings.

For Franuhoffer diffraction, the width of central maximum is the measure of the deviation. If the width of the beam is more than linear measure the obstacle (width in the case of slit and diameter of an objective for optical instruments), light will deviate more. If the width of the beam is either nearly equal to or smaller than the obstacle, it will be travelling straight. In this situation ray optics can be used. Thus, we can define a length, called Fresnel distance (Z_f) such that $Z_f = \frac{d^2}{\lambda}$, where d is the linear measure of the obstacle and λ is the wavelength of light. It defines the distance upto which bending is very less, and ray optics is applicable. However, one should not use this cirterion as a condition for using ray optics.

4.7 Comparison between Interference and Diffraction

In common, the patterns (fringes) obtained in both interference and diffraction are due to superposition of waves. Fundamentally, there are some differences between interference and diffraction, as given below

Interference	Diffraction
(1) It is obtained due to superposition of waves from different coherent sources. That is, it is the effect produced due to superposition of different wavefronts.	(1) It is obtained due to superposition of waves originated from the different parts of the same wavefront.
(2) Bright and dark all interference fringes are of equal width.	(2) Diffraction fringes are not of the same width. Central maximum is having the largest width, while width of maxima and minima decreases for higher order of diffraction.
(3) All bright fringes have equal intensities.	(3) Central maximum has highest intensity, and it decreases with higher order diffraction maxima.
(4) Interference dark bands are perfectly dark.	(4) Regions of minimum intensities may not be perfectly dark.

Illustration 5 : Angular width of a central maximum in a Franhoffer diffraction obtained by a single slit using light of wavelength 6000 Å is measured. If light of another wavelength is used, the angular width of the central maximum is found to be decreased by 30%. Find (i) the other wavelength (ii) If the experiment is repeated keeping the apparatus in a liquid, the angular width of central maximum decreases by the same amount (i.e. 30%), find its refractive index.

Solution : Angular width of central maximum is given by (1)

$$2\theta = \frac{2\lambda}{d} \Rightarrow \theta = \frac{\lambda}{d}$$

For first light, $\theta_1 = \frac{\lambda_1}{d}$ and for second light, $\theta_2 = \frac{\lambda_2}{d}$

$$\therefore \frac{\theta_2}{\theta_1} = \frac{\lambda_2}{\lambda_1} \qquad (2)$$

But θ_2 is 30% less that of θ_1

That is, θ_2 = 70% of $\theta_2 = 0.7\ \theta_1$

Using in equation (2) $\frac{\lambda_2}{\lambda_1} = 0.7$

$\therefore \lambda_2 = 0.7 \times 6000\ \text{Å} = 4200\ \text{Å}$

That is, wavelength in a liquid is 4200 Å.

$$n = \frac{\lambda_{air}}{\lambda_{liquid}} = \frac{6000}{4200} = 1.43.$$

Illustration 6 : Obtain the necessary condition to observe maxima in the case of Fraunhoffer diffraction in term of $\alpha\left(= \frac{\pi d \sin\theta}{\lambda}\right)$.

Solution : In the case of Fraunhoffer diffraction, intensity at a point is given by,

$$I = I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right) \qquad (1)$$

If at any point of maxima takes place, $\frac{dI}{d\alpha} = 0$

Using equation (1),

$$\frac{dI}{d\alpha} = I_0\left\{\frac{2\sin\alpha\cos\alpha}{\alpha^2} - \frac{2\sin^2\alpha}{\alpha^3}\right\} = 0$$

(Condition for maxima require $\frac{dI}{d\alpha} = 0$)

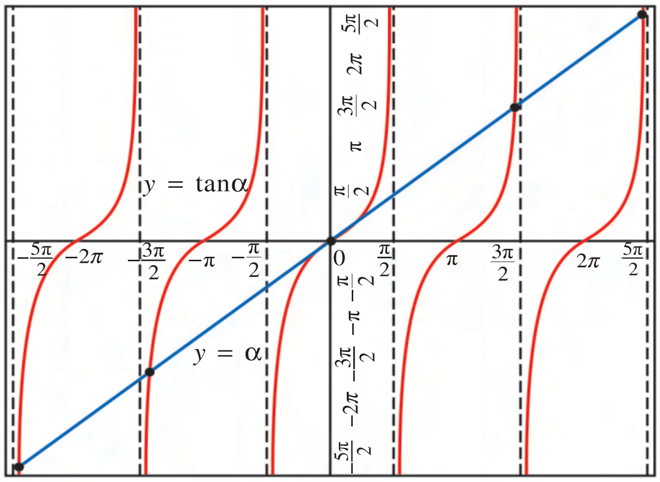
$$\therefore \frac{2\sin\alpha\cos\alpha}{\alpha^2} = \frac{2\sin^2}{\alpha^3}$$

$$\therefore \tan\alpha = \alpha \qquad (2)$$

Equation (2) gives the necessary condition for maxima to take place.

Only for Information : To find the value of α from equation (2) for which diffraction maxima occur, graph of $y = \tan\alpha$ and $y = \alpha$ are plotted. Intersections of these graphs give value of α (in rad) for maxima.

It also explains that why we had not considered $\alpha = \frac{\pi}{2}$ value in condition for maxima.



4.8 Resolving Power of Optical Instruments

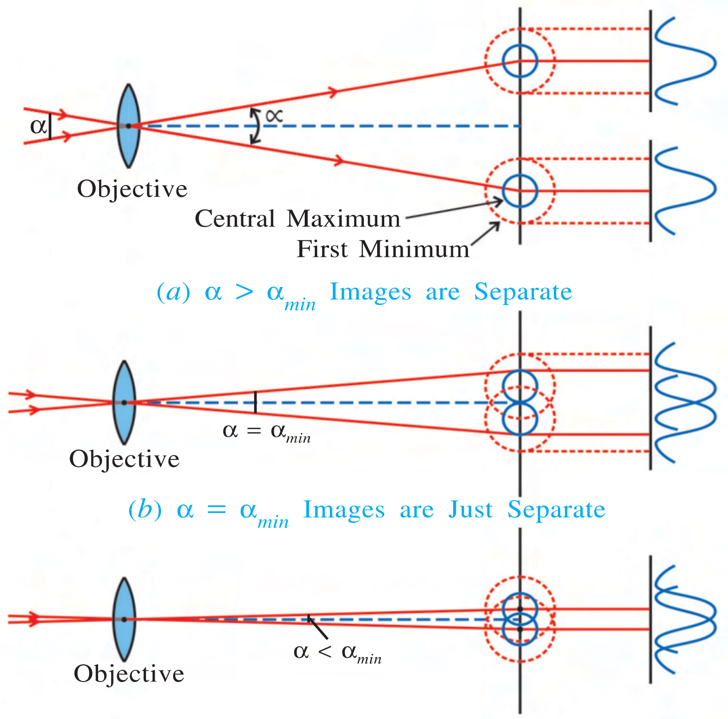
As studied in the previous semester, optical instruments are used to see an object clearly and comfortably. But when two objects or their images are very close to each other, they may appear as one. And it may not be possible for the eyes to see them as separate. Even optical instruments such as telescope or microscope used to see object have limitations in resolving two nearby objects on their images due to diffraction phenomenon. In this section, we will study resolving power of optical telescope and microscope.

Rayleigh's Criterion : When a beam of light (light waves) from a point object passes through the objective of an optical instruments, the lens acts like a circular aperture and produces a diffraction pattern (Airy's disc and Airy's rings) instead of sharp point image. If there are two point objects kept close to each other, their diffraction pattern may overlap. Then it may be difficult to distinguish them as separate. The criterion to get distinct and separate images of two closely placed point like objects was given by Rayleigh.

"The images of two point like objects can be seen as separate if the central maximum in the diffraction pattern of one falls either on the first minimum of the diffraction pattern of the other or it is at a grater separation."

For the case of circular aperture diffraction due to lens of diameter D, Rayleigh's criterion is given by, $\sin\theta \approx \theta = \frac{1.22\lambda}{D}$. Here, λ is wavelength of light.

4.8 (*a*) Resolving Power of Telescope : Suppose we are observing two nearby stars with the help of a telescope. The ray coming from these stars make an angle α at the lens of the telescope as shown in the figure 4.16. Since only limited parts of incident wavefronts can pass through the lens, lens acts as an obstacle, and produces diffraction. An image of stars appear as two central bright spots surrounded by alternate dark and bright rings of decreasing intensity as we go away from the central bright spots. From the figure 4.16 (*a*), it is obvious that, if angle α is large, the diffraction pattern will be quite distinct. Hence, the images of the stars will be seen as separate.



(*a*) $\alpha > \alpha_{min}$ Images are Separate

(*b*) $\alpha = \alpha_{min}$ Images are Just Separate

(*c*) $\alpha < \alpha_{min}$ Images Overlap

Figure 4.16 Resolution of Images

But if two stars are close to each other (figure 4.16. (*b*) and (*c*)) angle α will be very small and the diffraction pattern of both stars may mingle with each other. In this situation it is difficult to see both the stars distinctly and clearly.

"The ability of an optical instrument to produce distinctly separate images of two closely placed objects is called its resolving power (R.P.)"

It is clear from the above discussion for optical instruments like telescope and microscope that R.P. depends on an angle α. If diameter of an objective of telescope is D and its focal length is *f*, then the width of central maximum obtained by it is given by $f\left(\frac{1.22\lambda}{D}\right)$. Here, λ is the wavelength of incident light. Width of central maximum on screen = $f\alpha$.

∴ The necessary minimum angle to see two images distinctly (α_{min}) is, $f\alpha_{min} = f\left(\frac{1.22\lambda}{D}\right)$

$$\therefore \alpha_{min} = \frac{1.22\lambda}{D} \qquad (4.8.1)$$

Here, α_{min} is known as angular resolution of the telescope, while its inverse is known as resolving power or geometrical resolution.

$$\text{Thus, R.P. of telescope,} = \frac{1}{\alpha_{min}} = \frac{D}{1.22\lambda} \qquad (4.8.2)$$

Since R.P. of telescope is directly proportional to the diameter of its objective, telescopes with large objective lens are used to see very far closely placed celestial objects.

For example, angular resolution of Hubble telescope is 0.1" (0.1 second), while angular resolution of human eye is approximately 1' – 2' (1 to 2 minutes).

4.8 (*b*) Resolving Power of Microscope : Image of a point like object by an objective of microscope is shown in figure 4.17. Let diameter of the lens be D and its focal length be f.

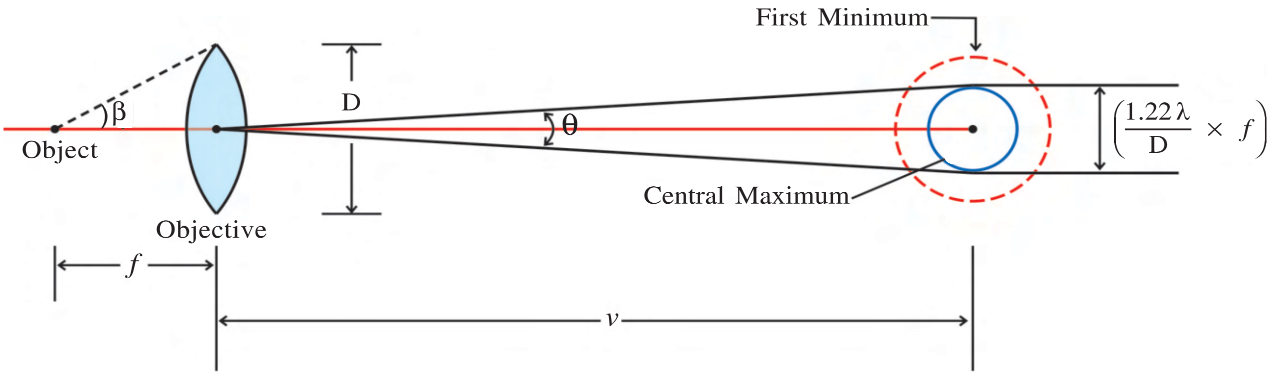


Figure 4.17 Image Formation by Microscope

As object distance is usually kept greater than that of f (remember theory of compound microscope of previous semester). Let an image distance be v. The angular width of central maximum due to the effect of diffraction is,

$$\theta = \frac{1.22\lambda}{D}$$

$$\therefore \text{ width of central maximum, } v\theta = \left(\frac{1.22\lambda}{D}\right)v \qquad (4.8.3)$$

If image of two point like objects are at a separation less than $v\theta$, then it will be seen as a mixed single object. It can be proved that a minimum distance (d_m) for which objects can be seen separately is given by,

$$d_m = \left(\frac{1.22\lambda}{D}\right)\frac{v}{m} \qquad (4.8.4)$$

When m = $\frac{v}{f}$ magnification. Substituting value of m in above equation,

$$d_m = \left(\frac{1.22\lambda}{D}\right)f \qquad (4.8.5)$$

From the figure 4.17, $\frac{\left(\frac{D}{2}\right)}{f} = \tan\beta$

$\therefore \frac{D}{f} = 2\tan\beta$. Using this in equation (4.8.5),

$$d_m = \left(\frac{1.22\lambda}{2\tan\beta}\right) \qquad (4.8.6)$$

For small angle β (in rad), $\tan\beta \approx \sin\beta$

$$\therefore d_m = \left(\frac{1.22\lambda}{2\sin\beta}\right) \qquad (4.8.7)$$

Reciprocal of d_m known as R.P. of microscope. That is,

$$\text{R.P. of microscope} = \frac{1}{d_m} = \left(\frac{2\sin\beta}{1.22\lambda}\right) \qquad (4.8.8)$$

Equation (4.8.8) is derived for air as medium between an object and objective lens. Instead, some medium with large refractive index (n) may be used between object and objective to increase the R.P. of microscope. In this situation, R.P. of microscope is given by $\left(\frac{2n\sin\beta}{1.22\lambda}\right)$. Here, the term $n\sin\beta$, is known as Numerical Aperture. Normally, appropriate type of oil immersion is used to increase the resolution. It is also true that R.P. of microscope is inversely proportional to wavelength λ.

Illustration 7 : In the following two cases upto what minimum distance two point like objects can be seen distinctly by a human eye ? (1) Distance between eye and objects is 25 cm and (2) Distance between eye and object is 5 m. Diameter of pupil of eye is 2.5 mm. Consider wavelength of light 5500 $\mathring{A}$.

Solution : Considering an eye as a simple microscope $d_{min} = \frac{1.22\lambda f}{D}$.

Here, f is the focal length of human eye. Remember that ciliary muscle of eye sets the focal length of the lens to the object distance.

(1) $d_{min} = \frac{1.22 \times 5500 \times 10^{-10} \times 0.25}{2.5 \times 10^{-3}} = 6.71 \times 10^{-5}$m

(2) $d_{min} = \frac{1.22 \times 5500 \times 10^{-10} \times 5}{2.5 \times 10^{-3}} = 1.34 \times 10^{-3}$m

Illustration 8 : Huble space telescope is at a distance 600 km from earth's surface. Diameter of its primary lens (objective) is 2.4 m. When a light of 550 nm is used by this telescope, at what minimum angular distance two objects can be seen separately ? Also obtain linear minimum distance between these objects. Consider these objects on the surface of earth and neglect effects of atmosphere.

Solution : $\alpha_{min} = \frac{1.22\lambda}{D} = \frac{1.22 \times 550 \times 10^{-9}}{2.4}$

$= 2.8 \times 10^{-7}$ rad

$= 0.058''$ ($\because$ $1'' = 4.85 \times 10^{-6}$ rad)

Linear distance between objects = α_{min}L,

where L = distance between telescopes and objects.

$\therefore$ linear distance between objects = $2.8 \times 10^{-7} \times 600 \times 10^{3}$

= 0.17 m

Illustration 9 : Calculate the useful magnifying power of a telescope of 11 cm objective. The limit of angular resolution of eye is $2'$ and wavelength of light used is 5500 $\mathring{A}$.

Solution : The magnifying power of a telescope is given by,

$M = \frac{D}{d}$, where D = diameter of the objective

d = diameter of the eyepiece.

For normal (useful) magnification, diameter of eyepiece should be equal to the diameter of the pupil (d_e) of the eye. Therefore, useful magnification is

$$M = \frac{D}{d_e} \quad (1)$$

From the equation of limit of resolution of telescope.

$$d\theta = \frac{1.22\lambda}{D}$$

$$= \frac{1.22 \times 5500 \times 10^{-10}}{11 \times 10^{-2}} = 6.1 \times 10^{-6} \text{ rad}$$

Limit of angular resolution of eye ($d\theta'$) is given as $2'$.

$$\therefore \ d\theta' = \frac{2 \times 3.14}{60 \times 180^{o}} = 5.815 \times 10^{-4} \text{ rad}$$

$$\therefore \ \text{Useful magnification } M = \frac{d\theta'}{d\theta} = \frac{5.815 \times 10^{-4}}{6.1 \times 10^{-6}}$$

$$= 95.3$$

4.9 Polarization

Interference and diffraction phenomenon have manifested wave nature of light. In fact, these both effects, are observed for any kind of waves whether longitudinal or transverse. In the previous chapter we have studied that light (visible part of electromagnetic spectrum) is transverse waves. Its transverse character can be experimentally verified through the polarization phenomenon. In the case of longitudinal waves, particles of the medium oscillate in the direction of propagation only. On the other hand, in transverse waves vibration of particles or field vectors are possible in all directions perpendicular to the direction of propagation. In a sense, transverse waves enjoy preference in oscillations perpendicular to wave propagation. Due to this preferential character of particle or field oscillation, we may define the concept of polarization, which gives information about the state of oscillations of particles or field vectors.

4.9 (*a*) Unpolarized and Plane Polarized Light : To consider the polarization phenomenon see the following figure 4.18.

Suppose an atom or molecule is at point O and emitting electromagnetic wave as shown in the figure. It can be seen that the directions of $\vec{E}$, $\vec{B}$ and the propagation of waves are mutually perpendicular. In an ordinary light source like bulb, there are large number of such atomic emitters. They all emit electromagnetic waves with their $\vec{E}$ vectors (also called light vectors) vibrating randomly in all directions perpendicular to wave propagation. It means that $\vec{E}$ of one wave is not parallel to $\vec{E}$ of another wave. (Again we consider only $\vec{E}$ vectors for further discussion.) Also, the waves emitted by different atoms of a source and propagating in the same direction form a beam of light. If such beam of light is assumed to be coming out of the plane of the paper, light vectors of its waves will be found in all random direction in a plane of paper. Such light is called Unpolarized Light.

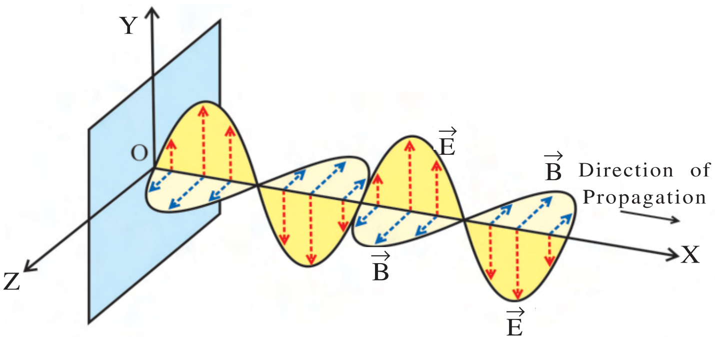


Figure 4.18 Propagation of Light

Such unpolarized light is schematically represented in figure 4.19 (*a*) and (*b*). For simplicity, we may resolve any light vector of unpolarized light into two perpendicular components (as shown in figure 4.19(*c*)) to the direction of propagation. However, we must remember that each of the wave in unpolarized beam of light is independently polarized.

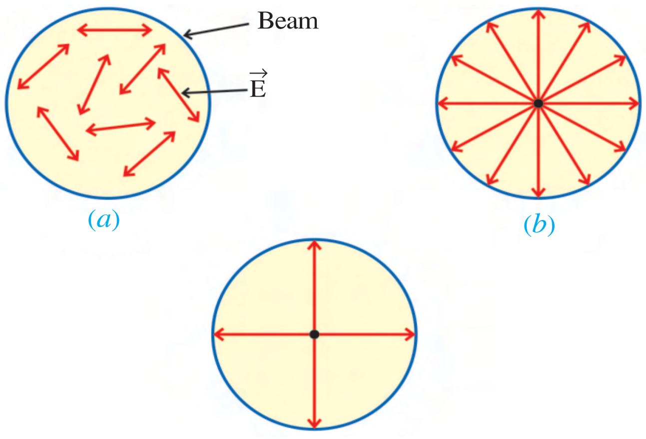


Figure 4.19 Unpolarized Light

"In a beam of light, if the oscillations of $\vec{E}$ vectors are in all directions in a plane perpendicular to the direction of propagation, then the light is called unpolarized light."

In 1815, Biot discovered that certain mineral crystals (like tourmaline) absorbs light selectively. This is called Selective Absorption or Dichroism. When light passes through tourmaline crystal freely transmit the light components which are polarized to a definite direction. While crystal absorbs light strongly whose polarization is perpendicular to this definite direction. This definite direction in a crystal is known as an optic axis. If the crystal is cut in proper size (1 to 2 mm thick) perpendicular components is totally absorbed (see figure 4.20). Hence, in the light emerging out of the tourmaline plate, which are parallel to the optic axis. Thus, emergent beam of light only coplanar and parallel $\vec{E}$ vectors are found. Such light is known as polarized light. Thus, tourmaline crystal is a natural polarizer or Polaroid.

"The beam of light in which light vectors are coplanar and parallel to each other is plane polarized or linearly polarized light."

The process by which getting the plane polarized light from unpolarized light is called polarization.

"The plane containing the direction of the beam and the direction of oscillation of $\vec{E}$ vectors is called the plane of oscillation (vibration)." In the figure 4.20, *abcd* is the plane of oscillation.

"A plane perpendicular to the plane of oscillation and passing through the beam of light is called the plane of polarization."

In above figure 4.20 $a'b'c'd'$ is the plane of polarization.

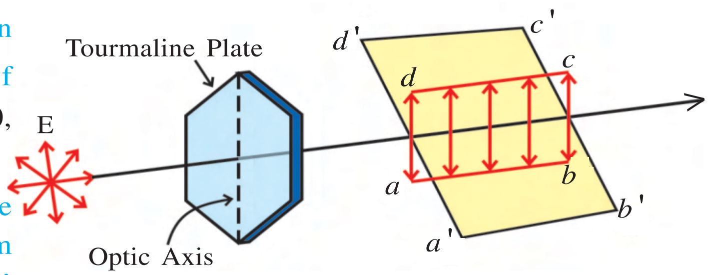


Figure 4.20 Polarization through Tourmaline Plate

4.9 (*b*) Malus' Law : The confirmation that the tourmaline acts as a polarizer can be checked as follows. Since tourmaline plate absorbs perpendicular components of $\vec{E}$ vectors, the intensity of emerging light is less than that of the incident unpolarized light. When tourmaline plate is rotated with an incident beam as an axis, intensity of emerging polarized light remains the same. This observation shows that in unpolarized light, in all directions in a plane perpendicular to the direction of propagation light vectors are uniformly distributed.

Now to analyze polarized light, another tourmaline plate B is arranged parallel to the plate A, as shown in the figure 4.21.

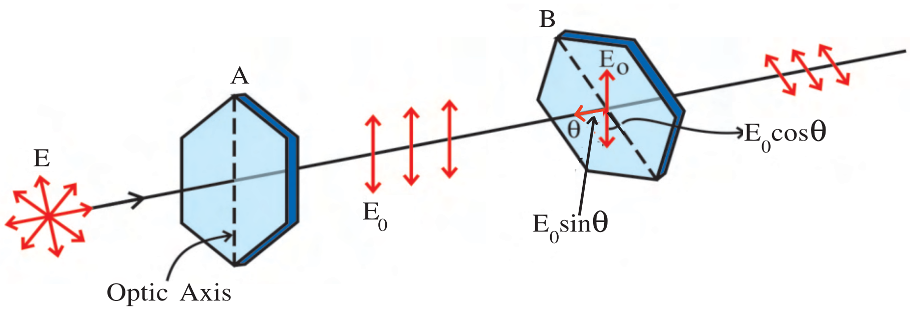


Figure 4.21 Polarized and Analyzer

An optic axis of plate B makes an angle θ with that of the plate A. In this situation, $\vec{E}$ vectors emerging from plate A (E_0) makes an angle θ with an optic axis of plate B. Therefore, we can resolve them into two components.

(1) $E_0\cos\theta$ parallel to the optic axis of plate B, and

(2) $E_0\sin\theta$ perpendicular to the optic axis of plate B.

Thus, only $E_0\cos\theta$ components will emerge out of plate B, while perpendicular components are absorbed. Since intensity is proportional to the square of amplitude, intensity of light incident on plate B is $I_0 \propto E^2_0\cos^2\theta$.

$$\therefore \frac{I}{I_0} = \cos^2\theta$$

or

$$\therefore I = I_0\cos^2\theta \qquad (4.9.1)$$

Equation (4.9.1) is known as Malus Law. It is obvious from above equation that if plate B is completely rotated, twice the intensity of emerging light is zero (corresponding to $\theta = \frac{\pi}{2}$ and $\frac{3\pi}{2}$) and twice it becomes maximum (corresponding to θ = 0 and π). This procedure will help us to verify whether the given light is polarized or not. Since tourmaline plate B is used to analyze a state of polarization of incident light, it is known as Analyzer.

4.9 (*c*) Nicol prism : In 1828 A.D. William Nicol made a Polaroid (polarizer and analyzer) from calcite crystal.

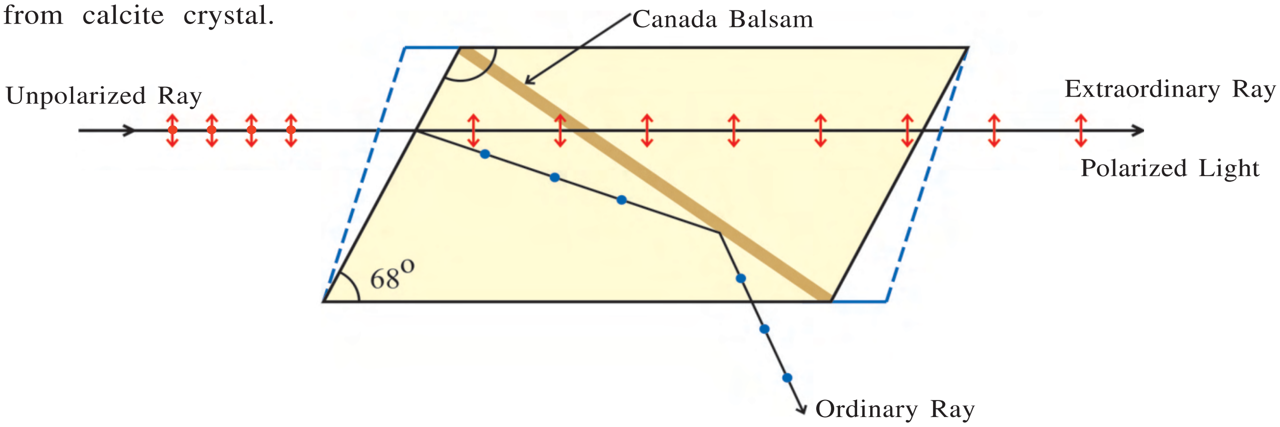


Figure 4.22 Nicol Prism

Nicol prism is made of two crystals of calcite. These crystals are cut at an angle of 68° with respect to its principal axis and then these pieces are joined with Canada balsam (a type of glue).

When unpolarized light is incident on such prism as shown in the figure, it divides into two rays, both rays are plane polarized. $\vec{E}$ vectors of one of the rays are perpendicular to plane as shown in the figure. This ray is called Ordinary Ray. The $\vec{E}$ vectors of another ray have oscillations parallel to the plane. This ray is called Extra Ordinary Ray. For these rays, refractive indaxes are $n_0 = 1.658$ and $n_e = 1.486$. The refractive index of Canada balsam is 1.55. As shown in the figure (4.22) the ordinary ray experiences total internal refraction at the surface of Canada balsam and comes out from one side of the prism while extraordinary ray comes out of the prism as plane polarized light.

4.9 (*d*) Polarization by Reflection and Brewster's Law : There are many methods of polarizing the light. We discussed one of them (with the help of tourmaline plate). Polarized light can also be obtained by reflection of light through transparent medium. In 1809, French scientist Malus found that when a ray of light is incident on surface of transparent medium, most of the $\vec{E}$ vectors in the reflected ray are perpendicular to the plane of incidence, that is reflected ray is partially polarized.

Here, the state of polarization of reflected ray depends on angle of incidence. Experimentally, it can be shown that when a ray of light is incident on a surface of transparent medium at some definite angle of incidence, reflected ray is found to be totally plane polarized. In this state all the $\vec{E}$ vectors in the reflected ray of light are parallel to each other and perpendicular to the plane of incidence. Such an angle of incidence is called Angle of Polarization of the given transparent medium. It depends on the type of the medium.

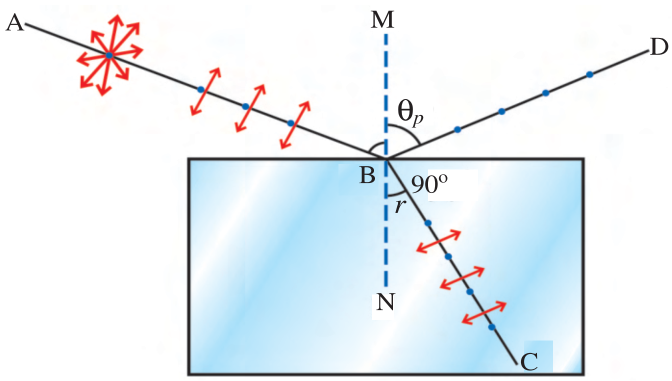


Figure 4.23 Polarization Through Reflection

A plane containing incident ray AB normal BM and a reflected ray BD is the plane of incidence in figure 4.23. The components of E, perpendicular to the plane of incidence are shown by (•). The components of $\vec{E}$ parallel to the plane of incidence are shown by (↔). The components perpendicular to the plane of incidence are known as σ components while components parallel to the plane of incidence are called π components.

When the angle of incidence is same as angle of polarization, only part of σ components are reflected. Hence, the reflected light is found to be totally plane polarized. In this situation π components are not found in reflected ray of light.

As in reflected ray of light a small part of σ components are present, it is very weak in comparision with the incident ray. At the surface of glass only 15% of σ components are reflected while 85% of σ components and all π components are refracted. Hence refracted ray is quite intense as compared to reflected ray.

Experimentally Brewster showed that when the reflected ray of light is totally plane polarized, the angle between reflected and refracted rays is 90°. An important result obtained from this experiment is known as Brewster's Law.

Brewster's Law : "When a ray reflected from a surface of transparent object is totally plane polarized, the tangent of the angle of incidence (angle of polarization) is equal to the refractive index of the material of the object."

i.e., $n = \tan\theta_p$ (4.9.2)

where n = refractive index of the medium and θ_p is the angle of polarization.

Proof : As shown in figure 4.23, $\angle MBD + \angle DBC + \angle r = 180^\circ$

$\therefore \theta_p + 90^\circ + r = 180^\circ$

$\therefore r = 90^\circ - \theta_p$ (4.9.3)

According to Snell's law, refractive index

$$n = \frac{\sin\theta_p}{\sin r} = \frac{\sin\theta_p}{\sin(90^0 - \theta_p)} = \frac{\sin\theta_p}{\cos\theta_p} = \tan\theta_p \quad (4.9.4)$$

Equation 4.9.4 is known as Brewster's law.

4.9 (*e*) Uses of Polarization : Historically polarization was used to determine the type of the light (transverse) for longitudinal waves the oscillations of the particles of medium being parallel to direction of propagation, the polarization of longitudinal wave is never possible.

From the state of polarization of light emitted by an object or scattered by it, properties of the objects can be studied.

With the help of polarization it is found that in the rings of saturn there are ice cyrstals.

By studying state of polarization of ultraviolet light scattered by different viruses, their shape and size can be known.

The polarization of light is also useful in studying atoms and nuclei. The method known as photo-elasticity is used to study property of stress and strain of glass or bakelite.

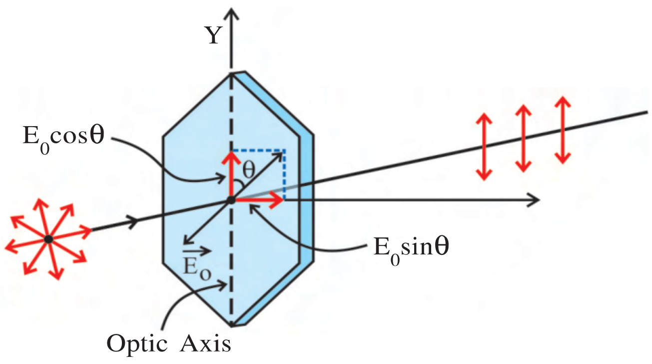
The type of sugar and concentration of its solution can be determined by passing plane polarized light through the solution of sugar. In LCD (Liquid Crystal Display) polarized light is used. They are used in calculators, watches and in the screens of laptops. To decrease the glare, sunglasses are also made from Polaroid.

Illustration 10 : Prove that when unpolarized light passes through a polarizer, the intensity of the transmitted light will be exactly half to the incident light.

Solution : As shown in the figure, let one such light vector make an angle θ w.r.t. optic axis. According to Malus' law, emergent intensity for this light vector will be,

$I = I_0\cos^2\theta$ (1)

where I_0 = intensity of the incident unpolarized light.



But we know that in unpolarized light, $\vec{E}$ vectors are distributed in all directions in a plane perpendicular to the direction of propagation. That is, all values of θ starting from 0 to 2π are equally possible.

Therefore, the average, emergent intensity is, given by

$$I_{ave} = \langle I \rangle = I_0 \left\langle \cos^2\theta \right\rangle$$

$$= \frac{I_0}{2\pi} \int_{\theta=0}^{\theta=2\pi} \cos^2\theta \, d\theta = \frac{I_0}{2\pi} \int_0^{2\pi} \left(\frac{1+\cos 2\theta}{2} \right) d\theta$$

$$= \frac{I_0}{4\pi} \left\{ [\theta]_0^{2\pi} + \left[\frac{\sin 2\theta}{2} \right]_0^{2\pi} \right\}$$

$$= \frac{I_0}{4\pi} \{(2\pi - 0) + 0\} = \frac{1}{2} I_0$$

That is, transmitted intensity is exactly half to the incident intensity.

Illustration 11 : A Plane polarized light is incident normally on the tourmaline plate. Its $\vec{E}$ vectors make an angle 60^o with the optic axis of the plate. Find the % difference between initial and final maximum values of $\vec{E}$ vectors.

Solution : According to Malus' law, $I = I_0 \cos^2\theta$

$$\therefore \frac{I}{I_0} = \cos^2(60^o) = (0.5)^2 = 0.25 = \frac{1}{4}$$

$$\therefore \frac{E^2}{E_0^2} = \frac{1}{4} \quad (\because \; I \propto E^2)$$

$$\therefore \frac{E}{E_0} = \frac{1}{2}$$

$$\therefore \frac{|E - E_0|}{E_0} = \frac{|1-2|}{2} = \frac{1}{2}$$

$$\% \Delta E = \frac{\Delta E}{E_0} \times 100 = \frac{1}{2} \times 100 = 50\%$$

Illustration 12 : A ray of light travelling in water is incident on a glass plate immersed in it. When the angle of incident is 51^o the reflected ray is totally plane polarized. Find the refractive index of glass. Refractive index of water is 1.33.

Solution : Angle of incidence, $\theta_p = 51^o$

Since at this incidence angle, reflected ray is totally plane polarized, using Brewster's law, refractive index of glass w.r.t. water is.

$$n' = \tan\theta_P = \tan 51^o = 1.235$$

$$\text{But, } n' = \frac{\text{refractive index of glass}(n_g)}{\text{refractive index of water}(n_w)}$$

$$\therefore n_g = n' n_w = 1.235 \times 1.33 = 1.64$$

Illustration 13 : A slit of width d is illuminated by white light. For what value of d will the first minimum for red light of wavelength $\lambda_R = 6500$ Å appear at $\theta = 15^o$? What is the situation for violet colour having wavelength $\lambda_V = 4333$ Å at the same point. $\sin 15^o = 0.2588$.

Solution : Since the diffraction occurs separately for each wavelength, we have to check condition for minima and maxima for each wavelength separately.

For the first minimum of red colour, $n = 1$, using equation,

$$d\sin\theta = n\lambda, \qquad (1)$$

$$\text{slit width, } d = \frac{n \times \lambda_R}{\sin\theta} = \frac{1 \times 6500 \times 10^{-10}}{\sin 15^o}$$

$$= \frac{6.5 \times 10^{-7}}{0.2588} = 2.512 \times 10^{-6} \text{ m}$$

For violet colour, since wavelength is different we have to check whether the condition for minimum or maximum will satisfy.

$$\text{Using } d\sin\theta = n'\lambda_V \qquad (2)$$

$$\therefore n' = \frac{d \sin\theta}{\lambda_V} = \frac{2.512 \times 10^{-6} \times 0.2588}{4333 \times 10^{-10}}$$

$$\therefore n' = 1.50$$

But to observe, minima, in equation (2), n' should be an integer. Thus, for violet colour condition for minimum does not satisfy.

$$\text{Using } d\sin\theta = (2n + 1)\frac{\lambda_V}{2},$$

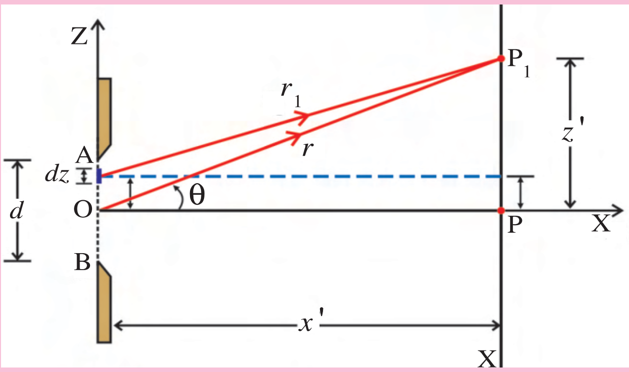
$$n' = \frac{d\sin\theta}{\lambda_V} - \frac{1}{2} = 1.5 - \frac{1}{2} = 1.0$$

This result suggests that for violet colour first maximum is observed.

Note : Irrespective of the width of the slit, at the position of first minimum of red colour, first maximum for violet colour takes place.

APPENDIX

For analysis of diffraction pattern in general (i.e. to know the intensity distribution and location of interference fringes) however, we ignore the converging lens and assume that the screen (C) is at very large distance. So the diffracted waves are considered to be effectively plane. However, it is tobe noted that even, while using the lens, situation remains the same. Since different secondary waves from the slit are passing through different thickness of the lens and therefore, they cover equal optical path length. (Optical path length in medium is equal to the product of refractive index of the medium and geometrical path length.)



Maxima and Minima Diffraction Due to Single Slit

As shown in the figure, the centre (O) of the slit is considered as the origin for Cartesian coordinate system. We further assume that slit AB is divided into large number of small segments (slit-segments) each of width dz. One such element is depicted at a distance z in the figure. We are now interested to find an equation for resultant intensity at different points on the screen due to superposition of waves from all such slit-segments.

The displacement at point P_1 due only to one such slit-element of width dz is given by,

$$de = E'\sin(\omega t - kr_1) \quad (1)$$

Where E' is the amplitude at point P_1. It is known that the larger the width dz, the larger is the amplitude E' (and therefore the intensity). That is, $E' \propto dz$ or $E' = A'dz$, where A' is the proportionality constant.

$$\therefore\ de = A'\sin(\omega t - kr_1)dz \quad (2)$$

Now, resultant displacement at point P_1 due to all slit-segments from B to A is,

$$e = A'\int_B^A \sin(\omega t - kr_1)dz = A'\int_{\frac{-d}{2}}^{\frac{+d}{2}} \sin(\omega t - kr_1)dz \quad (3)$$

from the figure, $r^2 = (x')^2 + (z')^2$

$\therefore\ (x')^2 = r^2 - (z')^2$

and $r_1^2 = x'^2 + (z' - z)^2$

$= (r^2 - z'^2) + (z' - z)^2$

$= r^2 - 2zz + z^2$

$$r_1^2 = r^2\left(1 - \frac{2z'z}{r^2} + \frac{z^2}{r^2}\right)$$

Since $r >> z$, $\frac{z^2}{r^2}$ is very small and it can be neglected, and the term $\frac{2z'z}{r^2}$ is very small compared to unity.

$$\therefore\ r_1^2 = r^2\left(1 - \frac{2z'z}{r^2}\right)$$

$$r_1 = \left(1 - \frac{2z'z}{r^2}\right)^{\frac{1}{2}}$$

Using Binomial theorem

$[(1 + x)^n \approx 1 + nx,\ x <<1]$, $r_1 \approx r\left(1 - \frac{1}{2}\frac{2z'z}{r^2}\right)$

$$\therefore\ r_1 = r - \frac{z'z}{r}$$

Also, from ΔOPP_1, $\sin\theta = \frac{z'}{r}$

$$\therefore\ r_1 = r - z\sin\theta \qquad (4)$$

Using equation (4) into (3),

$$e = A' \int_{-d/2}^{+d/2} \sin(\omega t - kr + kz\sin\theta)\, dz$$

$$= \frac{-A'}{k\sin\theta}\left[\cos(\omega t - kr + kz\sin\theta)\right]_{-d/2}^{+d/2}$$

$$= \frac{-A'}{\left(\frac{2\pi}{\lambda}\right)\sin\theta}\left[\cos\left\{(\omega t - kr) + \left(\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta\right)\right\}^{-d/2} - \cos\left\{(\omega t - kr) - \left(\frac{2\pi}{\lambda}\frac{d}{2}\sin\theta\right)\right\}\right] \qquad \left(\text{writing}\ k = \frac{2\pi}{\lambda}\right)$$

Using an identity, $\cos(\theta_1 + \theta_2) - \cos(\theta_1 - \theta_2) = 2\sin\theta_1\sin\theta_2$

$$e = \frac{A'\lambda}{2\pi\sin\theta}\left[-2\sin(\omega t - kr)\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)\right]$$

$$= \left\{\left(\frac{A'\lambda}{\pi\sin\theta}\right)\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)\right\}\sin(\omega t - kr) \qquad (5)$$

Thus, the resultant amplitude (E) at point P_1 is,

$$E = \left(\frac{A'\lambda}{\pi\sin\theta}\right)\sin\left(\frac{\pi d\sin\theta}{\lambda}\right)$$

or

$$E = A'd\left(\frac{\sin\alpha}{\alpha}\right),$$

where we have assumed $\frac{\pi d\sin\theta}{\lambda} = \alpha$ (6)

Since intensity is directly proportional to the square of amplitude, resultant intensity at point P_1 is,

$$I = A^2 d^2\left(\frac{\sin\alpha}{\alpha}\right)^2$$

$$I = I_0\left(\frac{\sin\alpha}{\alpha}\right)^2 \qquad (7)$$

with $I_0 = A'^2 d^2$ = maximum intensity (8)

SUMMARY

1. An imaginary surface passing through particles of the medium or points in the space oscillating in phase is known as wavefront. It is used to describe the wave propagation.
2. Huygen's principle suggests that every point of a wavefront behaves as an independent secondary source, and emits by itself secondary spherical waves.
3. For the isotropic medium new wavefront maintains its original shape.
4. The physical effect produced by superposition of two or more waves is called interference. Using the principle of superposition resultant displacement at a point where interference takes place can be found.
5. Light sources emitting light waves with equal frequencies and either with constant or zero initial phase difference are known as coherent sources, otherwise sources are known as incoherent sources of light.
6. Coherent sources can only produce stationary interference.
7. In general, two methods are used to obtain coherent cources. (1) Division of wave front and (2) division of amplitude.
8. **For Superposing Waves :**

   (1) Phase difference of $2n\pi$, $n = 0, 1, 2, ...$ on path difference of $n\lambda$, $n = 0, 1, 2, ...$ produce constructive interference.

   (2) Phase difference of $(2n - 1)\pi$, $n = 1, 2, ...$ or path difference of $(2n - 1)\frac{\lambda}{2}$, $n = 1, 2, ...$ produce destructive interference.
9. Distance between two consecutive dark or bright interference fringes is same, $\bar{x} = \frac{\lambda D}{d}$. All bright fringes are equally bright.
10. Diffraction is the effect produced due to the limited part of the wavefront.
11. For Fraunhoffer diffraction, condition for minima can be given as,

    path difference $= n\lambda$; $n = 0, 1, 2, 3, ...$

    Corresponding to different values of n,

    $n = 1 \Rightarrow$ First order minimum,

    $n = 2 \Rightarrow$ Second order mimimum etc., we get different order minima.
12. For maxima is Fraunhoffer diffraction, path difference $= (2n + 1)\frac{\lambda}{2}$, $n = 1, 2, 3,...$
13. Corresponding to different values of n, we get different order maxima.
14. From central or zeroth order maximum, towards the higher order maxima, intensity rapidly decreases. It also decreases in proportion with the width of the slit.
15. The ability of an optical instrument to produce two nearby objects clearly and separate is defined as resolving power of an instruments.
16. Only transverse waves show polarization effect.
17. Ordinary light sources produce unpolarized light.
18. Different techniques are available to convert unpolarized light into the polarized light.

EXERCISE

For the following statements choose the correct option from the given options :

1. The distance between two slits in Young's experiment is 0.2 mm. If the wavelength of light used is 5000 Å, the angular position of 3rd bright fringe from the central bright fringe is .... rad.

(A) 0.075 (B) 0.75 (C) 0.0075 (D) 0.057

2. In Young's experiment the distance between two slits is 0.4 cm and the distance of the screen from the slits is 100 cm. If the wavelength of the light used is 5000 Å, the distance of 4th dark fringe from the central bright fringe is ....

(A) 4.37×10^{-2} cm (B) 4.37 mm (C) 8.74×10^{-2} cm (D) 8.74 mm

3. The distance between two slits in Young's experiment is 0.1 *m*m and the distance of the screen from the slits is 100 cm. If the wavelength of light is 5000 Å the width of the fringe is ....

(A) 5 mm (B) 2.5 mm (C) 2.5 cm (D) 5 cm

4. In Young's experiment the distance between two slits is halved and the distance between the screen and slits is doubled. The width of the fringe ....

(A) remains the same (B) becomes half (C) becomes double (D) becomes 4 times

5. A diffraction is formed with red light. If red light is repalced by blue light, ...

(A) the pattern does not change.

(B) the maxima and minima are narrow and more crowded.

(C) the maxima and minima are broadened and become distinct.

(D) diffraction pattern disappears.

6. In Young's experiment if transparent thin sheets are palced in front of two thin slits such that the central bright fringe remain at the same position. Thickness and refractive index of both sheets are t_1 and t_2, and n_1 and n_2, respectively. In this case, ...

(A) $\frac{t_1}{t_2} = \frac{n_1}{n_2}$ (B) $\frac{t_2}{t_1} = \frac{n_2}{n_1}$ (C) $\frac{t_1}{t_2} = \frac{(n_2-1)}{(n_1-1)}$ (D) $\frac{t_2}{t_1} = \frac{(n_2-1)}{(n_1-1)}$

7. A plate of refractive index 1.5 is placed in the passage of one ray in Young's experiment. If the central fringe is bright, the minimum thickness of the plate is ....

(A) 2λ (B) λ (C) $\frac{\lambda}{3}$ (D) $\frac{2\lambda}{3}$

8. To determine the position of a point like object precisely, ... light should be used.

(A) polarized (B) long wavelength (C) short wavelength (D) intense

9. The angular spread of central maximum, in diffraction pattern, does not depend on ....

(A) the distance between the slit and sources (B) wavelength of light

(C) width of slit (D) frequency of light

10. In Fraunhoffer diffraction by a single slit, the width of the slit is 0.01 cm. If the wavelength of light incident normally on the slit is 6000 Å the angular distance of second maximum from the mid line of central maximum is .... rad.

(A) 0.015 (B) 0.15 (C) 0.075 (D) 0.030

11. Detailed information can be obtained by the oil immersion objective of a microscope, because the objective has ....

(A) large value of magnification (B) greater value of resolution

(C) large diameter (D) none of the above

12. A person finds that the sun rays reflected by the still surface of water in a lake are polarized. If the refractive index of water is 1.327, the sun will be seen at the angle of .... with the horizon.

(A) 57^o (B) 75^o (C) 37^o (D) 53^o

13. Ordinary light incident on a glass slab, at the polarizing angle, suffers a deviation of 22^o. in the medium. The value of angle of refraction is ....

(A) 74^o (B) 22^o (C) 90^o (D) 34^o

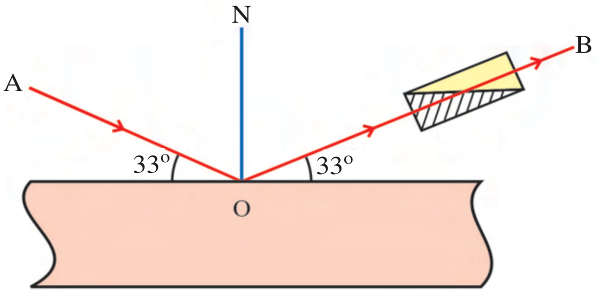
14. The ratio of resolving power of telescope, when lights of wavelengths 4000 Å and 5000 Å are used, is ....

(A) 16 : 25 (B) 5 : 4 (C) 4 : 5 (D) 9 : 1

15. The diameter of the lens of a telescope is 1.22 m. The wavelength of light is 5000 Å. The resolving power of the telescope is .... m^{-1}.

(A) 2×10^5 (B) 2×10^6 (C) 2×10^2 (D) 2×10^4

16. AO is a ray incident on a glass having refractive index 1.54, as shown in the figure. A Nicol prism is appropriately kept in the path of the reflected ray OB. Now, the Nicol prism is rotated. The intensity of light emferging from the Nicol Prism ....



(A) becomes zero and remains zero.

(B) slightly increases and decreases.

(C) does not change.

(D) decreases gradually and becomes zero and then again increases.

17. Unpolarized light falls on two polarizers placed one on top the other. What must be the angle between the characteristic directions (optic axis) of the polarizer if the intensity of the transmitted light is one third of the incident beam from source.

(A) 54.7^0 (B) 35.3^0 (C) 0^0 (D) 60^0

ANSWERS

1. (C)	**2.** (A)	**3.** (B)	**4.** (D)	**5.** (B)	**6.** (C)
7. (A)	**8.** (C)	**9.** (A)	**10.** (A)	**11.** (B)	**12.** (C)
13. (D)	**14.** (B)	**15.** (B)	**16.** (D)	**17.** (B)	

Answer the following questions in brief :

1. State Huygen's principle.
2. What is interference ?
3. State the principle of superposition.
4. What are coherent source ?
5. Give relation between optical path length and geometrical path length.
6. What is Airy's disc ?

7. Define resolving power of an optical instrument.

8. State Rayleigh's criterion.

9. Define plane of polarization.

10. Define linearly polarized light.

Answer the following questions :

1. Explain the use of wavefront to understand wave propagation.

2. Prove that the distance between consecutive dark and bright fringes in interference pattern is given by $\frac{\lambda D}{2d}$.

3. Explain central maximum obtained due to single slit Frahunhoffer diffraction.

4. Determine the width of central maximum in Franhoffer diffraction.

5. Explain the importance of Fresnel distance.

6. Give two points of comparison for interference and diffraction pattern.

7. Define unpolarized light and polarized light.

8. With diagram, give construction of Nicol prism.

9. State and prove Brewster's law.

10. Give uses of polarization.

Solve the following questions :

1. Two coherent line sources are 0.7 *m*m apart. If the centre of the fourth dark fringe of the interference pattern formed by the light emitted from them, on a screen placed at a distance of 1 m, is at 3 mm from the centre of the central bright fringe. Find the wave length of the monochromatic light used. [**Ans. :** 6000 $\mathring{A}$]

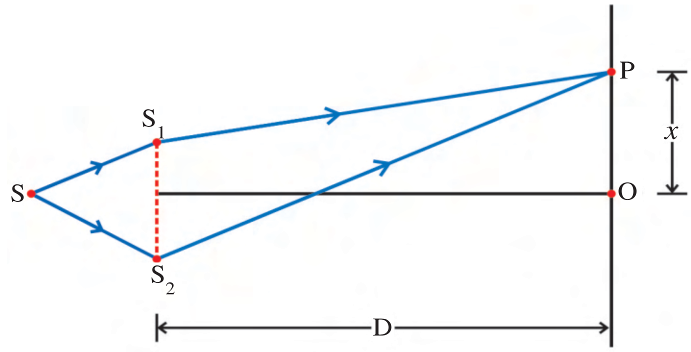
2. In an Young's experiment, the distances between two slits and that between slits and the screen are 0.05 cm and 1 m, respectively. Find the distance between 3^{rd} bright and 5^{th} dark fringes. Take the wavelength of light equal to 5000 $\mathring{A}$. [**Ans. :** 1.5 mm]

3. In Young's experiment fifth bright fringe produced by light of 4000 $\mathring{A}$ superposes on the fourth bright fringe of an unknown wavelength. Find the unknown wavelength.

   [**Ans. :** 5000 $\mathring{A}$]

4. In Young's experiment, the distance between two slits is 1 mm and the distance between two consecutive bright fringes is 0.03 cm. Now, on displacing the screen away from the slits by 50 cm, the distance between two consecutive dark fringes is doubled. Find the wavelength of light used. [**Ans. :** 6000 $\mathring{A}$]

5. If the difference of time taken by two waves emitted from coherent sources to reach a point is an integral multiple of the period of the wave show that the constructive interference will occur at that point.

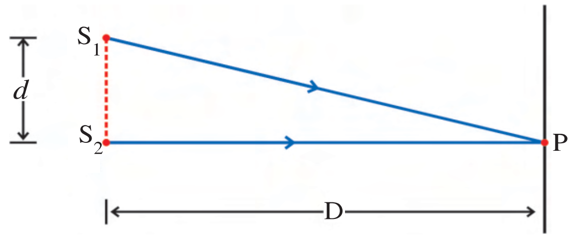
6. 

As shown in the figure, for interference by two rays is such that $SS_2 - SS_1 = 0.25\ \lambda$, where λ is the wavelength of the light used, obtain the conditions for constructive and destructive interference at point P.

7. In Young's double slit experiment, if the distance between two slits is double, than the wavelength of light used. Prove that a maximum 5 bright fringes will be obtained on the screen.

8. In Young's experiment a beam of light of wavelength 6500 Å and 5200 Å is used. Find the minimum distance from the central bright fringe where bright fringes produced by both the wavelength get superposed. The distance between two slits is 0.5 mm and the distance between the slits and the screen is 100 cm. [**Ans. :** 0.52 cm]

9. White light is used in Young's double slit experiment, as shown in the figure. At a point on the screen directly in front of slit S_2, certain wavelengths are producing destructive interference (i.e. they are missing in the diffraction pattern). Find these wavelengths, corresponding to first and second order diffraction. Here, $d << D$.



[**Ans :** (i) $\frac{d^2}{D}$, $n = 1$, (ii) $\frac{d^2}{3D}$, $n = 2$]

10. Three light waves are superposed at a certain point, where their electric field components are given as $E_1 = E_0 \sin\omega t$, $E_2 = E_0 \sin(\omega t + 60^\circ)$, $E_3 = E_0 \sin(\omega t - 30^\circ)$. Find their resultant $E(t)$ at that point. (1) Find resultant amplitude E_R by resolving $\vec{E}$ into sine and cosine components in the phasor diagram. (2) Through resultant vector in the phase diagram, phase can be found. [**Ans. :** $E(t) = E_R \sin(\omega t + \beta)$ with $E_R = 2.4E_0$, $\beta = 8.8^\circ$]

11. In Fraunhoffer diffraction, the wavelength of light incident normally on the slit is $\frac{d}{2}$ where d is the width of the slit. What will be the number of bright fringes formed on an infinitely extended screen placed at any distance from the slit.

[**Ans. :** 3 maxima are formed]

12. Light of wavelength 5000 Å is incident on a slit of width 2 mm in Fraunhoffer diffraction. Find the width of second maximum on the screen placed at the focal plane of the convex lens of a focal length 100 cm. The lens is placed close to the slit. [**Ans. :** 0.025 cm]

13. An apparatus for Young's experiment is immersed in a liquid of refractive index 1.33. The distance between two slits is 1 mm and that between slits and screen is 1.33 m. The wavelength of light used is 6300 Å in air.

(1) Find the distance between two consecutive bright fringes. (2) Keeping the apparatus in the liquid, one of the slits is covered with a glass plate of refractive index 1.53. If in this condition the first order dark fringe is displaced in the position of zeroth order bright fringes. Find the thickness of the plate. [**Ans. :** (i) 0.63×10^{-3} m (ii) 1.57×10^{-6} m]

•

5

ATOMS

5.1 Introduction

Greek philosophers were the first to propose that all matters consist of very small particles, called atoms (meaning indivisible). However, this idea did not get scientific support until a (British) chemist, John Dalton (1803), performed series of experiments with various chemicals, and showed that the matter indeed consists of elementary lumpy particles, the atoms. He proposed that (1) atom is the smallest (fundamental) indivisible part of an element, which takes part in chemical reactions, and (2) each element is made up of one kind of atoms only. However, no further progress was made of this atomic theory of matter and related questions until the end of nineteenth century, when the experimental evidences began to accumulate.

For instance, Jean Perin discovered negatively charged electrons. Later, J.J. Thomson found the ratio of the charge (e) and mass (m) of electrons, while Milikan had measured the charge of an electron.

On the other hand, Henry Bacqurel and Madam Curie had discovered radioactivity in some heavy elements (atoms) Rutherford's experiment on radioactivity established that in radioactive radiations, positively charged α-particles are also present along with electrons. These positive charged α-particles are found to be a few thousand times heavier than electrons.

Thus, these observations have proved that atoms are in fact divisible, and more fundamental negative and positive particles are found to be the building blocks of atoms.

It was also established that atoms are electrically neutral, i.e. they should contain same amount of negative and positive charges.

Further, it was also known at that time that condensed matter (like solids, liquids and even dense gases) emit electromagnetic radiations at all temperatures. In these radiations, a continuous distribution of several wavelengths with different intensities is present. In contrast, electromagnetic radiations emitted (either due to electrical discharge or on heating) from rarefied gases were found to be of certain discrete wavelengths.

Also, values of the emitted wavelengths were found to be the characteristics of the gas. Since, in rarefied gases, average distance between atoms is large, the radiation emitted can be considered due to individual atoms. Similarly, when atoms were exposed to radiation of continuous wavelengths, they absorb certain discrete characteristic wavelengths only.

Based on these experimental findings, natural questions which arise are :

(1) What do atoms consist of ?

(2) If they are made up of negative and positive charges, how are they arranged inside an atom ?

(3) What is the reason for stability of an atom ?

(4) Why do atoms emit or absorb radiation of some definite discrete wavelengths only, depending on their types ?

(5) Does this discrete nature of emission or absorption of electromagnetic radiation by atoms have any similarity to photon-theory of black-body radiation ? (Chapter 7, of Semester III.)

Thomson's Plum Pudding Model : The discoveries of electrons, alpha (α) particles and charge neutrality of an atom have suggested that an atom must be made up of equal quantity of positive charge and electrons. This lead Thomson to the following atomic model.

According to him, positive charge is distributed uniformly in a small spherical space of atoms and electrons are embedded inside it in a definite manner. Since this is like the seeds of watermelon embedded in its pulp, the model was named as Watermelon model or Plum Pudding Model.

However, this model has some drawbacks. According to the laws of electrostatics, stationary stable charge distribution is not possible. Electrons in atoms experience Coulombian force due to positive charge in atom. They should perform accelerating motion according to the Newton's second Law of Motion. But according to Maxwell's electromagnetic theory, an accelerated charge continuously emits radiation for indefinite time (i.e., energy). How can a stable atom emit continuously and indefinitely energy ? And even if it is so, it is in contrast to the experimental observation of emission of discrete wavelengths.

To solve these problems of his atomic model, Thomson thought that electrons remain stationary unless they are disturbed from outside. He also tried different arrangements of electrons in different atoms. However, he could not explain the radiation of discrete wavelengths.

Nevertheless, he could estimate the size of atoms to be of the order of 10^{-10}m in radius from the wavelengths of emitted radiations.

5.2 Rutherford's Eperiment of Alpha (α) Particles Scattering and his Atomic Model

Ernst Rutherford, a former student of J. J. Thomson, had performed many experiments on alpha (α) particles emitted by some radioactive elements. In his preliminary observations he found that α-particles, after passing through a slit kept in a chamber, incident on a photographic plate do not give rise to a sharp and distinct image of the slit showing scattering of α particles all over the space. But when the chamber was almost evacuated, the images became sharp. From this observation Rutherford concluded that the α-particles must be scattered by the air particles present in the chamber.

On the other hand, H. Geiger had studied scattering of α-particles, by a thin foil of metal. He found that α-particles are scattered at a very small angle, while passing through a thin foil.

Later, Rutherford assigned an experiment to Geiger and his young student E. Marsden to find whether α-particles get scattered at larger angles or not.

Geiger-Marsden's Experiment : A schematic diagram of Geiger-Marsden experiment is shown in figure 5.1.

Here, a radioactive source $_{83}Bi^{214}$ emits, α-particles of energy 5.5 MeV. Using lead blocks, they are collimated into a narrow beam, and directed on to a thin metal foil (F) of gold. Thickness of the foil was kept 2.1×10^{-7} m. The foil scatters α-particles. The scattered α-particles were observed through a detector consisting of circular zinc sulphide (ZnS) screen (a scintillation screen) and a microscope. Here, an arrangement rotate the microscope around the screen and α-particles scattered at different angles can be received. Every α-Particles which strikes ZnS screen gives a momentory bright spot. By counting the number of was made to bright spots at different angles in a given interval of time the number of α-particles can be known.

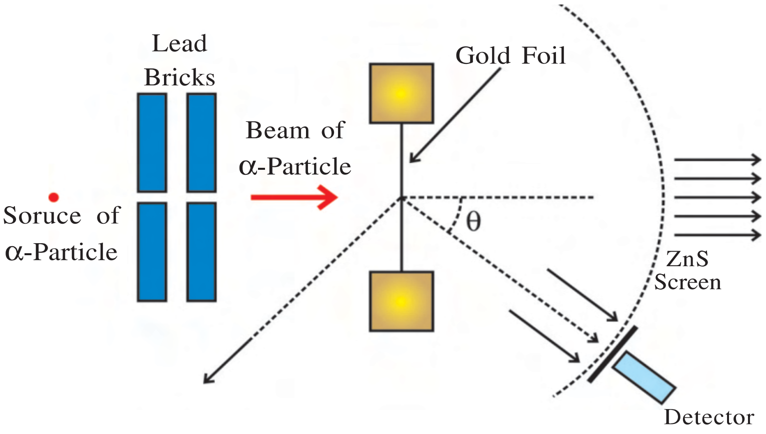


Figure 5.1 Geiger-Marsden Experiment

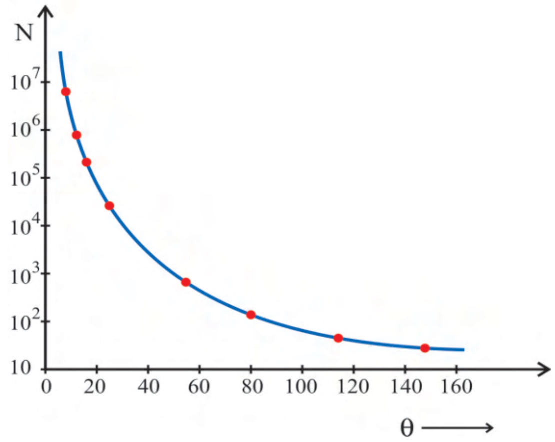


Figure 5.2 A Graph of Number of Scattered α-Particles Versus Scattering Angel

A typical graph of the total number of α-particles (N) scattered at different angles (θ), in a given interval of time is shown in figure 5.2. In the graph, experimental results are shown by dots while the theoretical result due to Rutherford is represented by the continuous line. Majority of the α-Particles (about 10^5 or more) were, scattered at very small angle (< 15°), while only 0.1% (~80) of the incident α-particles were scattered at large angle (~150°). The number of α-particles scattered in backward direction (θ ≈ 180°) was about only 1 out of 10^4.

Rutherford's Explanation : Rutherford suggested that since large number of α-particles are scattered at very small angles, atoms must be largely hollow. For very few backward scattering of α-particles he argued two points. First, since the metal foil is very thin, possibility of multiple scattering of α-particles resulting into backward scattering is almost zero. Second, then it is reasonable to assume that in order to deflect the α-particle backwards, it must experience a large repulsive force, and this force could be provided if the large (almost all) mass of the atom and its entire positive charge were concentrated in a small central region. He called this central region as the nucleus. In this case, the incoming α-particle depending on its kinetic energy could get close to central positive charge without penetrating it, and then gets scattered at very large angle (~180°).

Charge of α-particle is $+2e$ and total positive charge in the nucleus of gold is Ze, where Z(= 79) is atomic number of gold and e is an electronic charge. Since the nucleus of gold is about 50 times heavier than an α-particle, it remains stationary during the scattering process. Under this circumstance, the trajectory of α-particle can be calculated using Newton's second law of motion and the Coulomb's law for electrostatic force of repulsion between α-particle and nucleus of the foil. Magnitude of Coulombian force is given by,

$$F = \frac{1}{4\pi\epsilon_0}\frac{(2e)(Ze)}{r^2} \quad (5.2.1)$$

This force is directed alone the line joining the α-particles and the nucleus.

In addition, Rutherford had derived an expression for number of α-particles scatterd at different angle (θ), as follows.

Only for Information : $N = N_0 ns\left(\frac{Ze^2}{mv^2}\right)\frac{1}{\sin^4\left(\frac{\theta}{2}\right)}$

where, N_0 = number of α-particles incident

N = number of α-particles scattered at an angle θ

S = thickness of the foil

n = number of atoms per unit volume

m = mass of one α-particle

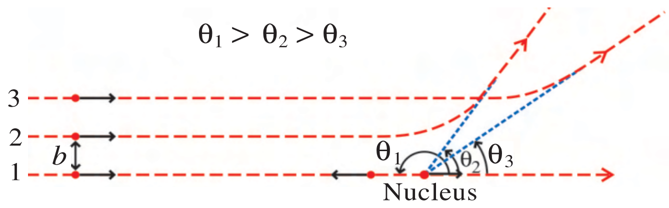


Figure 5.3 Trajectory of α-particles

Result obtained using above equation by Rutherford is shown as continous line in figure 5.2, which matches very well with experimental results (dots) in the graph. However, the magnitude and direction of the force on α-particle continuously changes as it approaches the nucleus as shown in figure 5.3.

Obviously, the trajectory of α-particles depends on the initial perpendicular distance of its velocity vector from the nucleus.

"The perpendicular distance of the initial velocity vector of the α-particle from the centre of the nucleus is known as the impact parameter (b)."

"The minimum distance of the α-particle from the centre of the nucleus for a case when $b = 0$ (head-on collision) is known as the closest distance of approach."

It can be seen from figure 5.3 that the larger the value of b, the smaller is the scattering angle (θ). For curve -1, the impact parameter is zero and it experiences the head-on collision. Therefore, scattering angle of such α-particles will be very large. It was found that α-particles, with highest kinetic energy can go nearest to the nucelus only at a distance about 10^{-15} m. This determines the approximate radius of the nucleus as of the order of 10^{-15} m. (1 *fm*, fermi = 10^{-15} m). Thus, the size of the nucleus is nearly 10^5 times smaller than the size of an atom (which is of the order of 10^{-10} m).

According to Kepler's planetary model, planets being light revolve round the sun under the effect of strong attractive Gravitational force. Based on Kepler's planetary laws, Rutherford further proposed that light electrons revolve in a circular orbit round the nucleus.

Rutherford's Atomic Model : He proposed that the entire mass and positive charge of an atom resides in a very small region at its centre, the nucleus. Negatively charged electrons move round the nucleus in a circular orbit. This is called Rutherford's planetary model of atom or the nuclear model of atom.

Drawbacks of Rutherford's Atomic Model : According to classical physics, an electron depending on its kinetic energy can revolve around the nucleus in any orbit out of infinite possible orbits. In other words, there is no constraint on the radius of the orbit of electron. Electron moving in a circular orbit performs an accelerated motion. But an accelerated charge radiates energy in the form of electromagnetic radiation. If the electron revolving around the nucleus continuously radiates energy, its energy and hence its radius should go on decreasing. Hence, such an orbit would not be circular, rather it will be spiral terminating at its nucleus, as shown in figure 5.4.

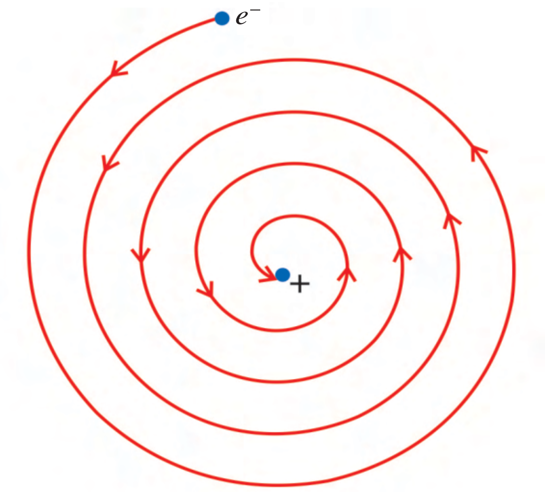


Figure 5.4 Spiral Orbit of Classical Electron

In this case the atom cannot remain stable. Thus, Rutherford's atomic model failed to explain the stability of atom.

Illustration 1 : In a Geiger-Marsden experiment, what is the distance of closest approach to the nucleus (minimum distance from nucleus) of a 7.7 MeV α-particle before it comes to rest for a moment and reverse its direction ?

Solution : Let d be the distance between the α-particle and the gold nucleus, when α-particle stops for a moment before returning back. When α-particle stops for a moment its kinetic energy is zero, and it is all converted into potential energy.

$\therefore$ Potential energy at distance d = Kinetic energy of the α-particle at large distance (7.7×10^6 eV)

$$\therefore \quad \frac{1}{4\pi\epsilon_0}\frac{(2e)(Ze)}{d} = 7.7 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

For gold, Z = 79 and $\frac{1}{4\pi\epsilon_0} = 9 \times 10^9$ MKS

$$\therefore \quad d = \left(\frac{1}{4\pi\epsilon_0}\right)\frac{2e^2 \times (79)}{(7.7 \times 1.6 \times 10^{-13})}$$

$$= \frac{2 \times (9 \times 10^9) \times (1.6 \times 10^{-19})^2 \times 79}{(12.32 \times 10^{-13})}\frac{1}{n^3}$$

$$= 2.95 \times 10^{-14} \text{ m}$$

$\therefore \; d = 29.5 \; fm \approx 30 \; fm$

Note : Actual radius of the gold nucleus is about 6 fm.

5.3 Atomic Spectra

As shown in the figure 5.5, on passing an electrical discharge through atomic gas at low pressure (i.e. rarefied gas), atoms of the gas get excited and emit radiations. These radiations consist of some definite wavelength which are characteristics of nature of the element.

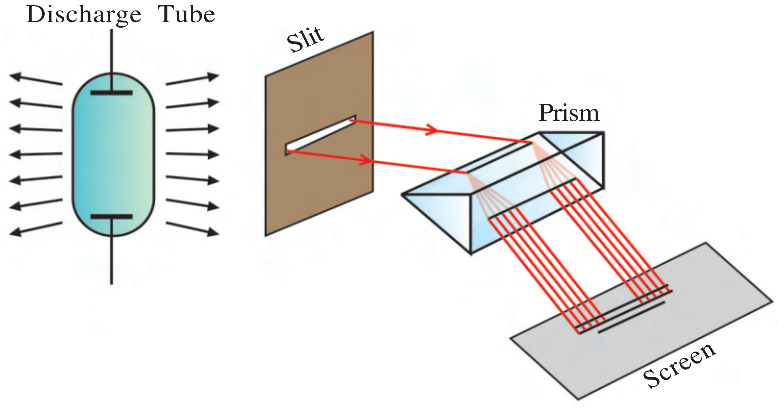


Figure 5.5 Atomic Spectra through Electrical Discharge

By the end of 19^{th} century it was established that specific groups of the lines of the spectrum can be formed according to their frequencies or wavelengths. In any such group, the wavelength of the spectral lines can be calculated by taking difference of a series of terms in a common formula. The spectral lines falling in a group constitute a spectral series.

The atomic spectra of gases consists of several spectral series.

First such series in the visible region of H spectrum was discovered by Swedish school teacher Johann Jakob Balmer in 1885. This is called the Balmer series (see figure 5.6). The line with largest wavelength (6563 Å) is called H_{α}, the next with wavelength (4861 Å) is known as H_{β}, etc. As the wavelength decreases, the lines appear closer together and are weaker in intensity.

Balmer gave a simple empirical formula for the wavelengths of Balmer series as,

$$\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right), \; n = 3, 4, 5, ... \qquad (5.3.1)$$

Here, R is known as Rydbery constant. Its experimental value is 1.09737×10^7 m^{-1}. Wavelength of H_{α} line in Balmer series can be obtained by taking $n = 3$. Similarly, the wavelengths of the successive lines can also be found by taking $n = 4, 5, ...$, respectively.

Other spectral series for hydrogen were subsequently discovered, as depicted in figure 5.7.

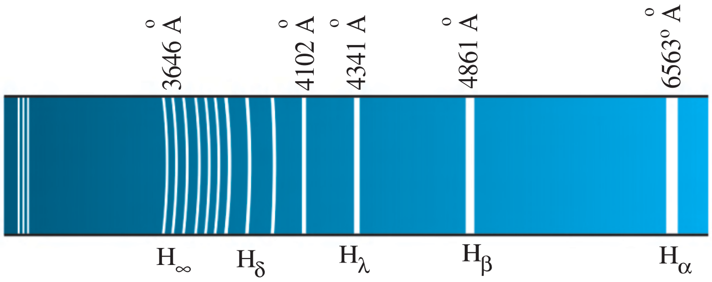


Figure 5.6 Balmer Series

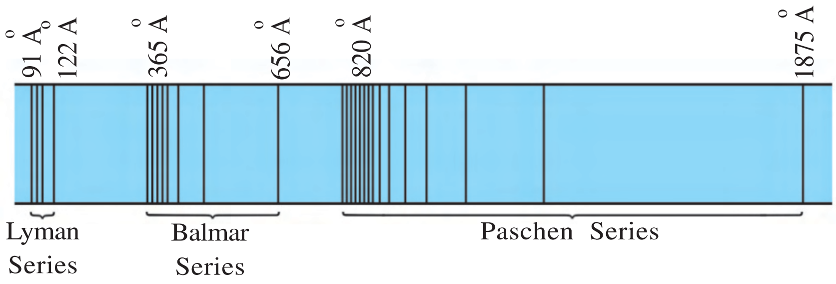


Figure 5.7 Some Spectral Series of Hydrogen Atom

They are known after their discoverers, as below.

Lyman series (in ultraviolet region)

$$\frac{1}{\lambda} = R\left(\frac{1}{1^2} - \frac{1}{n^2}\right), \quad n = 2, 3, \ldots$$

Paschen series (in near infrared region)

$$\frac{1}{\lambda} = R\left(\frac{1}{3^2} - \frac{1}{n^2}\right), \quad n = 4, 5, \ldots$$

Brackett series (in infrared region)

$$\frac{1}{\lambda} = R\left(\frac{1}{4^2} - \frac{1}{n^2}\right), \quad n = 5, 6, \ldots$$

Pfund series (in far infrared region)

$$\frac{1}{\lambda} = R\left(\frac{1}{5^2} - \frac{1}{n^2}\right), \quad n = 6, 7, \ldots$$

It is possible to write single formula for all these spectral series of hydrogen atom as,

$$\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right) \tag{5.3.2}$$

Taking appropriate integral values of m and n in the above formula, wavelengths for different spectral lines of different series can be found.

Later it was found that there are only a few elements (hydrogen, He^+, Li^{++} etc.) whose spectra can be represented by simple formula like equation (5.3.2). Similar spectral series of atoms having more complicated structures than hydrogen atom, however, requires more involved equation then (5.3.2), though equation (5.3.2) fits to discrete emission of certain frequencies or wavelengths, they do not give any physical reasoning to this observation.

5.4 Bohr's Atomic Model and Hydrogen Spectra

Niels Bohr (1913) made certain modifications in Rutherford's atomic model. Combining the ideas of newly developing quantum hypothesis by Plank and later by Einstein in explaining photoelectric effect. He introduced the theory as hypotheses.

Hypothesis 1 : Out of all the orbits permitted by the classical mechanics an electron can exist around the nucleus only in those orbits in which its orbital angular momentum is an integral multiple of $\frac{h}{2\pi}$. These orbits are known as **stationary** or **stable orbits**. The electron in a stable orbit does not radiate energy, and therefore the electron can revolve steadily in such orbit. Here, h is Planck's constant. Its value is 6.625×10^{-34} Js.

Hypothesis 2 : When an electron makes a transition from a stable orbit with energy E_i to another stable orbit with a lower energy E_k, it radiates $E_i - E_k$ amount of energy in the form of photon of electromagnetic radiation of frequency f such that $E_i - E_k = hf$. Similarly, when an electron absorbs a quantum of frequency f, it makes a transition from lower energy state (E_k) to higher energy state (E_i).

Combining the classical model of Rutherford and Bohr's hypotheses, equation for energy and stability of atom and their spectra can be explained as follows.

A schematic diagram of Bohr's atomic model is shown in the figure 5.8.

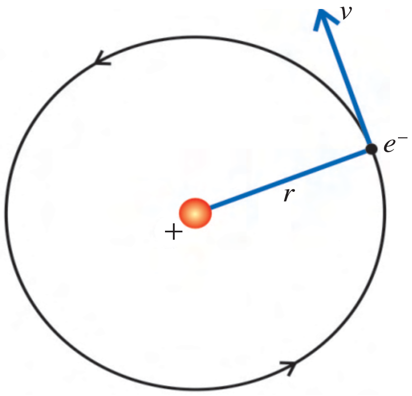


Figure 5.8 A Schematic Diagram of Bohr's Atomic Model

Suppose mass and charge of an electron revolving in a circular orbit with radius r are m and e, respectively. Let its linear speed in this orbit be v. The charge in the nucleus is Ze, where Z is the atomic number. Here, necessary centripetal force is provided by coulombian attractive force between an electron and the positive charge of the nucleus. Thus,

$$\text{Centripetal force, } \frac{mv^2}{r} = \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r^2} \qquad (5.4.1)$$

Here $\in_0$ = Permittivity of free space.

Using the first hypothesic of Bohr, the orbital angular momentum of an electron is given by,

$$l = mvr = n\frac{h}{2\pi} \quad (\because \ m\vec{v} \perp \vec{r}) \qquad (5.4.2)$$

Where n = 1, 2, 3, ... n, is called the principal quantum number. From equation (5.4.2.),

$$m^2v^2r^2 = \frac{n^2h^2}{4\pi^2}$$

$$\therefore \ v^2 = \frac{1}{m^2r^2}\frac{n^2h^2}{4\pi^2} \qquad (5.4.3)$$

Putting the value of v^2 from equation (5.4.3) into equation (5.4.1),

$$\frac{m}{r}\left(\frac{1}{m^2r^2}\frac{n^2h^2}{4\pi^2}\right) = \frac{Ze^2}{4\pi\epsilon_0 r^2}$$

Solving for r, we have

$$r = n^2\left(\frac{h^2\epsilon_0}{\pi Ze^2 m}\right) \qquad (5.4.4)$$

For the given atom, $r \propto n^2$ (5.4.5)

Total energy of an electron in the orbit is

E_n = Kinetic energy + Potential energy

$$= \frac{1}{2}mv^2 + \left(-\frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}\right)$$

But from equations (5.4.1) and (5.4.3.),

$$\frac{1}{2}mv^2 = \frac{1}{8\pi\epsilon_0}\frac{Ze^2}{r}$$

$$\therefore \quad E_n = \frac{1}{8\pi\epsilon_0}\frac{Ze^2}{r} - \frac{1}{4\pi\epsilon_0}\frac{Ze^2}{r}$$

$$E_n = -\frac{1}{8\pi\epsilon_0}\cdot\frac{Ze^2}{r} \qquad (5.4.6)$$

Substituting the value of r from equation (5.4.4), we have,

$$E_n = -\frac{Ze^2}{8\pi\epsilon_0} \times \frac{1}{\left(\frac{n^2h^2\epsilon_0}{\pi Zme^2}\right)}$$

$$\therefore \quad E_n = -\frac{Z^2me^4}{8\epsilon_0^2h^2} \times \frac{1}{n^2} \qquad (5.4.7)$$

Equation (5.4.7) gives an energy of an electron in n^{th} orbit of an atom with atomic number Z.

For hydrogen atom, Z = 1

$$E_n = -\frac{me^4}{8\epsilon_0^2h^2} \times \frac{1}{n^2} \qquad (5.4.8)$$

Putting the known values of m, e, ϵ_0 and h, we get,

$$E_n = -\frac{21.76\times10^{-19}}{n^2}\,\text{J} = -\frac{13.6}{n^2}\,e\text{V} \qquad (5.4.9)$$

$$\text{i.e.} \quad E_n \propto \frac{1}{n^2} \qquad (5.4.10)$$

Negative sign in equation (5.4.9) can be understood as follows. We have considered energy of a stationary electron as zero, when placed at an infinite distance from the positively charged nucleus (i.e. free electron). That is, when an electron is brought from infinite, work is done by the system under the action of coulombian attractive force. And equivalent amount of energy is stored in an electron as its potential energy. This is the binding energy of an electron in this specific orbit. Conversely, if we give a minimum energy E_n to an electron in an orbit with quantum number n it will become free.

5.5 Energy Levels and Hydrogen Spectra

By choosing different values of n in equation (5.4.9) we can obtain energies of an electron in different orbits having principal quantum number n. These are shown by horizontal lines on a vertical axis in figure 5.9.

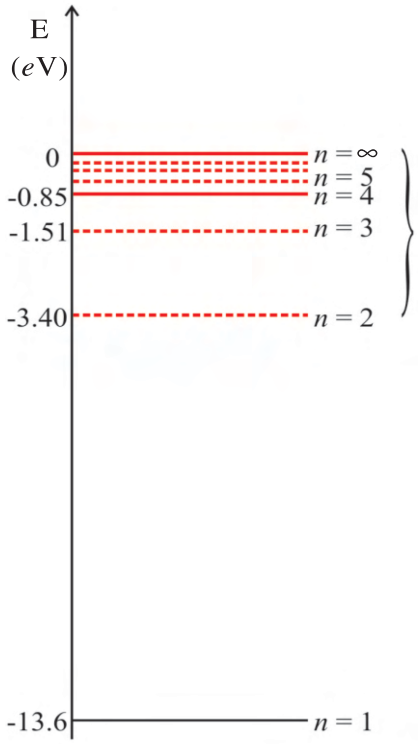


Figure 5.9 Energy Levels for Hydrogen Atom (Values are Only for Information)

Electron in an orbit with $n = 1$ has a minimum energy and it is said to be in its ground state. The successive energy states with values $n = 2, 3, 4,......$ are respectively known as the first excited state, the second excited state, etc.

Suppose an electron makes a transition from energy state E_i with $n = n_i$ to an energy state E_k with $n = n_k$. If $E_i > E_k$ then according to the second hypothesis of Bohr,

$$E_i - E_k = hf_{ik} \tag{5.5.1}$$

When f_{ik} is the frequency of the radiation emitted when the electron makes this transition.

Using equation (5.4.8),

$$E_i - E_k = hf_{ik} = \frac{me^4}{8\epsilon_0^2 h^2}\left(-\frac{1}{n_i^2} + \frac{1}{n_k^2}\right)$$

$$\therefore\ f_{ik} = \frac{me^4}{8\epsilon_0^2 h^3}\left(\frac{1}{n_k^2} - \frac{1}{n_i^2}\right)$$

But using an equation, $c = f\lambda$ wave number

$$\frac{1}{\lambda_{ik}} = \frac{f_{ik}}{c} = \frac{me^4}{8\epsilon_0^2 ch^3}\left(\frac{1}{n_k^2} - \frac{1}{n_i^2}\right) \tag{5.5.2}$$

Putting the known values of m, e, c, h and $\in_0$ prefactor in equation (5.5.2) can be calculated. It is known as Rydberg's constant.

$$R = \frac{me^4}{8\epsilon_0^2 ch^3} = 10973700\ m^{-1} \tag{5.5.3}$$

This is also defined as R_∞. This value is very close to the experimental value of R.

For Information Only : In the derivation of equation (5.4.8) we have assumed that the nucleus is very heavy (in principle, having infinite mass). However, the finite nuclear mass causes a slight variation in the Rydberg constant from atom to atom. In that case, it is defined as,

$R = \frac{\mu e^4}{8\epsilon_0^2 ch^3}$; where $\mu = \frac{mM}{(m+M)}$ is known as the reduced mass of electron. M is the mass of the nucleus. For example, for lightest hydrogen atom,

$R_H = 1096770\ m^{-1}$. This is just 0.055% smaller than the value in equation (5.5.3).

In general, Rydberg constant for any atom can be defined as,

$$R_{atom} = \frac{R_\infty}{\left(1 + \frac{m}{M_{atom}}\right)} \tag{5.5.4}$$

where m is mass of an electron and M is the mass of nucleus.

Combining equation (5.5.2) and (5.5.3)

$$\frac{1}{\lambda_{ik}} = R\left(\frac{1}{n_k^2} - \frac{1}{n_i^2}\right) \qquad (5.5.5)$$

Equation (5.5.4) is nothing but the equation (5.3.2), when $n_k \to m$ and $n_i \to n$. The wavelengths of different spectral lines, as discussed in section – 5.3, can be calculated by taking appropriate values of n_k and n_i. (See figure 5.10)

Success of the Bohr Model :

(1) Stability and energy of hydrogen-like atoms can be calculated.

(2) Atomic Spectra of Hydrogenic atoms (e.g., He^+, Li^{+2}, Be^{+3e}) can be explained.

(3) Later, Wilson and Sommerfeld have showed mathematically that the Planck's energy quantization rule and Bohr's angular momentum quantization rule are the special cases of more general quantization rule. This gave theoretical confirmation to Bohr's adhoc hypothesis.

(4) Due to difference in nuclear mass for different isotopes, Rydberg constants (R) are different. Based on this fact and using equation (5.5.2), Urey, Brickew-Edde and Murphy have found "heavy hydrogen" or "deuterium."

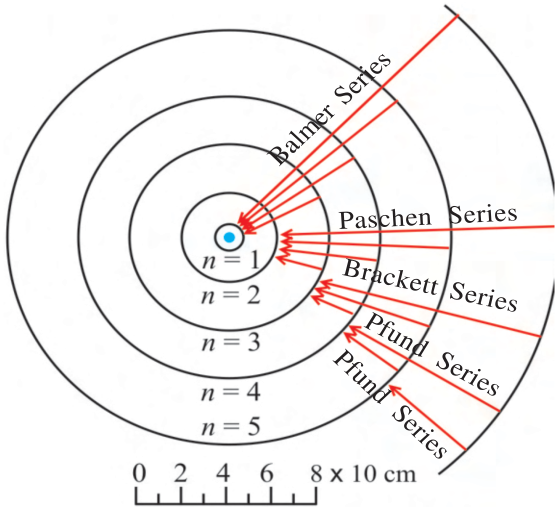


Figure 5.10 A Schematic Diagram of Hydrogen Spectra

For Information Only : In interstellar space and in plasma state (which can be characterized as a collection of almost equal number of positive ions and electrons) atoms are found with their valence electron in a large orbit (n is very high) far from the ion core. Such atoms are known as Rydberg atoms and such very closely spaced states are known as Rydberg state. Due to very large value of principal quantum number (n) in such atoms, the outermost electron feels an almost hydrogenic coulomb potential (due to point-like heavy nucleus with Z number of protons). Such atoms can also be explained by Bohr-like model, and their energy is given by similar equation to (5.4.8). More refined calculations showed that the energy of such atoms is given by,

$E_n \propto \frac{1}{(n-\Delta)^2}$, where Δ is known as quantum defect.

Limitations of Bohr Model :

(1) The orbits of electron need not be circular as orbits under inverse square force are in general elliptical. However, Sommerfeld had shown that circular orbit is a special case of more general elliptical orbits.

(2) Calculations of energies in these orbits make use of classical mechanics, while angular momenta are assumed to be quantized. Thus, it is an odd combination of the classical and quantum principles.

(3) Further, the model is also unable to explain the relative intensities of the spectral lines.

(4) When the spectral lines of the hydrogen spectra are observed with a spectrometer having very high resolving power each line is found to be made up of closely spaced more than

one line. This fine structure of spectral lines can not be explained on the basis of the Bohr model.

(5) The Bohr model is unable to explain the distribution and arrangement of electrons in atoms.

Illustration 2 : The spectroscopic values of the Rydberg's constant for the hydrogen and ionized helium are 109677.7 cm^{-1} and 109722.4 cm^{-1}, respectively. Calculate the ratio $\frac{e}{m}$ for an electron. The specific charge of the hydrogen ion is 96490 C/g.

Solution : Due to finite mass of nucleus, Rydberg's constant is different for different atoms.

It is given by, $$R_{atom} = \frac{R_\infty}{\left(1 + \frac{m}{M_{atom}}\right)} \quad (1)$$

where, R_∞ = 109737 cm^{-1} = Rydberg's constant for infinitely heavy nucleus.

m = mass of an electron.

M_{atom} = mass of the nucleus of an atom under consideration.

For H – atom, $$R_H = \frac{R_\infty}{\left(1 + \frac{m}{M_H}\right)} \quad (2)$$

and for ionized helium atom $$R_{He} = \frac{R_\infty}{\left(1 + \frac{m}{M_{He}}\right)}$$

It is found that $M_{He} \approx 4M_H$. Taking the ratio of equation (2) and (3),

$$\frac{R_H}{R_{He}} = \frac{\left(1 + \frac{m}{4M_H}\right)}{\left(1 + \frac{m}{M_H}\right)}$$

$$\therefore\ R_H \times \left(1 + \frac{m}{M_H}\right) = R_{He} \times \left(1 + \frac{m}{4M_H}\right)$$

$$\therefore\ \frac{m}{M_H}\left(R_H - \frac{1}{4}R_{He}\right) = R_{He} - R_H$$

$$\therefore\ \frac{m}{M_H} = \frac{R_{He} - R_H}{\left(R_H - \frac{1}{4}R_{He}\right)} = \frac{(109722.40 - 109677.70)}{(109677.70 - 0.25 \times 109722.40)}$$

$$\frac{m}{M_H} = 5.435 \times 10^{-4}$$

Now, $\frac{e}{m} = \frac{\left(\frac{e}{M_H}\right)}{\left(\frac{m}{M_H}\right)} = \frac{96490}{(5.435 \times 10^{-4})}$

$= 1.7753 \times 10^{8}$ C/g $= 1.7753 \times 10^{11}$ C/kg

Illustration 3 : In a hydrogen atom, show that the frequency of an electron in an orbit of principal quantum number n is given by $f = \frac{me^4}{4\in_0^2 h^3 n^3}$. Also, prove that for large values of the quantum number n, the radiation emitted in transition from a level $(n + 1)$ to a level n has the same frequency. Take, $R = \frac{me^4}{8\in_0^2 ch^3}$.

> **For Information Only :** According to Bohr's correspondence principle, for very large quantum number n, when atom makes transition such that change in principal quantum number of orbits $\Delta n = \pm 1$, then the quantum theory frequency and the classical orbit frequency become equal.

Solution :

According to Bohr's hypothesis, angular momentum of an electron is quantized. That is,

$$mvr = n\frac{h}{2\pi}$$

Using $v = r\omega$ in this equation,

$$\omega = \frac{nh}{2\pi mr^2} \qquad (1)$$

But, radius of electron orbit is given by

$$r = \frac{n^2h^2\in_0}{\pi me^2} \qquad (2)$$

Putting the value of equation (2) into (1),

$$\omega = \frac{\pi me^4}{2\in_0^2 h^3 n^3} \text{ or } f = \frac{me^4}{4\in_0^2 h^3 n^3} \qquad (3)$$

When electron makes transition from $(n + 1) \rightarrow n$ orbit, corresponding wave number is,

$$\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{(n+1)^2}\right]$$

$$= R\left[\frac{1}{n^2} - \frac{1}{n^2(1+\frac{1}{n})^2}\right] = R\left[\frac{1}{n^2} - \frac{1}{n^2} \times (1 + \frac{1}{n})^{-2}\right]$$

Since n is large, $\frac{1}{n} << 1$. Therefore, by keeping first two terms in Binomial expansion,

$$\frac{1}{\lambda} = R\left[\frac{1}{n^2} - \frac{1}{n^2} \times (1 - 2\frac{1}{n})\right] = R\left[\frac{1}{n^2} - \frac{1}{n^2} + \frac{2}{n^3}\right]$$

$$\frac{1}{\lambda} = \frac{2R}{n^3} = \frac{2me^4}{8\epsilon_0^2 ch^3} \cdot \frac{1}{n^3}$$

and corresponding frequency,

$$f = \frac{c}{\lambda} = \frac{me^4}{4\epsilon_0^2 n^3 h^3}$$ which is equation (3).

Illustration 4 : Angular velocity of an electron is n^{th} orbit in a hydrogen atom is given by, $\omega = \frac{\pi me^4}{2\epsilon_0^2 n^3 h^3}$. If we require the period of the electron in this orbit equal to that of geo-stationary satellites, what should be the value of n ? $m = 9.1 \times 10^{-31}$ kg, $e = 1.6 \times 10^{-19}$ C, $\epsilon_0 = 8.85 \times 10^{-12}$SI, $h = 6.625 \times 10^{-34}$ Js.

Solution : $\omega = \frac{2\pi}{T} = \frac{\pi me^4}{2\epsilon_0^2 n^3 h^3}$ (given)

$$\therefore\ n^3 = \frac{me^4 T}{4\epsilon_0^2 h^3}, \text{ where } T = 24 \times 3600 \text{ s}$$

$$\therefore\ n = \left[\frac{9.1 \times 10^{-31} \times (1.6 \times 10^{-19})^4 \times (24 \times 3600)}{4 \times (8.85 \times 10^{-12})^2 \times (6.625 \times 10^{-34})^3}\right]^{\frac{1}{3}}$$

$$= \left\{\frac{(5152702.46 \times 10^{-107})}{(91097.022 \times 10^{-126})}\right\}^{\frac{1}{3}} = (565.63 \times 10^{18})^{\frac{1}{3}}$$

$$n = 8.27 \times 10^6$$

Illustration 5 : Calculate the principal quantum number for which the radius of the orbit of the electron in Be^{+3} would be equal to that for the ground state of electron in a hydrogen atom. Also, compare the energy of the two states.

Solution : For Be, Z = 4 and for H, Z = 1.

Also,

$$r_n = \frac{h^2\epsilon_0}{\pi e^2 m}\frac{n^2}{Z}$$

$$\therefore\ r_n \propto \frac{n^2}{Z} \qquad (1)$$

For hydrogen atom in its ground state,

$$r_H \propto \frac{1^2}{1} = 1$$

Let $n = n$ be the principal quantum number for which radius of an electron orbit in Be^{+3} is equal to r_H.

i.e., $r_{Be^{+3}} \propto \frac{n^2}{4} = 1$

$\therefore \; n^2 = 4 \Rightarrow n = 2$

Thus, the radius of orbit corresponding to $n = 2$ is equal to that of ground state of H-atom.

Using an equation, $E_n = -\frac{Z^2 m e^4}{8\epsilon_0^2 n^2 h^2}$

$$E_n \propto \frac{Z^2}{n^2}$$

$$\therefore \; \frac{E_{Be^{+3}}}{E_H} = \frac{\left(\frac{4^2}{2^2}\right)}{\left(\frac{1^2}{1^2}\right)} = 4$$

i.e., $E_{Be^{+3}} = 4 \times E_H$

Illustration 6 : A mixture of ordinary hydrogen and its isotope, tritium (whose nucleus is approximately three times massive than ordinary hydrogen) is excited and its spectrum is studied. Calculate the shift in wavelength for the H_α lines in Balmer series. $R_\infty = 10973700 \text{ m}^{-1}$, mass of proton $M_H = 1.67 \times 10^{-27}$ kg, mass of electron, $m = 9.1 \times 10^{-31}$ kg.

Solution : Tritium is the isotope of hydrogen atom with one proton and two neutrons in the nucleus.

The wave numbers of the H_α line ($n = 3 \rightarrow n = 2$) of Balmer series are given by,

$$\frac{1}{\lambda_H} = R_H\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36} R_H \qquad (1)$$

$$\frac{1}{\lambda_T} = R_T\left(\frac{1}{2^2} - \frac{1}{3^2}\right) = \frac{5}{36} R_T \qquad (2)$$

Here, R_H and R_T are respectively the Rydberg constant for hydrogen and tritium atoms. From equations (1) and (2), shift (or difference) in their wavelengths is

$$\lambda_H - \lambda_T = \frac{36}{5}\left(\frac{1}{R_H} - \frac{1}{R_T}\right)$$

$$= \frac{36}{5}\left\{\frac{(1+\frac{m}{M_H})}{R_\infty} - \frac{(1+\frac{m}{M_T})}{R_\infty}\right\} \text{[using equation (5.5.4)]}$$

Where M_T = mass of tritium nucleus

$\approx$ 3 times mass of hydrogen nucleus, M_H

$$\therefore \lambda_H - \lambda_T = \frac{36}{5}\frac{m}{R_\infty}\left(\frac{1}{M_H} - \frac{1}{3M_H}\right) = \frac{36}{5} \times \frac{m}{R_\infty} \times \frac{2}{3M_H}$$

$$= \frac{72 \times 9.1 \times 10^{-31}}{15 \times 10973700 \times 1.67 \times 10^{-27}}$$

$$= 2.38 \times 10^{-10} \text{ m} = 2.38 \text{ Å}$$

5.6 Quantization of Energy and Momenta

We have seen that the emission spectra of hydrogenic atoms can be fitted to a common expression, $\frac{1}{\lambda} = R\left(\frac{1}{m^2} - \frac{1}{n^2}\right)$. The integers, occurring in these spectral series and discretness of emitted or absorbed wavelengths indicate the presense of something discrete in the atoms. Then one may curiously ask whether the discretness in atoms be treated/explained on the same footing to the quantum theory of black body radiation ?

Planck introduced quantum theory for radiation by quantizing the energy of an atomic oscillator, while Bohr, on the other hand, quantized the angular momentum of the electron moving in an orbit. For the stability of an atom, the electron in an atom should revolve only in non-radiating orbits (stationary orbits) around the nucleus. Further, we also know that the electron shows wave character in favorable circumstances, with its de Broglie wavelength, $\lambda = \frac{h}{mv}$. Here, h is the Planck's constant and $p = mv$ is its linear momentum.

Based on the quantum theory of Planck and dual nature of radiation and matter, we can understand how and why are physical quantities (like energy and momenta) quantized in an atom. Since the electron does not radiate in stationary orbits, the de Broglie wave associated with it must be a "stationary" wave. (Otherwise, energy will be carried away with the wave, and the path of an electron will be spiral ending into the nucleus.) Further, the wave representing the electron should be such that its value becomes the same at every point whenever it comes again to that point. Otherwise the probability of finding the electron at the same point will be different, which is not possible. This fact is picturized in figure 5.11 (*a*) and (*b*).

(*a*) (*b*)

Figure: 5.11 de Broglie's Interpretation of Bohr's Radius

Thus, out of all the orbits permitted by the classical physics, the **electron-wave** select only those orbits for which the circumference of the orbit is equal to an integral number (n) of the associated de Broglie wavelengths (figure 5.11 (*b*)). If it is not so, then the wave in each travel around the orbit will not be in phase and would interfere in such a way that their average intensity would be zero. In this case, an electron cannot be found in such an orbit.

Thus, necessary condition for permitted orbits (stationary orbits) is,

$$2\pi r = n\lambda \qquad (5.6.1)$$

where n = number of waves (integers)

r = radius of the orbit

and λ = wavelength of de Broglie wave = $\frac{h}{mv}$

$$\therefore\ 2\pi r = \frac{nh}{mv}$$

Now, angular momentum,

$$l = mvr = n\frac{h}{2\pi} \qquad (5.6.2)$$

With n = 1, 2, 3, ….. equation (5.6.2) shows the quantization of angular momentum of an orbital electron. Thus, the quantization rule for angular momentum, which was introduced in *adhoc* manner by Bohr to explain the stability of an atom, is found to be consistent with the wave character of electron and quantum theory of Planck.

Quantized angular momentum can now explain the discretness observed in radii for different orbits (equation 5.4.4) and energy of orbital electron (equation 5.4.8).

Illustration 7 : An elctron in hydrogen atom is revolving in an orbit with radius 5.29×10^{-11} m around the nucleus. Find the corresponding principal quantum number using the Bohr's quantum condition for allowed electronic orbit. $h = 6.625 \times 10^{-34}$ Js, $e = 1.6 \times 10^{-19}$ C, $\in_0 = 8.85 \times 10^{-12}$ MKS, $m = 9.1 \times 10^{-31}$ kg. Draw your inferences from the result.

Solution : For hydrogen atom (Z = 1), necessary centripetal force to the electron is provided by Coulombian force.

$$\frac{mv^2}{r} = \frac{1}{4\pi\in_0}\frac{e^2}{r^2} \quad (\because\ Z = 1)$$

$$\therefore\ v^2 = \frac{e^2}{4\pi\in_0 mr}$$

$$\therefore\ \text{linear momentum, } p = mv = \sqrt{\frac{me^2}{4\pi\in_0 r}}$$

Now, associated de Broglie wavelength,

$$\lambda = \frac{h}{mv} = \frac{h}{e}\sqrt{\frac{4\pi\in_0 r}{m}}$$

$$\therefore\ \lambda = \frac{6.625\times10^{-34}}{1.6\times10^{-19}}\sqrt{\frac{4\times3.14\times8.85\times10^{-12}\times5.29\times10^{-11}}{9.1\times10^{-32}}}$$

$$= 4.141 \times 10^{-15} \times \sqrt{64.617\times10^{8}}$$

$$\lambda = 3.328 \times 10^{-10}\ \text{m} = 3.328\ \text{Å} \qquad (1)$$

But, Bohr's quantum condition for allowed orbit requires,

$$2\pi r = n\lambda \qquad (2)$$

$$\therefore \; n = \frac{2\pi r}{\lambda} = \frac{2 \times 3.14 \times 5.29 \times 10^{-11}}{3.328 \times 10^{-10}}$$

$$= 9.98 \times 10^{-1}$$

$$\approx 1.0$$

Thus, according to Bohr's quantum condition, we say that electron is in ground state (n = 1). While according to de Broglie hypothesis, this means that there is one de Broglie wavelength that fit into the circumference of the orbit.

5.7 Excitation and Ionization Potential

Suppose an electron revolving in a stationary orbit of an atom absorbs specific energy, and jumps to an orbit of higher energy. This process is called excitation and the atom is said to be in the excited state. The energy absorbed to move electron from one orbit to the other is called excitation potential. For hydrogen atom,

1^{st} excitation energy ($n = 1 \rightarrow n = 2$) = 10.21 eV

2^{nd} excitation energy ($n = 1 \rightarrow n = 3$) = 12.09 eV

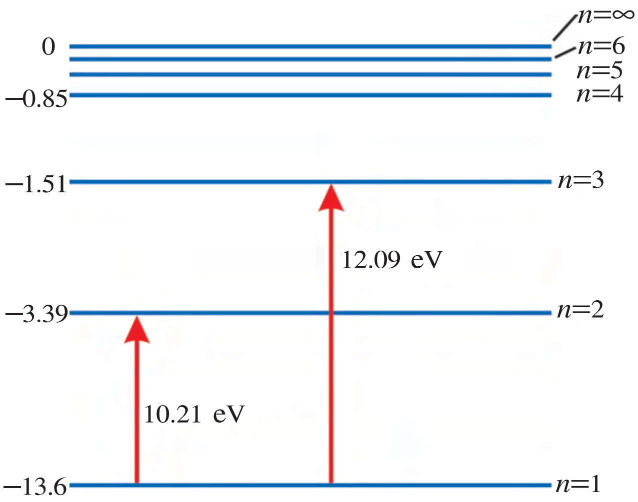


Figure 5.12 Excitation Energy (eV) for Hydrogen Atom

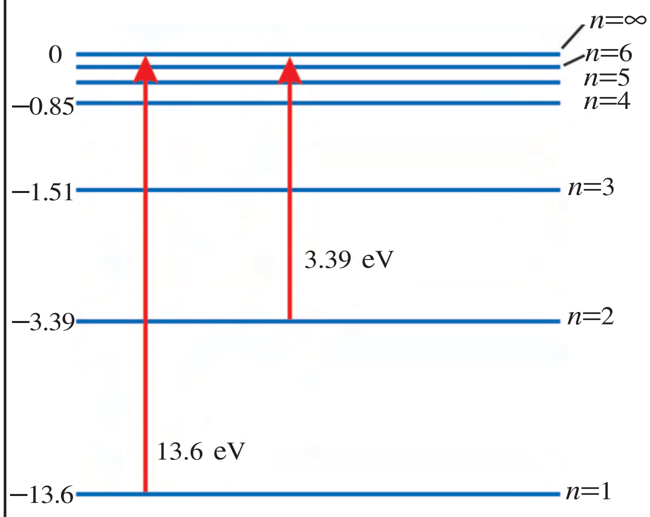


Figure 5.13 For Hydrogen Atom Ionization Energy (eV)

Similarly excitation energies for different transitions from different orbits in different atoms can be found. Figure 5.12 shows excitation energy for hydrogen atom.

If the energy supplied is large enough to remove an electron from the atom, then the atom is said to be ionized. The minimum energy needed to ionize an atom is called ionization energy. Remember in energy level diagram (see figure 5.13) energy is measured upwards, i.e. energy of free electron (in $n = \infty$ orbit) is taken as zero. The minimum amount of energy to release an electron from the ground state is called the first or principal ionization energy. The ionization of electrons from higher energy states is termed depending on their quantum numbers. For example, the second ionization energy (from level n = 2), third ionization energy (from level n = 3), etc.

1^{st} ionization energy ($n = 1 \rightarrow n = \infty$) = 13.6 eV

2^{nd} ionization energy (n = 2 to $n = \infty$) = 3.39 eV.

Figure 5.14 shows first ionization energy for different elements. It is to be noted that for closed shell rare gas elements ionization energy is highest in their respective groups, while alkali metals have the least. As one goes towards right (towards higher atomic number) in the same group ionization potential increases.

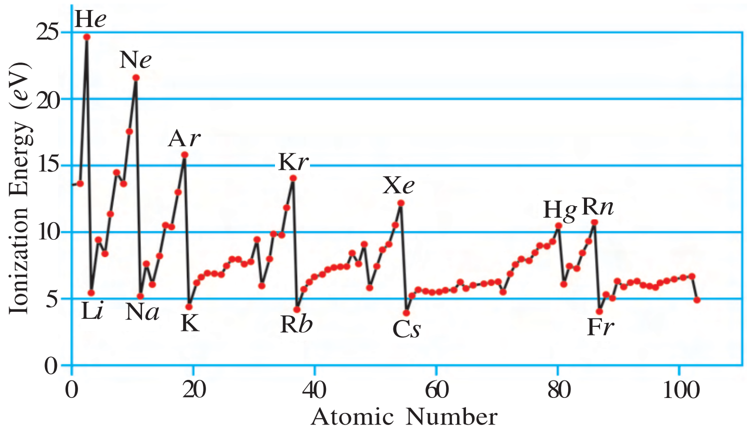
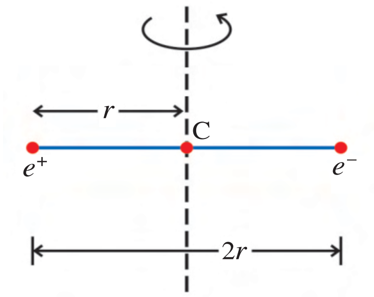


Figure : 5.14 First Ionization Energy for Different Elements (For information only)

Illustration 8 : A positronium–atom is a system of a positron (positive electron) and an electron, revolving about their centre of mass at their centre. Compare its ionization energy and emission wavelengths with those of hydrogen atom.

Solution : Since both the charge particles are moving about centre of mass (C), their kinetic energy can be calculated using the concept of reduced mass (μ).



$$\mu = \frac{m_+ m_-}{m_+ + m_-}$$

where m_+ = mass of positron

m_- = mass of electron

But, $m_+ = m_- = m$ (say)

$$\therefore \mu = \frac{1}{2}m$$

Total energy of the system, in ground state (i.e., $n = 1$) is

$$E = \frac{1}{2}\mu v^2 - \frac{e^2}{4\pi\epsilon_0 (2r)} \quad (1)$$

But $\mu v^2 = \frac{1}{2}mv^2 = \frac{1}{8\pi\epsilon_0}\frac{e^2}{r}$ From equation 5.4.1 and 5.4.3

$$\therefore \frac{1}{2}\mu v^2 = \frac{1}{16\pi\epsilon_0}\frac{e^2}{r} \quad (2)$$

$$\therefore E = -\frac{1}{16\pi\epsilon_0}\frac{e^2}{r} \quad (3)$$

Corresponding energy of hydrogen atom is,

$$E_H = -\frac{1}{8\pi\epsilon_0}\frac{e^2}{r} \text{ (see equation 5.4.6)} \quad (4)$$

$$\therefore E = \frac{1}{2}E_H$$

Since numerically first ionization energy is equal to the ground state energy, ionization energy of positronium-atom is half to the first ionization energy of hydrogen atom (+13.6 eV)

Also, emission spectrum has frequencies,

$f_{ik} = \frac{E_i - E_k}{h} = \frac{c}{\lambda_{ik}}$. Thus, wavelengths in emission spectrum is double to corresponding wavelengths in emission spectrum of hydrogen atom.

5.8 Emission and Absorption Spectra

In the experiment of a discharge tube filled with an atomic gas, the radiation of characteristic frequenices depending on the type of atoms is emitted and a line-spectrum is obtained.

When discharge takes place, the electrons of atoms of the gas get excited and go to higher energy states. In a time interval of about 10^{-8} sec electrons experience transition to lower energy states and emit radiation according to an equation $E_i - E_k = hf_{ik}$. The spectrum obtained in this way is called 'emission spectrum'. In 1870, Living and Dewar had noticed that the spectral lines of various elements could be grouped into distinct series. The simplest spectral series of hydrogen atom, Lyman, Balmer etc, can be examined by Bohr-like model. The next simplest spectra are of the "monovalent" atoms of alkalis, Li, Na, K etc., of the first column of the periodic table. But complexity in atomic spectera increases as we move towards the mid of the periodic table.

The general characteristics of atomic spectra can be summarized as follows :

(1) The position (Wavelengths) of spectral lines are characteristic of the atoms (i.e. the atomic number).

(2) The intensity of spectral lines grow as the concentration (Density) of atoms increases in the discharge tube.

(3) Intensity of spectral lines vary with temperature.

(4) The number of valence electrons in atoms determines the qualitative character of a spectrum (i.e. the number and type of series)

Let the atomic gas be at low temperature (e.g. room temperature) hence majority of the atoms are in ground state. Now suppose a radiation of continuous wavelengths is incident on atomic gas. Atoms absorb only specific amount of energy which is equivalent to energy difference of two allowed energy states. And corresponding wavelengths from the incident continuous radiation are absorbed. Thus, in the emergent radiation from the atomic gas, certain wavelengths, characteristic of the atoms are missing. These appear as dark lines in the emerging spectrum. Such a spectrum is known as 'absorption spectrum'.

For example, the radiation emitted by the lower layer of photosphere in the sun, which is at higher temperature is continuous. When this radiation passes through the outer part of the photosphere, which is at lower temperature, radiation of certain wavelengths are absorbed, and hence dark lines corresponding to these wavelengths are observed. These lines are called Fraunhoffer lines.

5.9 X-rays

X-rays were discovered by Rontgen in 1895. In the electromagnetic spectrum, the waves of wavelength between 0.001 to 1nm fall in this class. Coolidge designed a special kind of tube to have emission of X-rays in 1913, one such tube is shown in figure 5.15.

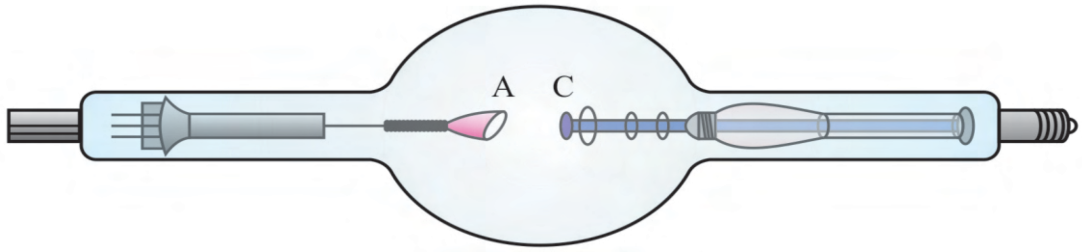


Figure 5.15 Coolidge tube for X-rays

Here, C is the cathode. When current is passed through the filament it gets heated and it heats the cathode which emits electrons through the process of thermionic emission. These electrons are accelerated between the cathode C and the anode A under the potential difference of 20 ~ 40 kV and allowed to fall on the anode. As a result, X-rays are emitted from the surface of the anode. Normally, the anode is made from a transition element (e.g. Mo)

X-ray Spectrum : Figure 5.16 shows the graph of wavelengths of X-rays emitted from the Mo target by 35 keV electrons against the relative intensity. Such a graph is called the X-ray spectrum corresponding to given element and energy of electrons.

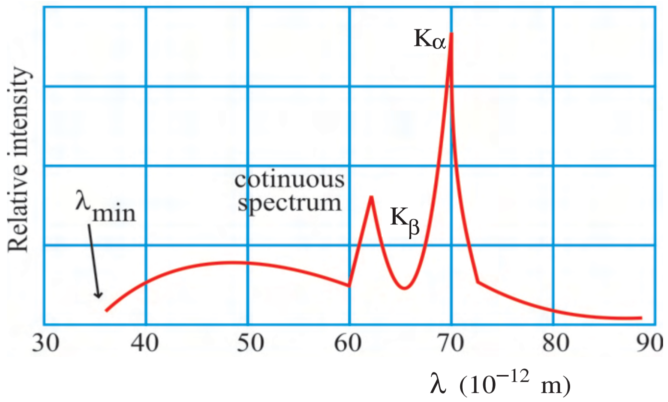


Figure 5.16 Characteristic Curve for X-Rays

Three characteristics of the graph are notable :

(1) The graph, starting from some minimum wavelength (λ_{min}) is continuous.

(2) The relative intensity is very large corresponding to some definite wavelengths.

(3) λ_{min} is a definite wavelength.

The continuous curve in the graph is called continuous X-ray spectrum. The peaks obtained for certain wavelengths indicates line spectrum. It is called the characteristic curve of given element.

Explanation of X-ray Spectrum : Highly energetic electrons enter the anode and collide with the atoms of the anode. They lose some energy during each collision. In this manner they go on losing energy during multiple collisions and emit X-rays of different frequencies and form a continuous spectrum of continuous frequencies (wavelengths).

Explanation of λ_{min} : If one or more electrons experience head-on collision with an atom of the anode in the beginning, its total kinetic energy completely gets converted into X-rays and hence X-ray of maximum frequency (minimum wavelength) is emitted.

If the electrons are accelerated under the potential difference V, their kinetic energy will be,

$$K = eV = \frac{1}{2}mv^2 \qquad (5.9.1)$$

Now, radiation energy = energy of photon = hf.

$$\therefore \; hf = eV$$

$$\therefore \; h\frac{c}{\lambda} = eV$$

$$\therefore \; \lambda = \frac{hc}{eV}$$

or $\lambda_{min} = \frac{hc}{eV}$ (5.9.2)

h = Planck's constant = 6.625×10^{-34} Js

c = Velocity of light = 3.0×10^{8} ms^{-1}

e = charge of an electron = 1.6×10^{-19} C

V = potential difference (V)

(**Note :** one photon is emitted due to each collision.)

Explanation of Characteristic X-ray Spectrum : Some of the incident electrons penetrate deep into the atoms of the anode and knock out the electrons from the atom from the inner shells. This creates vacancies. The electrons from outer shells experience transition to these vacancies and fill them up. The radiation of definite frequencies are emitted during such transitions.

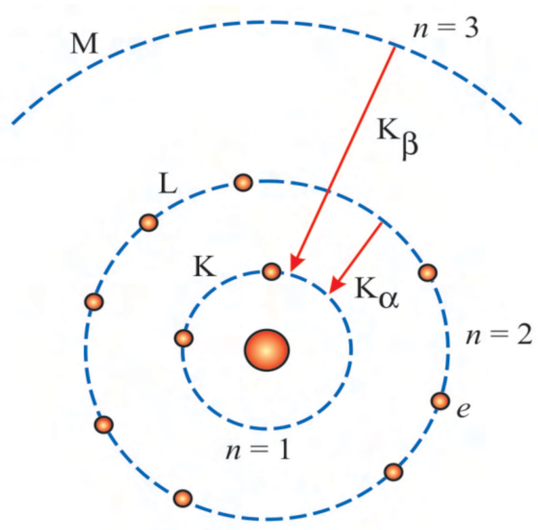


Figure 5.17 Electron Transition

The radiation is called K_{α} X-rays if it is emitted when the electron of K-shell corresponding to $n = 1$ is thrown out and the vacancy is filled by the electron from $n = 2$ (L-shell). Similarly the radiation is called K_{β} when it is emitted due to transition form $n = 3$ to $n = 1$. Thus, many lines of X-ray spectrum are obtained when electrons are thrown out from different shells and are replaced by electrons from different shells. The X-ray spectrum formed by such lines is called the characteristic spectrum. Such spectra depend on the type of element of the anode (target) as energies of electrons in K, L, M..... shells in the atoms of different types are also different. Hence, the wavelengths of K_{α}, K_{β}, L_{α},.... radiations are also different corresponding to different elements. That is why such curves are called characteristic spectrum for given element.

We can calculate the frequencies of the lines of characteristic X-ray spectrum with the help of a simple atomic model resembling Bohr's atomic model.

Suppose the atomic number of the element of the target is Z, i.e. there are Z electrons in its atom. Now in a multi-electron atom the electron under consideration in the presence of other electrons, cannot see (feel) the complete charge of the nucleus. That is, screening of the charge of the nucleus takes place due to other electrons and the electron under consideration can see only (Z − 1) e amount of charge on the nucleus.

Now from Bohr's atomic model,

Energy, $E_n = -\frac{mZ^2e^4}{8\varepsilon_0^2h^2}\frac{1}{n^2}$

In this equation taking Z = (Z − 1)

$$E_n = -\frac{m(Z-1)^2e^4}{8\varepsilon_0^2h^2n^2} \quad (5.9.3)$$

$$= -\frac{13.6(Z-1)^2}{n^2}\,eV, \quad (5.9.4)$$

where, $\frac{me^4}{8\varepsilon_0^2 h^2} = 13.6 \ eV$

To calculate the frequency of K_α radiation of a target of atomic number Z,

$$E_2 - E_1 = hf = 13.6 \ (Z - 1)^2 \left[\frac{1}{1^2} - \frac{1}{2^2}\right] \times 1.6 \times 10^{-19} \ J \quad (\because 1 \ eV = 1.6 \times 10^{-19} \ J)$$

$$\therefore f = \frac{13.6 \times 1.6 \times 10^{-19}}{6.62 \times 10^{-34}} \left[\frac{3}{4}\right] (Z - 1)^2$$

$$\therefore \sqrt{f} = \left(\frac{13.6 \times 1.6 \times 10^{-19} \times 3}{6.62 \times 10^{-34} \times 4}\right)^{\frac{1}{2}} (Z - 1)$$

$$\therefore \sqrt{f} = CZ - C = C(Z - 1) \tag{5.9.5}$$

where, $C = 4.965 \times 10^7 \ Hz^{\frac{1}{2}}$.

This equation represents a line, i.e. the graph of $\sqrt{f}$ versus Z is a straight line.

In fact in 1913, Moseley used a specially designed X-ray tube with a series of targets to obtain their characteristic spectra. Figure 5.18 shows the graph of the square root of frequency of K_α lines of 21 different elements versus their position in the periodic table.

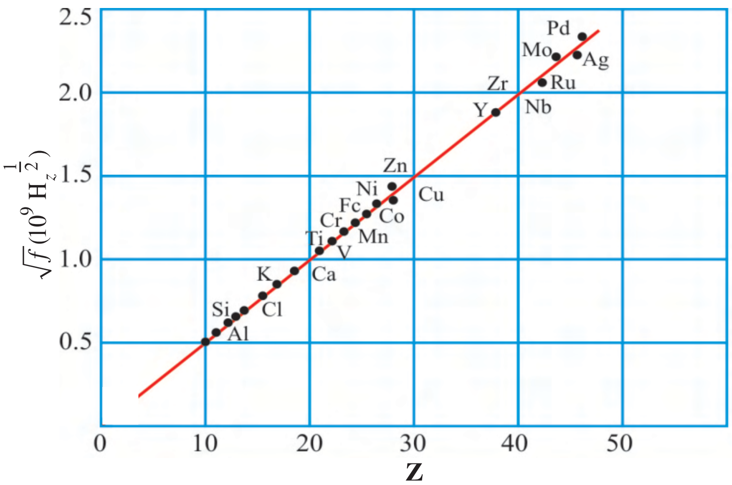


Figure 5.18 $\sqrt{f}$ **– Z Graph for K_α X-Ray Line**
(Only for Information)

This work of Moseley proved to be scientifically very important.

(1) At that (Moseley's) time the positions of different elements were decided from their atomic masses and not from their atomic numbers. With reference to the chemical properties these positions did not appear appropriate. Moseley suggested that the elements should be arranged with respect to their atomic numbers. On doing this, a relation between the chemical properties and positions of elements in the periodic table could be established.

(2) At that time, some places in the periodic table were missing. Atomic number Z of such missing elements could be decided from Moseley's work and such missing positions could be filled up with appropriate elements. The chemical properties of Lanthanide (or rare earth elements) were found to be very similar. So their positions in the periodic table could not be decided with certainty. This could be accomplished due to Moseley's work.

(3) In the periodic table, when the elements coming after uranium become available in sufficient quantity so that their x-ray spectra could be obtained, their positions were also fixed in the periodic table.

(4) As we have already seen earlier, the K_α radiation is associated with $n = 1$ shell which is nearest to the nucleus. Hence, information about the charge of the nucleus can also be obtained from such radiations.

(5) Ordinary emission or absorption spectra are associated with the transition of the valance electrons and hence no information regarding the charge of the nucleus can be obtained from them.

Illustration 9 : In X-ray tube the potential difference between the anode and the cathode is 20 kV and current flowing is 2 mA. Find,

(1) The number of electrons striking the anode in 1 s,

(2) The speed of electrons while striking the anode,

(3) Minimum wavelength (λ_{min}) emitted.

Solution :

(1) The number of electrons while striking the anode in 1 s.

$$n = \frac{I}{e} = \frac{2\times10^{-3}}{1.6\times10^{-19}} = 1.25 \times 10^{16}\ s^{-1}$$

(2) Speed of electrons, $v = \sqrt{\frac{2eV}{m}}$

$$= \sqrt{\frac{2\times1.6\times10^{-19}\times20\times10^{3}}{9.1\times10^{-31}}}$$

$$= \sqrt{7.033\times10^{15}}$$

$$= 8.386 \times 10^{7}\ ms^{-1}$$

(3) $$\lambda_{min} = \frac{hc}{eV} = \frac{6.625\times10^{-24}\times3\times10^{8}}{1.6\times10^{-19}\times20\times10^{3}}$$

$$= 6.21 \times 10^{-11}\ m = 0.621 \times 10^{-10}\ m = 0.621\ \mathring{A}$$

Illustration 10 : Calculate the atomic number of the element which gives minimum X-ray wavelength of 0.1 nm of K-series. $R = 1.09737 \times 10^{7}\ m^{-1}$.

Solution : From Moseley's law, the X-ray wavelength of K-series are given by,

$$\frac{1}{\lambda} = R(Z - 1)^2 \left(\frac{1}{1^2} - \frac{1}{n^2}\right) \qquad (1)$$

For minimum wavelength of the series, $n \to \infty$

$$\therefore \frac{1}{\lambda_{min}} = R(Z - 1)^2$$

$$\therefore (Z - 1) = \sqrt{\frac{1}{\lambda_{min} R}}$$

$$= \sqrt{\frac{1}{(0.1\times10^{-9})(1.09737\times10^{7})}}$$

$$= \sqrt{911.27}$$

$\therefore$ $(Z - 1) = 30.19$

$\therefore$ $Z = 31.19 \approx 31$ (Gallium)

SUMMARY

1. Atoms are divisible, and they are made up of more fundamental positive and negative charged particles.

2. Historically, Geiger-Marsden experiment on scattering of α-particles, and explanation of their results by Rutherford reveal the structure of an atom. It is known as Rutherford's atomic model. However, his model could not explain the stability of an atom.

3. The perpendicular distance of the initial velocity vector of the α-particle from the centre of the nucleus is known as impact parameter (b).

4. Neil Bohr through his hypothesis could explain the stability of an atom. However, Bohr's atomic model also found limitations in explaining certain details of atomic spectra.

5. However, Spectra of hydrogenic atoms can be explained by Bohr's model.

6. Radius of an electron in H-atom is given by,

$$r = \frac{h^2 \epsilon_0 n^2}{me^2 \pi Z} \Rightarrow r \propto \frac{n^2}{Z}$$

Energy of H-atom is given by,

$$E_n = \frac{-me^4 Z^2}{8\epsilon_0^2 n^2 h^2} \Rightarrow E_n \propto \frac{Z^2}{n^2}$$

7. Later, based on quantum theory of Planck and dual nature of radiation and matter, it was possible to prove quantization of orbital angular momentum of an electron, $l = n\hbar$.

8. Accidental discovery of X-rays by Rontgen in 1895, and extension of Bohr's atomic model by Moseley to understand K_α radiation were found to be very useful in arranging modern periodic table, to get information about the nucleus, etc.

For K_α line, $\frac{1}{\lambda} = R(Z - 1)^2\left(\frac{1}{1^2} - \frac{1}{n^2}\right)$.

EXERCISE

For the following statements choose the correct option from the given options :

1. An α-particle of 10 MeV is moving forward for a head on collision. What will be the distance of closest approach from the nucleus of atomic number Z = 50 ?

(A) 1.44×10^{-14} m (B) 2.88×10^{-14} m (C) 0.53×10^{-10} m (D) $\frac{0.53\times10^{-10}}{50}$ m

2. If the potential energy of the electron in the hydrogen atom is $\frac{-e^2}{4\pi\epsilon_0 r}$, then what is its Kinetic energy ?

(A) $\frac{-e^2}{4\pi\epsilon_0 r}$ (B) $\frac{e^2}{8\pi\epsilon_0 r}$ (C) $-\frac{e^2}{8\pi\epsilon_0 r}$ (D) $\frac{e^2}{4\pi\epsilon_0 r}$

3. According to Bohr's hypothesis, the angular momentum of the electron in any stationary orbit of radius r is proportional to ....

(A) r (B) $\frac{1}{r}$ (C) $\sqrt{r}$ (D) r^2

4. The radius of second orbit in an atom of hydrogen is R. What is its radius in third orbit ?

(A) 3R (B) 2.25R (C) 9R (D) $\frac{R}{3}$

5. The ratio of energies of electron in the first excited state to its second excited state in H-atom is ....

(A) 1 : 4 (B) 4 : 9 (C) 9 : 4 (D) 4 : 1

6. What is the angular momentum of an electron of Li-atom in $n = 5$ orbit.

(A) 5.27×10^{-1} Js (B) 6.625×10^{-34} Js (C) 1.325×10^{-34} Js (D) 16.56×10^{-34} Js

7. In an hydrogen atom, the radiation emitted is found to be in ultraviolet region due to transition between $n = 4$ to $n = 3$. During which of the following transitions will the light emitted be in infrared region?

(A) $2 \rightarrow 1$ (B) $3 \rightarrow 2$ (C) $4 \rightarrow 2$ (D) $5 \rightarrow 4$

8. The wavelength of the first line of Lyman series is λ. The wavelength of the first line in Balmer series is ....

(A) $\frac{27}{5}\lambda$ (B) $\frac{5}{27}\lambda$ (C) $\frac{9}{2}\lambda$ (D) $\frac{2}{5}\lambda$

9. The difference of wave numbers of lines with maximum and minimum wavelengths in Balmer series of hydrogen atom is .... m^{-1}. ($R = 1.097 \times 10^7$ m^{-1})

(A) 1.219×10^6 (B) 1.219×10^{-6} (C) 1219 (D) 1.219×10^7

10. For the first orbit of hydrogen atom the minimum excitation potential is .... C.

(A) 13.6 (B) 3.4 (C) 10.2 (D) 3.6

11. An electron with energy 12.09 eV strikes hydrogen atom in ground state, and gives its all energy to the hydrogen atom. Therefore, hydrogen atom is excited to .... state.

(A) fourth (B) third (C) second (D) first

12. In hydrogen atom, an electron makes transition from $n = 3$ to $n = 1$ state in time interval of 1.2×10^{-8} s calculate average toque (in Nm) acting on the electron during this transition.

(A) 1.055×10^{-26} (B) 4.40×10^{-27} (C) 1.7×10^{-26} (D) 8.79×10^{-27}

13. In which of the following system will the radius of the 2nd orbit be minimum?

(A) H-atom (B) Mg^{+11} (C) He^+ (D) B-atom.

14. The ionization potentials of hydrogenic ions A and B are V_A and V_B, respectively. Now if $V_B > V_A$ then ....

(A) $r_A > r_B$ (B) $r_A < r_B$ (C) $r_A = r_B$ (D) none of the above

Here, r is the radius of the ion.

15. The frequency of characteristic X-ray determines ...., property of the target.

(A) atomic weight (B) atomic number (C) melting point (D) conductivity

16. The operating voltage in Coolidge tube is 10^5 V. The speed of X-rays produced is .... ms^{-1}.

(A) 2×10^8 (B) 10^5 (C) 10^6 (D) 3×10^8

17. The wavelength of K_α spectral line is λ for an element of atomic number 43. The wavelength of K_α line for an element with atomic number 29 is .... λ.

(A) $\frac{43}{29}$ (B) $\frac{42}{28}$ (C) $\frac{9}{4}$ (D) $\frac{4}{9}$

18. If f_1, f_2 and f_3 are the frequencies corresponding to K_α, K_β and L_α X-rays for the given target. Then ....

(A) $f_1 = f_2 = f_3$ (B) $f_1 - f_2 = f_3$ (C) $f_2 = f_1 + f_3$ (D) $f_3 - f_2 = f_1$

19. A hydrogen atom absorbs 12.1 eV of energy and excited to higher energy level. How many photons are emitted during downward transition. Assume during each downward transition, one photon is emitted.

(A) 1 or 3 (B) 2 or 3 (C) 1 or 2 (D) 5 or more.

ANSWERS

1. (A)	**2.** (B)	**3.** (C)	**4.** (B)	**5.** (C)	**6.** (A)
7. (D)	**8.** (A)	**9.** (A)	**10.** (C)	**11.** (B)	**12.** (C)
13. (B)	**14.** (A)	**15.** (B)	**16.** (D)	**17.** (C)	**18.** (C)
19. (C)					

Answer the following questions in brief :

1. What is plum pudding model for atom ?
2. What is the use of lead blocks in α-particle scattering experiment due to Geiger and Marsden ?
3. What is the use of ZnS screen in α-particle scattering experiment due to Geiger and Marsden ?
4. What is impact parameter ?
5. What is the principal quantum number for third excited state ?
6. What are hydrogenic atoms ?
7. What are stationary orbits ?
8. Define excitation potential.
9. What happens to the intensity of spectral lines when conservation of atoms in discharge tube decrease ?
10. What are Fraunhoffer lines?
11. Who discovered X-rays ?
12. What is K_α line of X-rays ?

Answer the following questions :

1. Briefly discuss Thomson's plum pudding model.
2. Discuss only an experimental setup of Geiger-Marsden experiment for α-scattering.
3. Discuss only results of α-scattering experiment due to Geiger and Marsden.
4. Write drawback of Rutherford's atomic model.
5. What are atomic spectra? How are they obtained?
6. Write two hypothesis of Bohr's atomic model.

7. Using Bohr's atomic model, derive an equation for radius of orbit of an electron.
8. Explain energy level diagram of Hydrogen spectra.
9. Write an expression for energy of H-atom, using it, derive an equation,

$$\frac{1}{\lambda_{ik}} = \frac{me^4}{8\epsilon_0^2 ch^3}\left(\frac{1}{n_k^2} - \frac{1}{n_i^2}\right)$$ for electron transition from orbit $n = n_i$ to $n = n_k$.

10. Give two success of Bohr's atomic model.
11. Give two limitations of Bohr's atomic model.
12. Explain excitation potential.
13. Explain ionization potential.
14. Write difference between emission spectra and absorption spectra.
15. Give experimental arrangement for X-ray production through Coolidge tube.
16. Plot relative intensity versus wavelength graph for X-rays. Write characteristics of it.
17. Explain the origin of minimum wavelength in characteristic X-ray wavelength versus intensity graph.
18. Using Bohr's model, derive an expression, $\sqrt{f} = CZ - C$; where f is the frequency of K_α-line, Z is the atomic number of the target and C is constant.
19. Give two important points of Moseley's work on X-rays.

Solve the following examples :

1. How many revolutions does an electron perform before making transition from $n = 2$ to $n = 1$ state in hydrogen atom ? The average life time of first excited state in H-atoms is 10^{-8} s, $R = 1.097 \times 10^7$ m^{-1}. [**Ans. :** 8.23×10^6]
2. Calculate the wavelength and energy in eV of a photon emitted when in a hydrogen atom an electron makes a transition from the third excited state to the ground state.

   $R = 1.097 \times 10^7$ m^{-1}, $c = 3 \times 10^8$ ms^{-1} and $h = 6.625 \times 10^{-34}$ Js.

   [**Ans. :** 9.72×10^{-8} m, 12.76 eV]
3. Calculate the maximum wavelength of Balmer series in the hydrogen spectrum. Calculate the corresponding wave number. $R = 1.097 \times 10^7$ m^{-1}

   [**Ans. :** 6563 Å, 1.52×10^6 m^{-1}]
4. The H_α-line in Balmer series of the hydrogen spectrum has a wavelength 6563 Å. From this calculate the wavelength for the first line of the Lyman series (Ly_α).

   [**Ans. :** 1215 Å]
5. Wilson and Sommerfeld have defined a fine structure constant as, $\alpha = \frac{e^2}{4\pi\epsilon_0 \hbar c}$ in their atomic theory. Here, $\hbar = \frac{h}{2\pi}$. (i) Find dimension of α (ii) Find the value of $\frac{1}{\alpha}$ (iii) Express energy of hydrogen atom in terms of α (iv) Find the speed of electron in the orbit $n = 1$ in terms of α.

   [**Ans. :** (i) Dimension less (ii) 137 (iii) $\frac{-mc^2\alpha^2}{2n^2}$ (iv) αc]

6. At what temperature will the average molecular kinetic energy in gaseous hydrogen be equal to the binding energy of a hydrogen atom ? Boltzmann constant $k_B = 1.38 \times 10^{-23}$ JK^{-1}.

[**Ans. :** 1.051×10^5 K]

7. Calculate the energy required to remove an electron from He^+ ion.

[**Ans. :** 54.4 eV]

8. How much energy (in *e*V) should be given to an electron in the level $n = 2$ of a hydrogen atom, so that it emit the H_β-line of the Balmer series ? $R = 1.097 \times 10^7$ m^{-1}, $c = 3 \times 10^8$ ms^{-1}, $h = 6.625 \times 10^{-34}$ Js.

[**Ans. :** 2.56 eV]

9. Using the classical ideas, calculate the total energy of Li^+ – ion, calculate the percentage error with the experimental (198.09 eV) value. ($\frac{me^4}{8\epsilon_0^2 h^2} = 13.6$ eV)

[**Ans. :** 204 eV, 2.98 %]

10. A body of mass *m* is attached at one end of spring of force constant k. It is given a motion on a circular path of radius *r*. Assuming that there are integer number of waves representing this particle on the circumference of the circle and using Bohr's quantum conditions prove that the quantized energy is given by, $E_n = n\hbar\omega$; where n = integer, $\hbar = \frac{h}{2\pi}$ and angular frequency $\omega = \sqrt{\frac{k}{m}}$.

11. Estimate the value of the wavelength of K_α-line for silver (Z = 47).
$R = 1.09737 \times 10^7$ m^{-1}. [**Ans. :** 0.57 Å]

12. If the K, L and M energy levels of platinum are involved in emission of K_α and K_β lines of X-rays, corresponding energies are 78 keV, 12 keV and 3 keV, then find the wavelength of the K_α and K_β lines.
$h = 6.625 \times 10^{-34}$ Js, $c = 3 \times 10^8$ ms^{-1}, eV = 1.6×10^{-19} J.

[**Ans. :** $\lambda_{K_\alpha} = 0.188$ Å, $\lambda_{K_\alpha} = 0.165$ Å]

•

6

NUCLEUS

6.1 Introduction

We have seen in the previous chapter that the entire positive charge and almost the entire mass of the atom are concentrated in the small central region at its centre. This central region is called the nucleus. In this chapter we shall study about the constitution of nucleus, its dimension, mass, stability and the forces among its constituent particles, radioactivity, fission, fusion, energy produced in the stars etc.

6.2 Atomic Masses and The Constitution of Nucleus

You already know that the nucleus is made up of protons and neutrons, but the nucleus of the lightest* atom of hydrogen element is made up of a proton only as no neutron is present in it. The electric charge of proton has the same magnitude as that of an electron, that is 1.6×10^{-19} coulomb, but it is positive. Neutron has no charge. Proton and neutron each one is also called a nucleon. Instead of the entire atom, when we study the properties of its nucleus alone; that nucleus is also known as a nuclide. The nucleus of the atom of an element is symbolically represented as ${}_{z}X^{A}$ or ${}_{z}^{A}X$. Here X is the chemical symbol for the respective element. Z is the atomic number of that element. It shows the number of protons in the nucleus of the atom of that element. Moreover the atomic number also shows the position of that element in the Periodic Table. As the atom is electrically neutral, the number of electrons in it is also Z. A is called the atomic mass number or nucleon number of the nucleus of that element and it shows the total number of nucleons (proton and neutron) in the nucleus. A − Z = N is called the neutron number, which shows the number of neutrons in the nucleus. As for example, the nucleus of the atom of carbon element is represented as ${}_{6}^{12}C$. It contains 6 protons and (12 − 6 =) 6 neutrons. Moreover there are 6 electrons in the ${}_{6}^{12}C$ atom. In the nucleus of ${}_{82}^{208}Pb$ there are 82 protons and (208 − 82 =) 126 neutrons. And there are 82 electrons in the atom of ${}_{82}^{208}Pb$.

The atomic masses are extremely small as compared to 1 kg mass. e.g. The mass of ${}_{6}^{12}C$ atom is 1.992647×10^{-26} kg. It is more convenient to express such small masses in the unit called atomic mass unit instead of kg. It's symbol is u (also sometimes written as amu).

The twelfth part of the mass of unexcited ${}_{6}C^{12}$ atom is called 1 atomic mass unit.

Foot Note : * The lightest atom of H is ${}_{1}^{1}H$. Other atoms ${}_{1}^{2}H$ and ${}_{1}^{3}H$ are heavier than ${}_{1}^{1}H$.

$$\text{Thus, } 1\ u\ (\text{mass}) = \frac{1.992647 \times 10^{-26}}{12}\ \text{kg}$$

$$= 1.660539 \times 10^{-27}\ \text{kg}$$

For normal calculations it is also taken as 1.66×10^{-27} kg.

When the atomic masses of different elements are expressed in atomic mass unit (u), they are mostly found to be almost integer multiples of that of Hydrogen atom. But there are a few exceptions, e.g. the mass of the atom of Cl is 35.46 u. We will now see the reason for this.

The atomic masses are accurately measured with the help of an instrument called mass – spectrometer. In such experiments more than one type of atoms of the same element are found whose chemical properties are same but the masses are different. Such atoms of the same element but having different mass are called isotopes (In Greek, isotopes means same place). Such atoms occupy the same place in the periodic table. Thus in the different isotopes of the same element, the number of protons in their nuclei is the same but the number of neutrons is different and hence their masses are different. It was also found from experiments on mass spectrometer that in the objects found in nature, every element is made up of the mixture of its different isotopes and the relative proportions of such isotopes in the mixture are different for different elements. As for example, in case of Cl, the proportion of 34.98 u is 75.4% and that of 36.98 u is 24.6%. Hence the mass of Cl, atom is obtained from its weighted average.

$$\text{Thus, mass of Cl atom} = \frac{(75.4 \times 34.98) + (24.6 \times 36.98)}{100} = 35.47\ u$$

This value agrees with the experimental results.

[Usually the Cl atom with mass 34.98 u is written as ^{35}Cl and the one with mass 36.98 u is written as ^{37}Cl]. By subtracting the mass of electrons (mass of one electron is $m_e = 0.00055\ u$) from the mass of the atom, the mass of the nucleus is obtained.

In 1932 Chadwick, bombarded α-particles (They are nuclei of $_2He^4$ atoms. We shall further see about them.) on Be, and by applying law of conservation of energy and the law of conservation of mometum in the process, showed that the emitted particle in the process is electrically neutral, and its mass is almost equal to that of the proton. That particle was named as neutron. Chadwick was awarded the Nobel Prize in 1935 for this discovery of neutron. At present the mass of neutron is more accurately obtained as $m_n = 1.00866\ u = 1.6749 \times 10^{-27}$ kg.

The nuclei for which the neutron number (N = A – Z) is the same (but the Z values are different and A values are also different) are called the isotones of each other. The nuclei having the same atomic mass number (A = N + Z) are called the isobars of each other. For some nuclei Z values are same and A values are also same but their radioactive properties are different. They are called isomers of each other. $^{80}_{35}$Br has one pair of isomers. Prepare groups of isotopes, isotones and isobars from the nuclides give below :

$^{1}_{1}$H, $^{4}_{2}$He, $^{8}_{3}$Li, $^{9}_{4}$Be, $^{10}_{5}$B, $^{12}_{6}$C, $^{2}_{1}$H, $^{14}_{6}$C, $^{16}_{8}$O, $^{14}_{7}$N,

$^{13}_{6}$C, $^{233}_{92}$U, $^{235}_{92}$U, $^{238}_{92}$U, $^{214}_{82}$Pb, $^{214}_{83}$Bi, $^{3}_{1}$H, $^{3}_{2}$He, $^{7}_{3}$Li,

Illustration 1 : The masses of two isotopes of Boron ${}^{10}_{5}B$ and ${}^{11}_{5}B$ are respectively 10.01294 u and 11.00931 u. If the atomic mass of Boron is 10.811 u, find the proportion of these two isotopes.

Solution : If the proportion of ${}^{10}_{5}B$ is x %, the proportion of ${}^{11}_{5}B$ is $(100 - x)\%$

$$\therefore \; 10.811 = \frac{(x)(10.01294) + (100 - x)(11.00931)}{100}$$

$$\therefore \; 1081.1 = (10.01294 - 11.00931)\, x + 1100.931$$

$$\therefore \; 0.99637\, x = 19.831$$

$$\therefore \; x = 19.90 \; \%$$

$\therefore$ The proportion of ${}^{10}_{5}B$ would be 19.90 % and that of ${}^{11}_{5}B$ would be 80.10 %.

6.3 Nuclear Forces

In a tiny nucleus positively charged protons and charge-less neutrons are present. The coulomb repulsive forces between proton-proton act for all distances-small and large. Hence a natural question may arise as to how are they held tightly together in the tiny region like nucleus. Some other attractive force between the nucleons in the nucleus must be prevailing which should be large enough to overcome the effect of coulomb repulsive force and must be enough to hold them together.

Experiments have shown that inside a nucleus such a strong attractive force acts between two protons, between two neutrons and between proton-neutron. It is called strong (or nuclear) force. From the experiments from 1930 to 1950 a few characteristics known about this force are as under :

(1) Nuclear force is very much strong as compared to the coulomb force. (You also know that the coulomb force is very much strong as compared to the gravitational force.)

(2) This nuclear force quickly decreases to become zero for distances larger than a few femtometer [1 fm = 1 femtometer = 10^{-15} m. This distance is also known as 1 fermi (fm)]. Actually this force is attractive for distances greater than 0.8 fm (only up to small distance) and for distances smaller than 0.8 fm it is repulsive (!!)

(3) Nuclear force is not charge dependent. That is, the nuclear force between two protons, between two neutrons and between proton-neutron is almost the same. Hence proton and neutron are known with a common name nucleon.

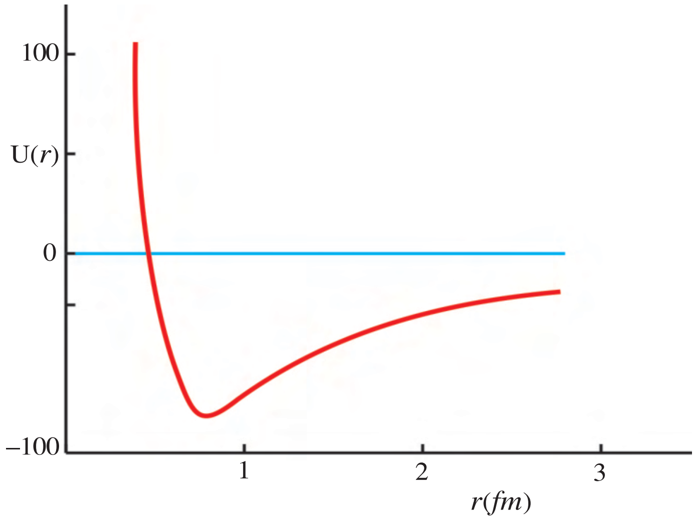


Figure 6.1 Potential Energy Between Two Nucleons

(4) This strong force is a short range force. Hence except in a small nucleus, a given nucleon can interact only with a few neighbouring nucleons very close to it, but not with all nucleons. This fact is often referred to as the saturation property of the nuclear forces. The nature of the graph of potential energy corresponding to the force between two nucleons against the distance is shown in the figure (6.1). For distance greater than 0.8 fm (there the force is attractive) this graph can be represented by

$U(r) = -g^2 \dfrac{e^{\frac{-r}{R}}}{r}$. R and g are constants. g^2 is called the strength parameter.

(5) The strong forces between the nucleons are presently not included in the category of fundamental forces. Neutron and proton are now considered as made up of quarks and the forces between the quarks are considered as fundamental forces. Such forces are ultimately resulted into nuclear forces between the nucleons. At present total six types of quarks are known to exist (They are named as up, down, charm, strange, top, bottom). Quarks are not separable from neutron or proton. A single isolated quark is not found in free state.

(6) Nuclear forces depend on the 'spin' of the nucleons.

(7) For gravitational force and the electrical force simple formula are obtained. But for nuclear force no simple formula is obtained.

6.4 Nuclear Radius

From Rutherford's experiments on α-particle scattering the nuclear radius was primarily estimated to be of the order of 10^{-14} m. Thereafter from modern experiments more accurate observations are obtained. The density of nuclear matter is not the same in the entire nucleus. The variation of density of nuclear matter (ρ) with distance (r) from its centre is shown in the figure 6.2. In the central region of the nucleus the density has the same value but in the surface region it gradually decreases. Thus nucleus does not have a definite surface but it has a characteristic average (or effective) radius R, which is given by the following formula.

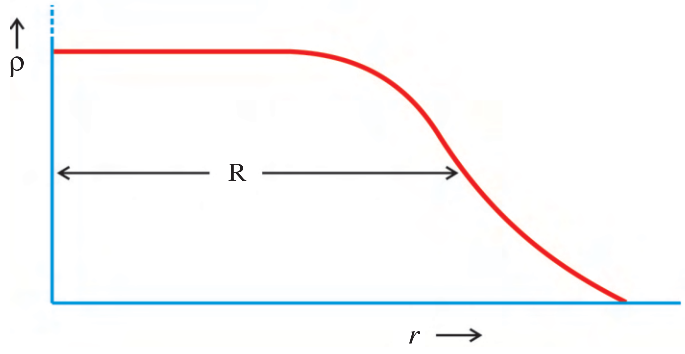


Figure 6.2 Variation in the Density of Nuclear Matter

$$R = R_0 \ A^{\frac{1}{3}}$$

where A is the atomic mass number and R_0 is a constant. The value of R_0, depends on the physical effects corresponding to which the experiments are performed. As for example, if the nuclear density is measured with the help of experiment on electron-nucleus collision then the distance from centre at which the density becomes 50% is taken as the average radius of the nucleus.

The radius obtained with the collision of α-particles and nucleus is called the interaction radius. But in both the types of the experiments the value of R_0 found in different cases is nearly 1.1 to 1.2 *fm*. For calculations we will take $R_0 = 1.1$ *fm*.

By obtaining R from value of A of the nucleus, its volume can be found. From its mass and the volume, its density can be found. For all nuclei the density is found to be nearly 2.3×10^{17} kg m^{-3}. This value is 2.3×10^{14} times that of water. By comparision it is found that nuclear radius is 10^{-4} times that of the atom and the nuclear volume is 10^{-12} times that of the atom but it contains 99.9% of the atomic mass. Hence its density is enormous.

For Information Only : The radius of the atom is nearly 10^4 times that of the nucleus. Hence there is vacant space near the nucleus which can accommodate other 9999 nuclei of its kind and then comes the electron. From this we feel that in ordinary matter – which is made up of atoms - there is too much empty space !!

6.5 Nuclear Stability

It has been found that in the stable nuclei of the atoms of the light elements the number of proton (Z) and the number of neutron (N) are equal or almost equal, while in the stable nuclei of heavy elements, the number of neutron is greater than the number of proton. As for example, ${}^{12}_{6}C$ is stable and contains equal number of proton and neutron. ${}_{82}Pb^{208}$ is a stable atom of heavy element, in which number of neutron exceeds that of proton by 44. Thus for stable nuclei of light elements the value of $\frac{N}{Z}$ is almost 1 while for stable nuclei of heavy elements $\frac{N}{Z}$ value is greater than 1.

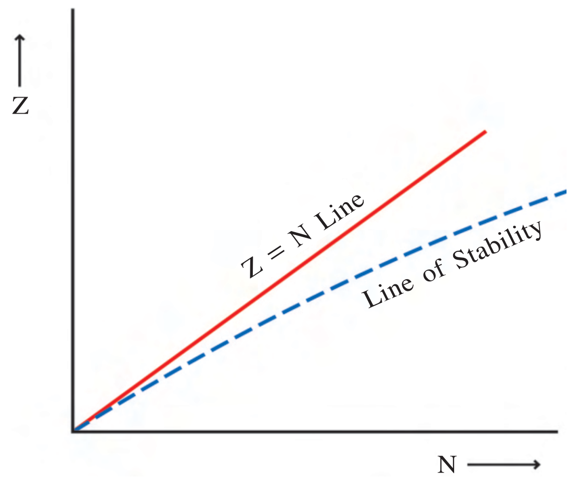


Figure 6.3 Z–N Graph for Stable Nuclei

In figure 6.3, the form of the graph of number of proton (Z) against that of neutron for some stable nuclei is shown by dotted line. Such a graph is called a nuclidic chart. This dotted line is called the line of stability.

The position of a stable nucleus is on or near this line and that of unstable nucleus is comparatively away from this line.

Morever, Z = N line is also shown in the figure. The initial small part of line of stability almost coincides with Z = N line. Thereafter the line of stability leans towards N-axis. It indicates that the value of N exceeds that of Z in heavy stable nuclides. The reason for this is as under :

In going from lighter to the heavier element, if one proton and one neutron are increased in the nucleus, then (i) that proton increases the coulomb repulsive force by interaction with all other protons in the nucleus (ii) that proton increases the attractive force by interaction through strong force with only a few neighbouring nucleons very close to it. (iii) the neutron also interacts with few neighboring nucleons through strong force to increase attraction. Hence the increase in the repulsive force is more than the increase in the attractive force. But if along with one proton, more than one neutron is added then enough attractive force can be produced to balance repulsive force and the stability of nucleus can be maintained. In light nuclei the numbers of proton and neutron are small, hence such a problem does not arise.

6.6 Mass-energy and Nuclear Binding Energy

Mass-energy : Before Einstein presented his special theory of relativity it was believed that in every process mass is conserved and the energy is also conserved separately. But Einstein showed from his special theory of relativity that mass can be converted into energy and energy can also be converted into mass and the relation between them is

$$E = mc^2 \qquad (6.6.1)$$

where, E = energy, m = mass which is transformed, c = velocity of light in vacuum. Thus mass m and energy mc^2 are equivalent to each other. Hence mass should also be considered as one form of energy.

In atomic and nuclear physics a unit of energy called electron volt (symbol : eV) is used.

"The change in the kinetic energy of an electron in passing through a potential difference of 1 volt is called 1 electron volt (eV) energy."

Here, it is clear that $1eV = 1.6 \times 10^{-19}$ J

Moreover, keV and MeV units are also used.

$1\ keV = 1$ kilo electron volt $= 10^3 eV = 1.6 \times 10^{-16}$ J

$1\ MeV = 1$ Million electron volt $= 10^6 eV = 1.6 \times 10^{-13}$ J

We can find the energy equivalent to 1 u mass by using the formula $E = mc^2$. In this way we get

$$1\ u\ (\text{mass}) \equiv 931.48\ MeV\ (\text{energy}) \tag{6.6.2}$$

Now in any process, instead of talking conservation of mass and conservation of energy separately we shall consider the conservation of energy and in the forms of energy we shall include the energy equivalent to mass.

Binding Energy of Nucleus : As a nucleus is made up of protons and neutrons, at first sight it appears that the mass of the nucleus would be equal to the sum of the masses of all of its protons and neutrons in the free state. But it is found that the mass of the nucleus is always less than the total mass of its constituents in the free state. This decrease in the mass is called the mass defect (Δm). If the mass of a nucleus $^A_Z X$ is M and we indicate the masses of proton and neutron in the free state as m_p and m_n, then always $M < Zm_p + Nm_n$ where $N = A - Z$ = number of neutrons, and

$$(Zm_p + Nm_n) - M = \text{mass defect } (\Delta m) \tag{6.6.3}$$

To explain this fact the equivalence between mass and energy is useful. If we form this nucleus from those constituent particles in the free state; the energy (Δmc^2) equivalent to mass defect (Δm) is generated and emitted out. Hence, if now we want to take all protons and neutrons of the nucleus; in the free state, this much energy has to be supplied from outside. Hence, the energy (Δmc^2), equivalent to the mass defect Δm is called the binding energy E_b of the nucleus. By dividing the binding energy E_b of the nucleus by its nucleon number (A), we get the average binding energy per nucleon $E_{bn}\left(= \frac{E_b}{A}\right)$. Thus E_{bn} is the average energy to be supplied per nucleon to make all constituents free from the nucleus.

The binding energy per nucleon is the measure of the stability of a nucleus.

Consider deuteron ($_1H^2$) nucleus. Its mass is 2.0141 u. Moreover the sum of the masses of the proton and the neutron in free state is 2.0165 u. Hence, the mass defect for $_1H^2$ is $\Delta m = 2.0165 - 2.0141 = 0.0024u$.

The energy equivalent to this mass defect is $0.0024 \times 931.48 = 2.24$ MeV. This energy is called the binding energy of ^2_1H. Thus to liberate proton and neutron from $_1H^2$, 2.24 MeV energy has to be supplied to it from outside. Conversely, if one proton and one neutron coalease to form, $_1H^2$, 2.24 MeV energy is emitted out. For $_1H^2$ nucleus the average binding energy per nucleon is $E_{bn} = \frac{2.24}{2} = 1.12 \frac{MeV}{\text{nucleon}}$.

The graph of binding energy per nucleon E_{bn} Vs. atomic mass number A is shown in the figure 6.4.

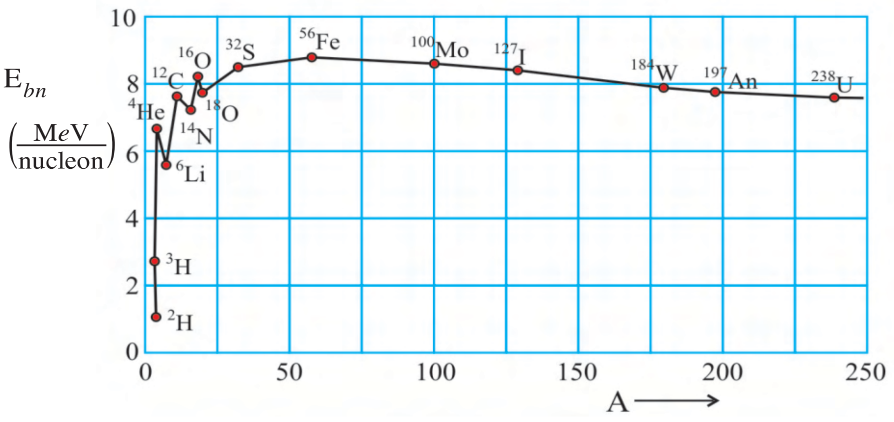


Figure 6.4 **Graph of $E_{bn} \rightarrow A$**

The notable points found from the graph are as follows :

(1) The graph rises quickly in the beginning. Thereafter it rises slowly. Near the nucleus of iron (A = 56), E_{bn} is found to be maximum and nearly equal to 8.8 $\frac{MeV}{nucleon}$. Thereafter the graph descends very slowly.

(2) For $A < 30$ and $A > 170$ the values of binding energy per nucleon are small.

(3) For nuclei with intermediate masses ($30 < M < 170$), the value of E_{bn} is almost constant. These nuclei are the most stable ones. Hence very much energy has to be supplied to liberate nucleons from them. The constancy of the binding energy per nucleon is the result of the nuclear forces to be short range. (That is, it is due to the saturation property of the nuclear forces.)

In a sufficiently large nucleus most of the nucleons reside in the interior part of the nucleus and the number a nucleons on the surface is less. Moreover every nucleon can interact only with a few neighbouring nucleons. Now if a nucleon is added to it, it does not interact with the nucleons in the interior and the number of nucleons on the surface is comparatively less. Hence the variation in E_{bn} is very small.

(4) The values of binding energy per nucleon for He^4, Be^8, C^{12}, O^{16}, ... are higher than those of their neighbouring nuclei. This fact indicates that there is a shell type structure (like the atom) even for a nucleus.

(5) The values of E_{bn} $\left(=\frac{E_b}{A}\right)$ are greater for the nuclei of intermediate masses, than those for nuclei heavier ($A > 170$) than them. Hence if such a heavy nucleus gets divided in two lighter nuclei then the binding energy per nucleon increases. Hence nucleons are more tightly bound with each other. This shows that energy is produced (released) in this process. This process is called nuclear fission.

Instead, if two nuclei (with $A < 10$) are fused to form a heavier nucleus then also the binding energy per nucleon is increased from earlier. Thus in this process also energy is produced. This process is called nuclear fusion.

Illustration 2 : (*a*) Find the binding energy per nucleon for $^{56}_{26}Fe$ nucleus from the data given below. (*b*) If the proton with least binding in this nucleus is emitted, $^{55}_{25}Mn$ nucleus is formed. Find the binding energy of this proton. Mass of proton m_p = 1.007825 *u*, mass of neutron m_n = 1.008665 *u*, mass of Fe nucleus M_{Fe} = 55.934939*u*, mass of M*n* nucleus *m* = 54.938046 *u*, 1 *u* = 931.494 M*e*V.

Solution : (*a*) $^{56}_{26}Fe$ nucleus has 26 protons and 30 neutrons. When these protons and neutrons are in free state their total mass = $Zm_p + Nm_n = 26\ m_p + 30\ m_n$

$$\therefore \text{ Mass defect } \Delta m = (Zm_p + Nm_n) - (M_{Fe})$$
$$= (26 \times 1.007825 + 30 \times 1.008665) - (55.934939)$$
$$= 0.528461\ u$$

$\therefore$ Binding energy E_b = energy equivalent to Δm = 0.528461 × 931.494 = 492.258 M*e*V

$$\therefore \text{ Binding energy per nucleon } E_{bn} = \frac{E_b}{A} = \frac{492.258}{56}$$

$$\therefore E_{bn} = 8.79\ \frac{MeV}{nucleon}$$

(*b*) If the proton is separated from $^{25}_{25}Fe$, the following reaction takes place.

$$^{56}_{26}Fe \rightarrow ^{55}_{25}Mn + ^{1}_{1}p$$

Total mass of M*n* and *p* = 54.938046 + 1.007825
= 55.945871 *u*

and mass of Fe is M_{Fe} = 55.934939 *u*

Thus in this reaction mass is increased. It shows that this reaction cannot occur spontaneously (naturally by itself) but if energy is supplied from outside, then only it can occur.

$\therefore$ Proton's binding energy = Energy to be supplied
= (energy equivalent to increase in mass)
= (55.945871 − 55.934939) (931.494) M*e*V
= 0.010932 × 931.494
= 10.18 M*e*V.

The energy to be supplied to separate a nucleon from a nucleus is called separation energy.

6.7 Natural Radioactivity

In 1895, scientist Rontgen discovered X-rays. Thereafter in 1896 in the study of knowing the relation of production of X-rays with the phenomenon of fluorescence, Becquerel found that radiations of certain specific properties are emitted naturally from uranium. This phenomenon was called the natural radioactivity. Those radiations were initially known as Becquerel rays.

Madam Curie and her husband Perie Curie separated two new elements from the ore of uranium called pitch blende. They were named as Polonium and Radium. These elements also possess the property of natural radioactivity and their activities are several times that of uranium.

Thereafter other scientists found that other heavy elements like Thorium, Actinium, also possess the property of radioactivity. Such elements are called radioactive elements and the radiations emitted from them are called radioactive radiations. Notable facts of this phenomenon are :

(1) The emission of radioactive radiations is spontaneous, instantaneous and continuous. It is not affected by external factors like change in temperature or pressure, presence of electric or magnetic field. Such parameters cannot stop the emission of radioactive radiations or cannot change the rate of emission.

(2) Even by chemically combining a radioactive element with any other element, the rate of emission of radiations is not affected.

These two points show that radioactivity is a nuclear phenomenon.

Actually the nuclei of heavy elements are unstable from their very birth in nature and during their attempts to acquire stability, they emit radioactive radiations.

This discovery can be considered very important in the development of modern physics.

6.8 Radioactive Radiations

Radioactive radiations are of three types : α–rays, β–rays, and γ–rays. From the information gathered by scientists from experiments their properties are found as under:

α-rays : α-rays are the material particles made up of 2 protons and 2 neutrons. It means that they are nuclei of Helium atom ($_2He^4$) only. They have $+2e$ electric charge. Their velocity depends on the nuclide emitting them.

β-rays : β-rays are electrons themselves. (But they have come by emission from the nucleus). Thus they are material particles. Their velocity also depends on the nuclide emitting them.

γ-rays : They are not material particles but are electromagnetic waves.

All these radioactive radiations affect the photographic plate, produce fluorescence; ionize the atoms of the medium through which they pass and penetrate upto a certain distance in a medium. The relative values of their ionizing–power and penetration power are shown in the Table 1.

Table 1

	α	β	γ
Ralative ionizing-power	10000	100	1
Relative penetration power	1	100	10000

6.9 Radioactive Constant and Activity

In a specimen of radioactive material, if the number of undisintegrated nuclei of an element at time t is N and thereafter if ΔN nuclei disintegrate in time-interval Δt, then $\lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t} = \frac{dN}{dt}$ is called the rate of disintegration, or the decay rate or activity I of that element, at time t. Activity means the number of nuclei decaying per unit time.

In this process it is found that the decay rate is proportional to the number of undisintegrated nuclei at that time.

$$\therefore \frac{dN}{dt} \propto -N \text{ [the negative sign indicates that as time passes N decreases]} \quad (6.9.1)$$

$$\therefore \frac{dN}{dt} = -\lambda N \quad (6.9.2)$$

$$\text{or } I = -\lambda N \quad (6.9.3)$$

Here λ is a constant, which is called the radioactive constant (or the decay constant) of the disintegrating element. Its unit is s^{-1}. Its value depends on the type of disintegrating element, but for different unstable isotopes of the same element, the values of λ are different.

A larger value of λ indicates that the rate of disintegration is greater. Such elements are short-lived. A smaller value of λ indicates that the rate of disintegration is smaller. Such elements are long-lived. External factors (like pressure, temperature, magnetic field, electric field.....) do not affect the value of λ.

In equation (6.9.2), $\frac{dN}{dt} = -\lambda N$, taking time-interval dt = unit; we get $\lambda = -\frac{dN}{N}$.

Hence we can interpret that "for the nucleus of a given element λ shows the probability of disintegration per unit time ."

Units of Activity : In memory of Becquerel, the SI unit of activity is Becquerel (B*q*). "Activity of a substance having 1 disintegration per second is called 1 Becquerel."

1 B*q* = 1 disintegration / sec.

A unit of activity determined in the memory of Madame Curie is known as Curie (C*i*). "The activity of a substance in which 3.7×10^{10} disintegrations per second take place is called 1 curie (C*i*)". For practical purposes millicurie and microcurie are also used as units.

$1 \text{ m}Ci = 10^{-3} \text{ C}i$, $1 \text{ µ}Ci = 10^{-6} \text{ C}i$

6.10 Exponential Law of Radioactive Disintegration

Suppose in a given sample, at time $t = 0$, the number of undisintegrated nuclei of a radioactive element is N_0 and at time $t = t$, it is N. At this time its rate of disintegration $\left(\lim_{\Delta t \to 0} \frac{\Delta N}{\Delta t} = \right) \frac{dN}{dt}$, is proportional to N and according to the equation (6.9.2),

$$\frac{dN}{dt} = -\lambda N$$

$$\therefore \frac{dN}{N} = -\lambda dt \quad (6.10.1)$$

Integrating on both the sides, we get

$$\ln N = -\lambda t + C \quad (6.10.2)$$

where C = constant of integration. For $t = 0$, $N = N_0$.

$$\therefore \ln N_0 = 0 + C = C \quad (6.10.3)$$

Substituting this value in equation (6.10.2), we get

$$\ln N = -\lambda t + \ln N_0$$

$$\therefore \ln N - \ln N_0 = -\lambda t$$

$$\therefore \; ln\left(\frac{N}{N_0}\right) = -\lambda t$$

$$\therefore \; \frac{N}{N_0} = e^{-\lambda t} \qquad (6.10.4)$$

$$\therefore \; N = N_0 e^{-\lambda t} \qquad (6.10.5)$$

Since $I \propto N$, we can also write $I = I_0 e^{-\lambda t}$ (6.10.6)

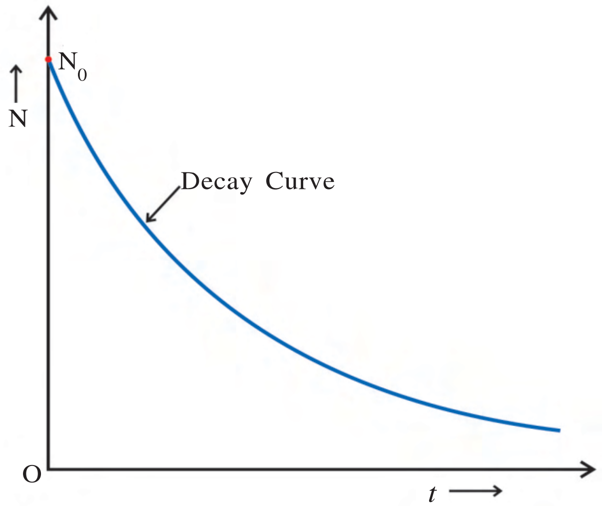


Figure 6.5 Decay Curve

Equation (6.10.5) is called the exponential law of radioactive disintegration. It shows that as time passes, the number of nuclei of a radioactive element decreases exponentially. Hence the activity also decreases exponentially. Actually, the activity (means the decay rate) is a more directly measurable quantity than the number of nuclei of radioactive element. For a given radioactive element the graph of N against t is shown in the figure. This curve is called the decay curve. It is clear that $I - t$ graph would also be similar to this.

6.11 Half Life $\tau_{\frac{1}{2}}$

As time passes, the number of nuclei of a radioactive element decreases. A quantity called half–life ($\tau_{\frac{1}{2}}$) related with this process is defined as under :

"The time-interval during which the number of nuclei of a radioactive element becomes half of its value at the beginning of the time-interval, is called the half life ($\tau_{\frac{1}{2}}$) of that element ".

According to this definition, in the exponential law of radioactive disintegration $N = N_0 e^{-\lambda t}$. when, $N = \frac{N_0}{2}$, we put time-interval t = half life $\tau_{\frac{1}{2}}$

$$\therefore \; \frac{N_0}{2} = N_0 e^{-\lambda \tau_{\frac{1}{2}}} \qquad (6.11.1)$$

$$\therefore \; 2 = e^{\lambda \tau_{\frac{1}{2}}}$$

$$\therefore \; ln\ 2 = \lambda \tau_{\frac{1}{2}}$$

$$\therefore \; (2.303)\ (log\ 2) = \lambda \tau_{\frac{1}{2}}$$

$$\therefore \; \tau_{\frac{1}{2}} = \frac{(2.303)(0.3010)}{\lambda}$$

$$\therefore \; \tau_{\frac{1}{2}} = \frac{0.693}{\lambda} \qquad (6.11.2)$$

Since the activity of a radioactive element is proportional to the number of undisintegrated nuclei N, it is clear that during the time-interval equal to the half-life the activity also becomes half. The half-lives of different radioactive elements range in a large interval, from 10^{-7}s to 10^{10} yr.

For an element with half-life of 10 yrs, if we mean that at 20 years all of its nuclei will disintegrate (that means, the existence of this element will be abolished), then it is not true. Actually at every 10 years number of its nuclei will become half (see figure 6.6) and even after a very long time, a certain number of nuclei of this element do survive. From this, we can represent as under :

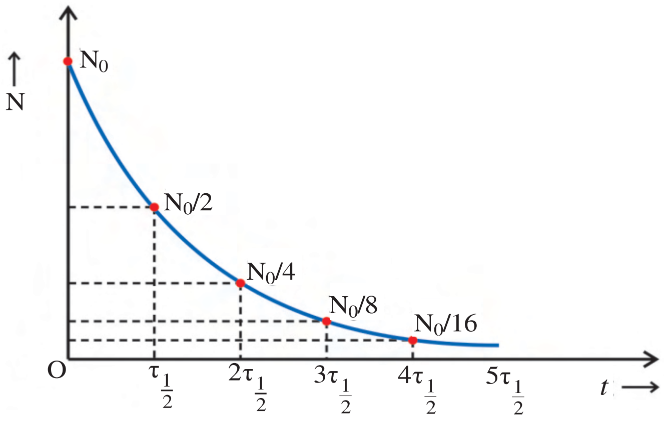


Figure 6.6 Half-Life

At time $t = 1\ (\tau_{\frac{1}{2}})$, $\left(\frac{N}{N_0}\right)$ becomes $\left(\frac{1}{2}\right)^1$

At time $t = 2\ (\tau_{\frac{1}{2}})$, $\left(\frac{N}{N_0}\right)$ becomes $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^1 = \left(\frac{1}{2}\right)^2$

At time $t = 3\ (\tau_{\frac{1}{2}})$, $\left(\frac{N}{N_0}\right)$ becomes $\left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^3$

$\vdots \qquad \vdots$

At time $t = n\ (\tau_{\frac{1}{2}})$, $\left(\frac{N}{N_0}\right)$ becomes $\left(\frac{1}{2}\right)^n$

i.e. At anytime t, $\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$

where $n = \frac{\text{given time}(t)}{\text{half life}(\tau_{\frac{1}{2}})}$ (6.11.4)

6.12 Mean Life (τ)

"The time-interval, during which the number of nuclei of a radioactive element becomes equal to the e^{th} part of its original number, is called the mean life or average life τ of that element." ($e = 2.718$)

In the exponential law of radioactive disintegration $N = N_0 e^{-\lambda t}$, when $N = \frac{N_0}{e}$, we can put t = mean life = τ.

$$\therefore \frac{N_0}{e} = N_0 e^{-\lambda\tau}$$

$$\therefore e = e^{\lambda\tau}$$

$$\therefore 1 = \lambda\tau$$

$$\therefore \ \tau = \frac{1}{\lambda} \tag{6.12.1}$$

Thus the mean life is equal to the reciprocal of the decay constant.

Radioactive elements having very short life-time (e.g. Plutonium) have half life very small compared to the age of the universe (1500 crore years). They have been decayed long ago and at present not found in nature (means their proportion would be extremely less). But they can be prepared by artificial nuclear processes.

From equations (6.11.2) and (6.12.1) it is clear that

$$\tau_{\frac{1}{2}} = (0.693)(\tau) \tag{6.12.2}$$

Here note that $\tau_{\frac{1}{2}} > \frac{\tau}{2}$

$$\text{Moreover, } \tau = \frac{\tau_{\frac{1}{2}}}{0.693} = 1.44\,\tau_{\frac{1}{2}} \tag{6.12.3}$$

A few cases have also been found that a certain nuclei of an element disintegrate by emitting α–particles and at that time certain other nuclei disintegrate by emitting β–particles. This is called branch disintegration. In this process, if the decay constant corresponding to α–particle emission is λ_α and that corresponding to β–particle emission is λ_β, then the total decay constant of that element would be $\lambda = \lambda_\alpha + \lambda_\beta$ and its mean life would be $\tau = \frac{1}{\lambda_\alpha + \lambda_\beta}$.

From this we get $\frac{1}{\tau} = \frac{1}{\tau_\alpha} + \frac{1}{\tau_\beta}$ where τ_α and τ_β are mean lives corresponding to emission of α–particles and β–particles respectively.

[From this we can also write $\frac{1}{\tau_{\frac{1}{2}}} = \frac{1}{\tau_{\frac{1}{2}(\alpha)}} + \frac{1}{\tau_{\frac{1}{2}(\beta)}}$ where $\tau_{\frac{1}{2}}$ = total (effective) half life]

Illustration 3 : In a specimen of uranium ore α–particles are emitted at the rate of $9.3 \times 10^5\ s^{-1}$. They are due to disintegration of ^{235}U nuclei, which is 0.72% in proportion in this specimen. If the half-life of ^{235}U is 7.04×10^8 yr, find the mass of this specimen of ore. (Take 1 yr = 3.16×10^7 s)

Solution : Activity of ^{235}U is given as $I = 9.3 \times 10^5 \frac{\text{decay}}{\text{s}}$.

From $\tau_{\frac{1}{2}} = \frac{0.693}{\lambda}$ we get $\lambda = \frac{0.693}{\tau_{\frac{1}{2}}} = \frac{0.693}{7.04 \times 10^8 \times 3.16 \times 10^7}\ s^{-1}$

$\therefore$ If number of undisintegrated nuclei of ^{235}U at this instant is N, then,

$I = \lambda N$ ...(neglecting negative sign)

$$= \left(\frac{0.693}{\tau_{\frac{1}{2}}}\right) N$$

$$\therefore\ N = \frac{(I)\left(\tau_{\frac{1}{2}}\right)}{0.693} = \frac{(9.3 \times 10^5)\ (7.04 \times 10^8 \times 3.16 \times 10^7)}{0.693}$$

$$= 3 \times 10^{22}$$

We know that 235 g of ^{235}U contains 6.02×10^{23} atoms.

Thus, mass of 6.02×10^{23} ^{235}U atoms is 235 g.

Then, mass of 3×10^{22} atoms is = m(suppose)

$$\therefore\ m = \frac{235 \times 3.0 \times 10^{22}}{6.02 \times 10^{23}}$$

$$\approx 12\ g$$

The proportion of ^{235}U in the ore is 0.72%. Thus, for 0.72 g of ^{235}U, mass of ore is 100 g, then for 12 g of ^{235}U, mass of ore = M (suppose)

$$\therefore\ M = \frac{100 \times 12}{0.72} = 1666\ g$$

$$= 1.666\ kg$$

Illustration 4 : In a sample of radioactive element, after $\frac{1}{\lambda}$ time (where λ = decay constant),

(1) What percent of initial amount remains undisintegrated ?

(2) What percent is disintegrated ?

Solution : (1) Number of undisintegrated nuclei at time t is,

$$N = N_0 e^{-\lambda t} = N_0\, e^{-\lambda(\frac{1}{\lambda})} \qquad (\because\ t = \frac{1}{\lambda})$$

$$= N_0 e^{-1} = \frac{N_0}{e}$$

At this time, the fraction of initial amount remaining undisintegrated, is

$$\frac{N}{N_0} = \frac{1}{e} = \frac{1}{2.718} = 0.368$$

$\therefore$ The percentage remaining undisintegrated is

$$= \frac{N}{N_0} \times 100 = 0.368 \times 100 = 36.8\%$$

(2) : At this time if number of nuclei disintegrated is

N', Then $N' = N_0 - N$

$$= N_0 - \frac{N_0}{e} = N_0\left(\frac{e-1}{e}\right)$$

$$= N_0\left(\frac{1.718}{2.718}\right) = N_0\ (0.632)$$

$\therefore$ At this time, fraction of initial amount disintegrated $= \frac{N'}{N_0} = 0.632$

$\therefore$ At this time, percentage disintegrated $= \frac{N'}{N_0} \times 100 = 63.2\%$

Illustration 5 : Half life of a radioactive element is 0.693 hr. What time would it take to disintegrate 80% of its nuclei ?

Solution : $\tau_{\frac{1}{2}} = 0.693$ hr, $\lambda = \frac{0.693}{\tau_{\frac{1}{2}}}$

If $N_0 = 100$, then 80 will disintegrate and $N = 20$ will remain undisintegrated.

From, $N = N_0 e^{-\lambda t}$

$20 = 100 e^{-\lambda t}$

$\therefore \frac{1}{5} = e^{-\lambda t}$

$\therefore 5 = e^{\lambda t}$

$\therefore ln\ 5 = \lambda t$

$\therefore (2.303)(log_{10}\ 5) = \left(\frac{0.693}{\tau_{\frac{1}{2}}}\right) t$

$\therefore (2.303)(0.6990) = \left(\frac{0.693}{0.693}\right) t$

$\therefore t = 1.61$ hour

Illustration 6 : Suppose the rate of production of element B from a radioactive element A is α = constant. If at $t = 0$ time; the number of atoms of B is N_0 and element B is also radioactive with decay constant λ, show that the number of atoms of B at time t is $N = \frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}]$

Solution : Rate of production of element B from A is α = constant. If number of atoms of element B at time t is N; the rate of disintegration of B at that time $= -\lambda N$.

$\therefore$ Rate of change of number of atoms of B is

$$\frac{dN}{dt} = \alpha - \lambda N$$

$$\therefore \frac{dN}{\alpha - \lambda N} = dt$$

$$\therefore \int_{N_0}^{N} \frac{dN}{\alpha - \lambda N} = \int_0^t dt$$

$$\therefore \left(-\frac{1}{\lambda}\right) [ln(\alpha - \lambda N)]_{N_0}^{N} = [t]_0^t$$

$$\therefore [ln(\alpha - \lambda N) - ln(\alpha - \lambda N_0)] = -\lambda[t - 0]$$

$$\therefore\ ln\frac{\alpha-\lambda N}{\alpha-\lambda N_0} = -\lambda t$$

$$\therefore\ \frac{\alpha-\lambda N}{\alpha-\lambda N_0} = e^{-\lambda t}$$

$$\therefore\ \alpha - \lambda N = (\alpha - \lambda N_0)e^{-\lambda t}$$

$$\therefore\ \lambda N = \alpha - (\alpha - \lambda N_0)e^{-\lambda t}$$

$$\therefore\ N = \frac{1}{\lambda}[\alpha - (\alpha - \lambda N_0)e^{-\lambda t}]$$

Illustration 7 : In the mixture of two elements A and B having decay constants 0.1 day^{-1} and 0.2 day^{-1} respectively; initially the activity of A is 3 times that of B. If the initial activity of the mixture is 2 m*Ci*, find the activity of it after 10 days.

Solution : $\lambda_A = 0.1\ day^{-1}$, $\lambda_B = 0.2\ day^{-1}$

$(I_0)_A = 3(I_0)_B$

At $t = 0$ time, activity of the mixture is

$I_0 = (I_0)_A + (I_0)_B = (3I_0)_B + (I_0)_B$

$\therefore\ 2 = 4(I_0)_B \quad \therefore\ (I_0)_B = 0.5$ m*Ci*

$\therefore\ (I_0)_A = 1.5$ m*Ci*

At time t, activity of A is $I_A = (I_0)_A \cdot e^{-\lambda_A t} = (1.5)(e)^{-(0.1)(10)}$

$$= \frac{1.5}{e} = \frac{1.5}{2.718} = 0.552\ \text{m}Ci$$

At time t, activity of B is $I_B = (I_0)_B \cdot e^{-\lambda_B t} = (0.5)[e^{-(0.2)(10)}]$

$$= \frac{0.5}{e^2} = \frac{0.5}{2.718^2} = 0.067\ \text{m}Ci$$

At time t, total activity of the mixture

$I = I_A + I_B = 0.552 + 0.067 = 0.619$ m*Ci*

Illustration 8 : A solution containing radionuclide ^{24}Na having half life of 15 hr and activity of 1 microcurie is injected in the blood of a person. After 5 hr, the activity of 1 cm^3 volume of sample of his blood shows activity of 296 disintegration/min. Find the total volume of blood in the person. 1 curie = 3.7×10^{10} disintegration/s.

Solution : Initial activity of ^{24}Na is

$I_0 = 1.0\ \mu Ci = 1.0 \times 10^{-6} \times 3.7 \times 10^{10}$

$= 3.7 \times 10^4$ disintegration / s.

$$\lambda = \frac{0.693}{\tau_{\frac{1}{2}}} = \frac{0.693}{15 \times 3600}\ s^{-1}$$

$I_0 = \lambda N_0$

$\therefore\ N_0 = \frac{I_0}{\lambda} = \frac{(3.7\times10^4)\cdot(15\times3600)}{0.693}$

$= 2.883 \times 10^9$ = total initial number of ^{24}Na nuclei.

After 5 hr, if the number of radionuclide in 1 cm^3 sample of blood is N, and activity is $\frac{296}{60}$ disintegration/s; then

From $I = \lambda N$

$$N = \frac{I}{\lambda} = \frac{296}{60} \times \frac{15\times3600}{0.693}$$

$= 3.844\ 10^5$

= Number of Na – nuclei in 1 cm^3 sample after 5 hr.

If in this 1 cm^3 sample of blood, the initial number of radio nuclide is N_0', then

$$\left(\frac{N}{N_0'}\right) = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} = \left(\frac{1}{2}\right)^{\frac{5}{15}} = \left(\frac{1}{2}\right)^{\frac{1}{3}}$$

$\therefore\ N_0' = (N)(2)^{\frac{1}{3}} = (N)(1.269)$

$= (3.844 \times 10^5)(1.269)$

Thus for N_0' radionuclide – volume of blood is 1 cm^3

Then, for N_0 radionuclide – volume of blood = ?

$$\therefore\ \text{Volume of blood} = \frac{N_0}{N_0'} = \frac{2.883 \times 10^9}{(1.269)(3.844 \times 10^5)}$$

$= 5.91 \times 10^3\ cm^3$

= 5.91 litre

Illustration 9 : In a sphere of 10^2m radius, radioactive material emits β^--particles at the rate of $5 \times 10^7\ s^{-1}$. If 40% of these emitted β^--particles escape from the sphere, how long would it take to raise the potential of the sphere from 0 to 16 V ? (Take $k = 9 \times 10^9$ SI unit)

Solution : Out of emitted β^--particles the number of β^--particle leaving the sphere in one second $= n = (0.4)(5 \times 10^7)$.

$= (2 \times 10^7)\ s^{-1}$

$\therefore$ The number of β^--particles leaving the sphere in t seconds $= n \times t$.

$\therefore$ In t seconds, the charge acquired by the sphere $Q = n \times t \times e$ .... (positive)

If the potential due to it is V, then

$$V = \frac{kQ}{R} = \frac{k(n\times t\times e)}{R}$$

$$\therefore\ 16 = 9 \times 10^9 \frac{(2\times10^7)(t)(1.6\times10^{-19})}{10^2}$$

$$\therefore\ t = \frac{16\times10^2}{9\times2\times1.6\times10^{-3}} = 55578 \text{ s} = 15.438 \text{ hr.}$$

6.13 α-decay

In the process of radioactivity, the unstable nucleus of a radioactive element disintegrates and forms a new nucleus. The disintegrating nucleus is called the parent nucleus and the newly formed nucleus is called the daughter nucleus.

Most of the nuclei with Z > 83 emit α-particles. As an illustration ${}_{92}U^{238}$ nucleus, emits α-particle and converts into ${}_{90}Th^{234}$.

This process can be written as :

$${}_{92}U^{238} \rightarrow {}_{90}Th^{234} + {}_{2}He^{4} \qquad (6.13.1)$$

Thus, in the process of α-decay, as compared to parent element the value of atomic number of daughter element is 2 unit less and the atomic mass number is 4 unit less.

All nuclei in the substance emitting α-particles, do not together emit α-particle at the same time. This process is related to probability. Hence no α-particle after its formation in the nucleus is emitted immediately or all α-particles are not together emitted. Moreover spontaneous emission of α-particle is possible only if mass of ${}_{92}U^{238}$ nucleus is greater than the sum of the masses of ${}_{90}Th^{234}$ nucleus and the α-particle. If it is not so this process cannot occur spontaneously (but can occur by giving energy from outside). We can verify this fact by obtaining masses of nuclei with the help of standard table.

It is clear that in this case the energy emitted is equal to $[M_U - (M_{Th} + M_\alpha)]c^2$, where M is the mass of the respective nuclide.

6.14 β-decay

In the process of β-decay, a nucleus spontaneously emits electron or the positron. Positron has the same charge as that of electron but it is positive, and its other properties are exactly identical to those of electron. Thus positron is the anti-particle of electron. Positron and electron are respectively written as β^+ and β^- or ${}^{0}_{+1}e$ and ${}^{0}_{-1}e$ or e^+ and e^-. Known illustrations of β-decay are

$${}_{15}P^{32} \rightarrow {}_{16}S^{32} + {}^{0}_{-1}e + \bar{\upsilon} \qquad (6.14.1)$$

(electron) (antineutrino)

$${}_{11}Na^{22} \rightarrow {}_{10}Ne^{22} + {}^{0}_{+1}e + \upsilon \qquad (6.14.2)$$

(positron) (neutrino)

Compared to the parent element, the atomic number of the daughter element is one unit more in β^--decay and one unit less in β^+-decay. In both cases the atomic mass number of the daughter element is the same as that of the parent element. Along with the emission of e^+, a particle called neutrino and with emission of e^-, a particle called anti neutrino are emitted. Neutrino and anti neutrino are the anti particles of each other. They are electrically neutral and their mass is extremely small as compared to even that of electron. Their interaction with other particles is negligible and hence it is extremely difficult to detect them. They can pass without interaction even through very large matter (even through the entire earth). They have $\frac{\hbar}{2}$ spin $\left(\hbar = \frac{h}{2\pi}\right)$.

In β-decay the electron is emitted from the nucleus (and not from the extra nuclear electronic orbits). Electrons do not reside in a nucleus, then how can they be emitted from nucleus ? In fact, a neutron in a nucleus disintegrates into a proton and an electron and this newly born electron (it can born in a nucleus but cannot stay there) is immediately expelled out which we call β^-particle. In a nucleus which contains more neutrons than that required for stability, a neutron disintegrates and a β^- is emitted.

$$n \rightarrow p + e^- + \bar{\upsilon}$$

If the proton is converted into neutron, e^+ is emitted.

$$p \rightarrow n + e^+ + \upsilon$$

6.15 γ-decay

There are energy levels of nuclei similar to the energy levels of the atoms. Also like the atom, when a nucleus makes a transition form a higher energy level to the lower energy level, a photon with energy equal to their difference is emitted. The energy levels of the nuclei are of the order of M*e*V. Even when the energy difference between such levels is 1 M*e*V, the wavelength of the emitted photon is obtained in the region of γ-rays. The following calculation will clarify this.

From hf = 1 M*e*V, $\frac{hc}{\lambda} = (1 \times 10^6)\ (1.6 \times 10^{-19}\ \text{J})$

$$\therefore\ \lambda = \frac{hc}{(1\times10^6)(1.6\times10^{-19})}$$

$$\therefore\ \lambda = \frac{(6.6\times10^{-34})(3.0\times10^8)}{1\times10^6\times1.6\times10^{-19}} = 12.37 \times 10^{-13}\text{m} = 0.0012\ \text{nm}$$

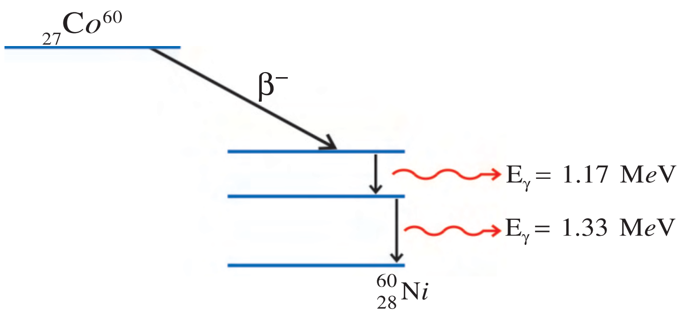


Figure 6.7 γ-decay

This value of λ falls in the region of γ-rays. Thus this radiation is γ-ray. When a nucleus emits α or β-particle, the daughter nucleus is mostly in an excited state. Such a daughter nucleus emits γ-photon by making one or more transitions.

As an illustration, when ${}^{60}_{27}Co$ by emitting β^--particle converts into ${}^{60}_{28}\text{N}i$, the ${}^{60}_{28}\text{N}i$ nucleus is in the excited state and by successive transitions it emits γ-ray photons of energies 1.17 M*e*V and 1.33 M*e*V.

Illustration 10 : If by successive disintegration of ${}^{238}_{92}\text{U}$, the final product obtained is ${}^{206}_{82}\text{P}b$, how many α and β particles are emitted ?

Solution : Suppose in this process x, α-particles and y, β-particles are emitted.

$$\therefore \ {}^{238}_{92}U \rightarrow {}^{206}_{82}Pb + x\left({}^{4}_{2}He\right) + y\left({}^{0}_{-1}e\right)$$

Comparing atomic mass numbers on both the sides,

$$238 = 206 + x(4) + y(0)$$

$$\therefore \ x = 8$$

Now comparing atomic numbers on both the sides,

$$92 = 82 + 2x + y(-1)$$

$$= 82 + 16 - y$$

$$\therefore \ y = 6$$

Thus, in this process 8 α-particles and 6 β-particles are emitted.

6.16 Nuclear Reactions

In 1919, Rutherford showed that by bombarding suitable particles of suitable energy on a stable element; that element can be transformed into another element. Such a reaction is called artificial nuclear transmutation. When he bombarded α-particles on nitrogen, he found that nitrogen was converted into oxygen. This process can be written as under :

$${}^{14}_{7}N + {}^{4}_{2}He \rightarrow {}^{17}_{8}O + {}^{1}_{1}H + Q \qquad (6.16.1)$$

Such processes, in which change in the nucleus takes place, are called nuclear reactions. Here Q is called the Q-value of the nuclear reaction and it shows the energy produced (released) in the process. Such reactions are also shown symbolically as $A + a \rightarrow B + b + Q$ or A (a, b) B.

Here, A is called the target nucleus,

a is called the projectile particle,

B is called the product nucleus,

and b is called the emitted particle.

The energy Q, liberated in the process is equal to the energy equivalent to the decrease in mass in the process.

$$Q = [m_A + m_a - m_B - m_b]c^2 \qquad (6.16.2)$$

where m is the mass of the respective particle.

The energy so produced appears as the increase of kinetic energy in the reaction. If $Q > 0$, the reaction is called exoergic reaction. If $Q < 0$, the reaction is called endoergic. It is self evident that endoergic reaction cannot occur spontaneously but can occur only if sufficient energy is supplied.

In nuclear reactions, it is necessary that the momentum, the electric charge and the energy each one is conserved.

The conservation of charge can be seen from the atomic numbers. Moreover, the sums of the atomic mass numbers before and after the reaction are equal, but the mass can change. In short, we will note that the Q value of the reaction = energy equivalent to decrease in mass in the reaction = increase in the kinetic energy.

Illustration 11 : Usually in laboratory, neutrons are obtained by bombarding α-particles, emitted from ${}^{226}Ra$, on ${}^{9}_{4}Be$ through the reaction ${}^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + {}^{1}_{0}n$. The energy of these α-particles is 4.78 MeV. Find the maximum kinetic energy of neutron.

[Take $M_\alpha = 4.002603\ u$, $M_{Be} = 9.012183\ u$, $M_c = 12.000000\ u$, $m_n = 1.0086\ u$, $1\ u = 931.494$ MeV]

Solution : ${}^{9}_{4}Be + {}^{4}_{2}He \rightarrow {}^{12}_{6}C + {}^{1}_{0}n$

Applying law of conservation of energy,

$(M_{Be} + M_\alpha)c^2 + K_\alpha = (M_c + m_n)c^2 + K_n + K_c$

Here as neutron is getting maximum kinetic energy, the kinetic energy of carbon K_c must be zero. (Since Be is the target $K_{Be} = 0$ is taken)

$\therefore$ (9.012183 + 4.002603)(931.494) + 4.78 = [12.000000 + 1.0086] × 931.494 + K_n

$\therefore$ $K_n = 10.54$ MeV

Illustration 12 : ${}^{241}Am$ in a steady state emits α-particle and the reaction ${}^{241}Am \rightarrow \alpha + {}^{237}Np$ takes place. Using following data, find the kinetic energy of α-particle.

$M_{Am} = 241.05682\ u$, $M_\alpha = 4.002603\ u$, $M_{Np} = 237.04817\ u$, $1\ u = 931.474$ MeV

Solution : According to law of conservation of energy

we get, $(M_{Am})c^2 = (M_\alpha - M_{Np})c^2 + K_f$

where K_f = Total kinetic energy of final products (α and Np)

$\therefore\ K_f = (M_{Am} - M_\alpha - M_{Np})c^2$

= energy equivalent to mass difference

= [241.05682 − 4.002603 − 237.04817] × 931.474 MeV

= 5.6326 MeV

According to the conservation of momentum,

$0 = \vec{P}_\alpha + \vec{P}_{Np}$ (∵ momentum of Am = 0)

$\therefore\ P_{Np} = P_\alpha$ (in magnitude)

$\therefore$ Total kinetic energy $K_f = \dfrac{p_\alpha^2}{2M_\alpha} + \dfrac{p_{Np}^2}{2M_{Np}}$ (Kinetic Energy = $\dfrac{p^2}{2m}$)

$$= \frac{p_\alpha^2}{2M_\alpha} + \frac{p_\alpha^2}{2M_{Np}} \quad (p_{N_p} = P_\alpha)$$

$$= \frac{p_\alpha^2}{2}\left[\frac{1}{M_\alpha} + \frac{1}{M_{Np}}\right] = \frac{p_\alpha^2}{2}\left[\frac{M_{Np} + M_\alpha}{M_\alpha M_{Np}}\right]$$

$\therefore$ Kinetic energy of α-particle $= \dfrac{p_\alpha^2}{2M_\alpha} = \dfrac{K_f \cdot M_{Np}}{M_{Np} + M_\alpha} = \dfrac{(5.6326)(237.04817)}{237.04817 + 4.002603}$

$$= 5.539 \text{ MeV}$$

Illustration 13 : In the reaction ${}^{A}_{Z}X \rightarrow {}^{A-4}_{Z-2}Y + {}^{4}_{2}He + Q$ of the nucleus X at rest, taking the ratio of mass of α-particle M_α and mass of Y-nucleus M_Y as $\frac{M_\alpha}{M_Y} = \frac{4}{A-4}$, show that the Q-value of the reaction is given by $Q = K_\alpha\left(\frac{A}{A-4}\right)$, where K_α = kinetic energy of α-particle.

Solution : Q-value of reaction = energy equivalent to mass-difference.

$$= (M_X - M_Y - M_\alpha)c^2$$

= increase in kinetic energy

$$= (K_\alpha + K_Y) - 0 \quad (\because \text{ X was steady})$$

$$= \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y v_Y^2 \qquad (1)$$

From conservation of momentum,

$$M_Y \vec{v}_Y + M_\alpha \vec{v}_\alpha = 0$$

$$\therefore \; M_Y v_Y = M_\alpha v_\alpha \text{ (in magnitude)} \qquad (2)$$

$$\therefore \; v_Y = \left(\frac{M_\alpha}{M_Y}\right) v_\alpha$$

Substituting this value in equation (1),

$$Q = \frac{1}{2}M_\alpha v_\alpha^2 + \frac{1}{2}M_Y\left(\frac{M_\alpha}{M_Y}\right)^2 v_\alpha^2$$

$$= \frac{1}{2}M_\alpha v_\alpha^2\left[\frac{M_\alpha}{M_Y} + 1\right] = K_\alpha\left(\frac{4}{A-4} + 1\right)$$

$$= K_\alpha\left(\frac{A}{A-4}\right)$$

6.17 Nuclear Fission

In 1932, Chadwick discovered neutron. Thereafter, Fermi suggested that as the neutron is electrically neutral it does not have to face the coulomb repulsive forces and hence by bombarding neutron on the nucleus it can go deep into the nucleus. Thus, it is a good projectile.

When Hann and Strassman, bombarded thermal neutron (of energy $\approx 0.04eV$) on compounds of uranium, they found ${}_{56}Ba^{144}$ in the newly formed radioactive elements. They were surprised with this result. Meitner and Frisch found that when thermal neutron is bombarded on uranium nucleus, it disintegrates uranium nucleus in two almost equal parts and in this process enormous energy is produced (released). This process was named nuclear fission.

In the fission of uranium, many different product nuclei have been obtained.

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{144}_{56}Ba + {}_{36}Kr^{89} + 3\,{}_{0}n^{1} + Q$$

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{133}_{51}Sb + {}^{99}_{41}Nb + 4\,{}^{1}_{0}n + Q$$

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}_{38}Sr^{94} + 2\,{}^{1}_{0}n + Q$$

The product nuclei obtained by the fission are called the fission fragments, the neutrons are called the fission neutrons and the energy is called the fission energy. In the above reaction, 60 different nuclei are obtained as fission fragments, having Z values between 36 and 56. The probability is maximum for formation of nuclei with A = 95 and A = 140. The fission fragments are radioactive and by successive emission of β^--particles result in stable nuclei.

The neutrons produced in this reaction are fast (almost 2 M*e*V energy).

The Q-value of this reaction, that is, the energy produced is very large like almost 200 M*e*V per fission. This energy is obtained due to conversion of mass difference, between the reactants and the products, into energy. Initially this energy is in the form of the kinetic energy of the fission fragments and the neutrons which eventually transforms into the heat energy in the surrounding material.

In a nuclear reactor which produces electric power such successive nuclear fission processes take place but in the controlled form, while in the nuclear bomb such successive processes occur in the uncontrolled manner and produces explosion.

The theoretical explanation of the nuclear fission reaction is given by "liquid drop model of the nucleus", in which a nucleus is compared with a drop of liquid.

6.18 Nuclear Chain Reaction and Nuclear Reactor

Nuclear Chain Reaction : In the previous article, we have seen that in the fission process of $^{235}_{92}U$ by a slow neutron, one or more neutrons are emitted. For every fission average $2\frac{1}{2}$ neutrons are obtained per fission. The reason for the fraction which appears here is that in certain fission processes, we get 4 or 3 or 2 neutrons emitted. In 1939 Fermi suggested that with the help of neutrons produced in this way if fission of other uranium nuclei is accomplished then, we get still more energy and still more neutrons. A series of such processes is called nuclear chain reaction. If such a process is properly controlled, then energy can be obtained continuously at steady rate. Nuclear reactor is the illustration of this. If such a process occurs in uncontrolled manner, then the energy produced causes explosion. Nuclear bomb is the illustration of this.

Now, we shall see about the difficulties encountered in the success of such nuclear chain reaction and their removal.

(1) Fission neutrons are fast (average energy is 2 M*e*V). They should be stopped from escaping from the fission material. Moreover, they should be slowed down and converted into thermal neutron (energy almost 0.04 M*e*V), to become suitable for fission.

To stop neutrons from escaping, neutron reflecting surfaces are used and in the arrangement of fission material the surface/volume ratio is kept low, because the leakage process of neutrons is a surface process. To slow down the neutrons, materials known as moderator are kept along with the fission material. Normal water (H_2O), heavy water (D_2O), Graphite, Beryllium etc. are good moderators. They slow down the neutrons but do not absorb them.

In producing fission of $^{235}_{92}U$ slow neutrons are more effective as compared to fast neutrons.

(2) In such a chain reaction enormous heat energy is produced and the temperature is likely to become 10^6K. Hence the fission material, moderator etc. should be cooled and that heat energy should be converted into the useful form. For this coolants are used.

Water, molten sodium metal, gases etc are passed as coolants through the tubes in fission chamber.

(3) In a nuclear chain reaction the ratio of the number of neutrons produced at any stage to the number of neutrons incident at that stage is called the multiplication factor K. It is a measure of the growth of number of neutrons. When K = 1, the reactor is said to be critical. If K becomes grater than 1, the reactor is said to be in super critical state. In this state the rate of reaction and the energy abruptly increase and explosion takes place. If K becomes less than 1 (sub critical state), the process slows down and eventually stops and we cannot get energy continuously with uniform rate. Hence in order to control the value of K, rods of materials which can absorb neutrons like Cadmium and Boron are kept in the fission material. These rods are controlled automatically.

If the value of K tends to be more than 1, the rods go deeper in fission material to absorb more neutrons. If the value of K tends to be less than 1, these rods automatically rush outside to decrease absorption of neutrons. These rods are called control rods.

When all these requirements are together fulfilled, energy is obtained continuously at a steady rate from the reactor.

Nuclear Reactor : It works on the principle of controlled nuclear chain reaction. A schematic diagram of a typical nuclear reactor power plant is shown in the figure 6.8. In order that fission can be accomplished with a thermal neutron, $^{235}_{92}U$ is taken as a fuel. But in natural Uranium its proportion is only 0.7% and that of $^{238}_{92}U$ is 99.3%. By specific processes, the proportion of $^{235}_{92}U$ is made nearly 3%. Such uranium is called enriched uranium. When $^{238}_{92}U$ absorbs a neutron, $^{239}_{94}Pu$ is formed by the following processes.

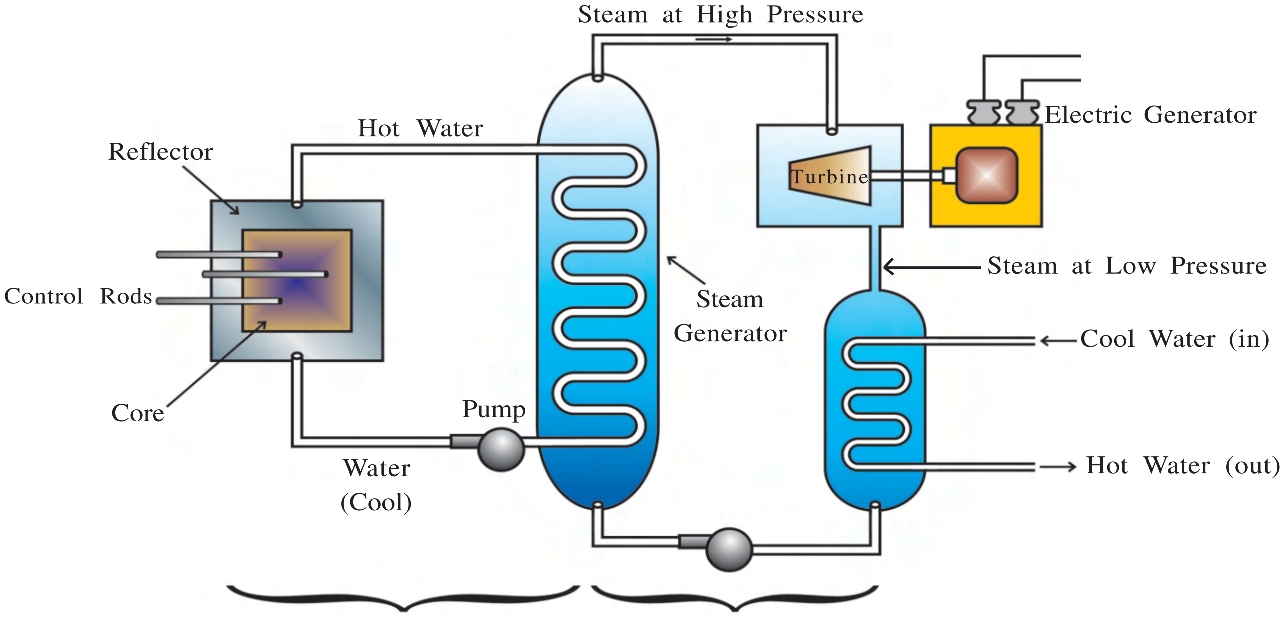


Figure 6.8 Nuclear Reactor

$$^{238}_{92}U + ^{1}_{0}n \rightarrow ^{239}_{92}U \rightarrow ^{239}_{93}Np + _{-1}e^{0} + \bar{\nu}$$

$$^{239}_{93}Np \rightarrow ^{239}_{94}Pu + _{-1}e^{0} + \bar{\nu}$$

This plutonium is intense radioactive and is fissionable by slow neutron.

Materials used as fuel and moderator are kept in the core part of the reactor. Fission occurs in the fuel material. Here normal water is used as moderator and also as coolent. This is called pressurised water reactor. Water is pushed into the core of the reactor using a pump. When it comes out of the core, its temperature becomes 600 K at 150 atm pressure. It is then passed into the steam generator. The steam of very high pressure produced in it, operates the turbine, which produces electric power.

After moving the turbine the pressure of the steam decreases. This steam is then cooled and converted into water and it is again pushed into the reactor with the pump. In order to control the multiplication factor K, control rods are kept.

6.19 Thermonuclear Fusion in Sun and other Stars

The Sun has been emitting energy at a tremendous rate of 3.8×10^{26} J/s since almost 500 crore years. The origin of such enormous energy remained unknown to scientists for many years. But from the study of nuclear physics, explanation of this has been obtained.

Energy is obtained by the fission of a heavy nucleus. But conversely, when two proper light nuclei are fused at a very high temperature to form a heavy nucleus, then also enormous amount of energy is produced. Such a process is called thermonuclear fusion. For example when Helium nucleus is made from protons or deuterons, much energy is produced. In the Sun and other stars the energy is produced by thermonuclear fusion.

In the Sun, energy is produced by a process called proton – proton cycle which occurs according to the following stages.

$$^{1}_{1}H + ^{1}_{1}H \rightarrow ^{2}_{1}H + _{+1}e^{0} + \nu + 0.42 \text{ MeV} \quad (6.19.1)$$

$$_{+1}e^{0} + _{-1}e^{0} \rightarrow 2\gamma + 1.02 \text{ MeV} \quad (6.19.2)$$

$$^{2}_{1}H + ^{1}_{1}H \rightarrow ^{3}_{2}He + \gamma + 5.49 \text{ MeV} \quad (6.19.3)$$

$$^{3}_{2}He + ^{3}_{2}He \rightarrow ^{4}_{2}He + ^{1}_{1}H + ^{1}_{1}H + 12.86 \text{ MeV} \quad (6.19.4)$$

When first three reactions occur twice, two $^{3}_{2}He$ nuclei are produced and between them the fourth reaction takes place. As a result of all reactions $4\,^{1}_{1}H$ and $2\,_{-1}e^{0}$ produce one α-particle, 2ν and 6γ-photons. Here energy equal to $2 \times 0.42 + 2 \times 1.02 + 2 \times 5.49 + 12.86 = 26.7$ MeV is liberated. Moreover, another process called carbon-nitrogen cycle is also suggested for the energy produced in the stars. We shall not go into the details of it at present.

It will take further 500 crore years for all of hydrogen in the core of the Sun to burn out and become helium. Thereafter, since the combustion of hydrogen is stopped, sun will start colling down and will collapse (contract) due to its own gravitation. This will again raise the temperature of its core and the outer envelope will expand, and thus the Sun will turn into a red giant. If the temperature will again rise to 10^{8} K, then the combustion of He will take place to form C. By further evolution of such a star, still higher temperature will be reached and by other fusion processes other heavy elements will be formed. But even with the fusion processes progressing further elements heavier than those near the peak of the binding energy curve will not be formed.

Attempts are in progress to produce energy (electric power) constantly and continuously in many countries of the world by a controlled chain reaction of nuclear fusion. They have not reached to the stage of great success. In India, such a research is going on in the Institute for Plasma Research (IPR) at, Bhat near Ahmedabad.

Illustration 14 : Assume the Sun to be completely made up of protons. When four protons fuse to form $^{4}_{2}He$ nucleus in a proton-proton cycle occurring in the Sun, 6.7 MeV energy is released per proton. The total output power of the Sun is 3.9×10^{26}W. Consider this power to be constant and mass of the sun equal to 2.0×10^{30} kg. How long will the Sun take to be fully converted into $^{4}_{2}He$ particles ?

[Take Mass of a proton = 1.67×10^{-27} kg, 1 yr = 3.16×10^{7} s]

Solution : Total mass of the Sun = 2.0×10^{30} kg

$$\therefore \text{ The number of protons in the Sun } = \frac{2.0 \times 10^{30}}{1.67 \times 10^{-27}} = 1.2 \times 10^{57}$$

Total output power of the Sun = 3.9×10^{26} J s^{-1}

Energy obtained per proton = 6.7 MeV = $6.7 \times 10^{6} \times 1.6 \times 10^{-19}$ J

If N is the number of protons destroying per sec, the total energy per second will be,

$(N)(6.7 \times 10^{6} \times 1.6 \times 10^{-19}) = 3.9 \times 10^{26}$

$$\therefore N = \frac{3.9 \times 10^{26}}{6.7 \times 10^{6} \times 1.6 \times 10^{-19}} = 3.6 \times 10^{38} \text{ protons destroy per s.}$$

Thus, 3.6×10^{38} protons take -1 s to destroy

then 1.2×10^{57} proton take $-t$ s to destroy

$$\text{where } t = \frac{1.2 \times 10^{57}}{3.6 \times 10^{38}} = 0.33 \times 10^{19} \text{ s} = \frac{0.33 \times 10^{19}}{3.16 \times 10^{7}} \text{ yr} = 1.044 \times 10^{11} \text{ yr}$$

$$= 104.4 \text{ Billion Year}$$

6.20 Nuclear hazards

The energy produced by nuclear fission and nuclear fusion is found to be useful in many ways. But devastating calamities can also occur due to them. The distructive effects of atom bomb have already been experienced by the mankind.

Although getting power from a nuclear reactor is beneficial; the waste products from it are intense radioactive and hence are harmful to the living bodies. No satisfactory solution is still found to store them or to dispose off. Moreover, accidents occurring in such a reactor can cause distruction in the surrounding. The large scale devastation in the surrounding of a reactor at Chernobyl in Ukraine in April 1986, due to an explosion in it is the illustration of it.

At present a huge amount of nuclear weapons are present on earth. They are capable of destroying all forms of life on earth for several times over. Not only that but its products will make this earth unfit for life for ever.

Theoretical calculations reveal that due to extravagant use of nuclear energy the radioactive waste will hang in the earth's atmosphere like the clouds and will absorb the solar radiation and a "nuclear winter" will be produced on the earth.

SUMMARY

1. The entire positive charge and almost entire mass of the atom is concentrated in the nucleus.

2. In ${}^{A}_{Z}X$ or ${}_{Z}X^{A}$, Z shows the atomic number and A shows the atomic mass number of the element. A − Z = N shows the number of neutron in the nucleus. The masses of the atom and the nuclei are expressed in the unit called the atomic mass unit (symbol : amu or u). The twelfth part of the mass of unexcited ${}^{12}_{6}C$ atom is called 1 u mass.

   1 u (mass) = 1.66×10^{-27} kg. Nuclei having equal Z values but different A values are called isotopes. Nuclei having same number of neutrons (N = A − Z) are called isotones of each other.

   Nuclei having same values of atomic mass number (A = N + Z) are called isobars of each other. Nuclei having equal Z and also equal A, but having different radioactive properties are called isomers of each other.

3. A strong attractive force acts which after balancing the repulsive force between protons, can tightly hold all nucleons together in a nucleus. Since such a nuclear force is short range, every nucleon can interact only with a few neighbouring nuclei (saturation property).

   Basically the forces between quark–quark ultimately result into nuclear forces. Nuclear forces depend on the 'spin' of nucleons.

4. The characteristic average radius of the nucleus is given by $R = R_0 A^{\frac{1}{3}}$, where A = atomic mass number. $R_0 = 1.1 f_m$ = constant. The density of nucleus is = 2.3×10^{17} kg m^{-3}.

5. In stable nuclei of light elements, the number of protons (Z) and the number of neutrons (N) are equal or almost equal, while in heavy stable nuclei, the number of neutron is greater than the number of proton.

6. According to Einstein's special theory of relativity, mass and energy can be transformed into each other. Mass m is equivalent to mc^2 energy. $E = mc^2$, where c = velocity of light in vacuum.

   "The change in the kinetic energy of an electron while passing through a potential difference of 1 Volt is called 1 eV (electron volt) energy."

   1 keV = $10^3 e$V, 1 MeV = $10^6 e$V

   1 u (mass) = 931.48 MeV (energy).

   The mass of the nucleus is always slightly less than the total mass of its constituents in the free state. This mass difference is called the mass defect Δm. The energy equivalent to it is $E_b = (\Delta m)c^2$ and it is called the binding energy of the nucleus. By dividing the binding energy with the total number of nucleons, we get the average binding energy per nucleon; $E_{bn}\left(=\frac{E_b}{A}\right)$. It is the measure of the stability of the nucleus. The maximum value of E_{bn} is found for nucleus of Fe and it is 8.8 MeV/nucleon. For nuclei of intermediate masses the value of E_{bn} is almost constant. For nuclei heavier or lighter than them, E_{bn} has smaller values. The nuclear structure is shell type. By the fission of heavy nuclei like U, energy is produced. It is called nuclear fission. Energy is also produced by the fusion of light nuclei. It is called nuclear fusion.

7. Becquerel found that Uranium spontaneously and continuously emits from itself radiations of specific properties. This phenomenon is called natural radioactivity. Madam Curie obtained other radioactive elements-Radium and Polonium–from the ore of uranium. Radioactivity is a nuclear phenomenon.

8. α rays are the nuclei of ${}^{4}_{2}He$ atoms. β-rays are electrons only. γ rays are not material particles but are electromagnetic waves. They all, produce fluorescence, affect photographic plate, can produce ionization and can penetrate.

9. In a given sample the number of nuclei disintegrating per unit time at an instant is called the rate of disintegration (or activity I) of that element at that instant and it is proportional to the number of undisintegrated nuclei at that instant. $\frac{dN}{dt} = -\lambda N$.

λ is called the decay constant or radioactive constant. It depends on the type of the radioactive element. It remains constant throughout the life of that element. The SI unit of activity is Becquerel (Bq). "The activity of a substance in which 1 disintegration occurs per second is called 1 Becquerel." "The activity of a substance in which 3.7×10^{10} disintegrations occur per sec is called 1 curie (Ci)."

$1\ mCi = 10^{-3}\ Ci$, $1\ \mu Ci = 10^{-6}\ Ci$

10. From rate of disintegration $\frac{dN}{dt} = -\lambda n$, the number of undisintegrated nuclei at time t is obtained as $N = N_0\, e^{-\lambda t}$. It is called the exponential law of radioactive disintegration. The graph of $N \rightarrow t$ is called the decay curve.

11. The time-interval during which the number of nuclei of a radioactive element (N) reduces to half its value at the beginning of the interval N_0, is called half-life $\tau_{\frac{1}{2}}$, of that element.

$$\tau_{\frac{1}{2}} = \frac{0.693}{\lambda}. \text{ If } \frac{\text{given time}(t)}{\text{half life}(\tau_{\frac{1}{2}})} = n,$$

the number of nuclei at time t is given by

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^n$$

12. "The time interval during which the number of nuclei of a radioactive element reduces to e^{th} part of the initial value is called the average life (τ) of that element. (e = 2.718)

$$\tau = \frac{1}{\lambda},\ \tau_{\frac{1}{2}} = (0.693)(\tau)$$

$$\tau = \frac{\tau_{\frac{1}{2}}}{0.693} = 1.44\,\tau_{\frac{1}{2}}$$

13. In α dacay, the atomic number and the atomic mass number of the daughter element are respectively 2 units and 4 units less than those of the parent element.

$${}_{Z}^{A}X \rightarrow {}_{Z-2}^{A-4}Y + {}_{2}^{4}He$$

The energy produced in this process is equal to $(M_X - M_Y - M_{He})c^2$.

14. The atomic number of the daughter element is one unit more in β^--decay and one unit less in β^+-decay than that of the parent element. In both the cases, there is no change in the atomic mass number.

$${}_{Z}^{A}X \rightarrow {}_{Z+1}^{A}Y + {}_{-1}^{0}e + \bar{\nu} \qquad (\beta^- \text{-decay})$$

$${}_{Z}^{A}X \rightarrow {}_{Z-1}^{A}Y + {}_{+1}^{0}e + \nu \qquad (\beta^+ \text{-decay})$$

ν and $\bar{\nu}$ are respectively neutrino and anti-neutrino. As their interaction with matter is negligible, their detection is very difficult. They are electrically neutral and have extremely small mass and $\frac{\hbar}{2}$ spin.

In β^--decay, a neutron in a nucleus disintegrates into proton and electron and this newly born electron gets emitted instantaneously.

15. The energy levels of the nucleus are of the order of MeV. When the nucleus makes a transition between these levels, electromagnetic waves of energy of the order of MeV are emitted. They are γ–rays.

16. Rutherford found that if suitable particles of suitable energy are bombarded on a stable element, that element gets transformed into another.

$${}_{7}^{14}N + {}_{2}^{4}He \rightarrow {}_{8}^{17}O + {}_{1}^{1}H + Q \quad (\alpha\text{-particle})$$

Q is called the Q value of this reaction. It shows the energy emitted in this process. It is equivalent to the mass difference occurring in the reaction. A reaction with $Q > 0$ is called exoergic reaction and the one with $Q < 0$ is called endoergic reaction. In these reactions momentum, electric charge and energy are conserved.

17. From the experiments of Hann and Stranssman as well as Meitner and Frisch, it was found that when uranium is bombarded by a thermal neutron, it breaks the uranium nucleus into two almost equal parts and enormous amount of energy is produced in this process. This phenomenon was named as nuclear fission.

$${}_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{92}U^{236} \rightarrow {}_{56}Ba^{144} + {}_{36}Kr^{89} + 3\,{}_{0}n^{1} + Q$$

In such a process other elements are also produced. 60 such different nuclides can be formed, which have Z values between 36 to 56. These fission fragments are intense radioactive. The neutrons produced in this process are fast (energy almost 2 MeV). The energy produced in this process is tremendous (nearly 200 MeV).

18. In the fission of ${}_{92}U^{235}$ nucleus by slow neutron, more than one neutron are produced. By using them for fission of other uranium nuclei, we get still more energy and still more neutrons. A series of such processes is called nuclear chain reaction. By controlling this process, energy is obtained from nuclear reactor continuously at

uniform rate. For this, the neutrons produced in the reaction should be stopped from escaping and should be slowed down also. For this neutron reflecting surfaces and moderators respectively are used. Moderator slows down neutron but does not absorb them. H_2O, D_2O, graphite, berelium etc are good moderators. Since excessive heat is produced, the temperature is likely to rise to 10^6 K. Hence with the help of coolant materials the fuel and moderator are cooled. H_2O, molten sodium metal, gas are used as coolants. In a fission chain reaction the ratio of number of neutrons emitted to the number of neutrons incident at any stage is called the multiplication factor K. For K = 1 the reactor is said to be critical. If K > 1, the reactor is said to be in the supercritical state, and it causes explosion. If K tends to be less than 1, then reactor gradually stops. For controlling the value of K, rods of C*d* and B are kept in the fission material. They can absorb neutron and can move automatically.

19. In the Sun and other stars energy is produced by the process of nuclear fusion. When light nucliei (e.g. proton) are coaleased to form a heavier nucleus (e.g. He) at a very high temperature, enormous energy is produced. Such a reaction is called thermonuclear fusion.

Such energy is produced in the Sun by a process called proton–proton cycle. Attempts are made throughout the world to get energy by a controlled chain reaction of nuclear fusion.

20. The waste-products from nuclear reactor are intense radioactive and hence dangerous to the living world. Moreover accidents are also likely to occur in the reactor.

EXERCISE

For the following statements choose the correct option from the given options :

1. How many protons, neutrons and nucleons respectively is the $^{206}_{82}Pb$ nucleus made up of ?

(A) 82, 206, 288 (B) 206, 82, 288 (C) 82, 124, 206 (D) 124, 82, 206

2. Which are the isotope, isotone and isobar nuclei respectively of $^{12}_{6}C$ from among $^{14}_{6}C$, $^{12}_{5}B$, $^{13}_{7}N$?

(A) $^{14}_{6}C$, $^{13}_{7}N$ $^{12}_{5}B$ (B) $^{12}_{5}B$, $^{14}_{6}C$, $^{13}_{7}N$ (C) $^{13}_{7}N$, $^{12}_{5}B$, $^{14}_{6}C$ (D) $^{14}_{6}C$, $^{12}_{5}B$, $^{13}_{7}N$

3. If the radii of $^{27}_{13}Al$ and $^{64}_{30}Zn$ nuclei are R_1 and R_2 respectively, then $\frac{R_1}{R_2}$ = ...

(A) $\frac{27}{64}$ (B) $\frac{3}{4}$ (C) $\frac{9}{16}$ (D) $\frac{13}{30}$

4. The binding energy per nucleon for deuteron ($^{2}_{1}H$) nucleus is 1.1 MeV and that for $^{4}_{2}He$ nucleus is 7 MeV. If two deuteron nuclei fuse to form $^{4}_{2}He$ nucleus, how much energy will be produced ?

(A) 11.8 MeV (B) 23.6 MeV (C) 26.9 MeV (D) 32.4MeV

5. Which is the necessary and sufficient condition for an element to be naturally radioactive?

(A) Z > 50 (B) Z > 60 (C) Z > 70 (D) Z > 83

6. Which one of the following is true for the relative ionizing power of α, β and γ ?

(A) It is maximum for α particle (B) It is maximum for β Particle.

(C) It is maximum for γ radiation (D) It is equal for α, β and γ

7. During the life time of a radioactive element as time passes the number of its nuclei decreases and along with that ....

(A) activity and λ go on decreasing (B) activity and λ go on increasing

(C) activity decreases but λ remains constant

(D) activity decreases but λ increases.

8. Half-life of a radioactive element is 5 min. At the end of 20 min. its .... % quantity will remain undisintegrated.

(A) 93.73 (B) 75 (C) 25 (D) 6.25

9. After a time interval equal to 3 half-lives; how many times would (*a*) the activity of a radioactive element be, of its initial activity ? Or (*b*) mass of a radioactive element be, of its initial mass or (*c*) number of nuclei of a radioactive element be of its initial number ?

(A) 2^3 (B) 3^2 (C) $\frac{1}{3^2}$ (D) $\frac{1}{2^3}$

10. By the disintegration of ${}_{94}Pu^{241}$, the element which is produced is also radioactive and disintegrates. In such a series total 8 α-particles and 5 β-particles are emitted and then the process stops. Which is the final element produced?

(A) ${}_{83}Bi^{209}$ (B) ${}_{82}Pb^{209}$ (C) ${}_{83}Bi^{214}$ (D) ${}_{82}Pb^{214}$

11. 1 g radioactive element reduces to $\frac{1}{3}$ g after 2 days. After total 6 days how much mass will remain ?

(A) $\frac{1}{27}$ g (B) $\frac{1}{6}$ g (C) $\frac{1}{9}$ g (D) $\frac{1}{12}$ g

12. Binding energy per nucleon for ${}^{n}_{z}P$ and ${}^{2n}_{z'}Q$ are x and y respectively. How much energy would be absorbed in the process ${}^{n}_{z}P + {}^{n}_{z}P = {}^{2n}_{z'}Q$?

(A) $2nxy$ (B) $2ny + 2nx$ (C) $2nx - 2ny$ (D) $\frac{2nx}{2ny}$

13. If the number of undisintegrated nuclei at time t is given by $N = N_0e^{-\lambda t}$, what is the number of nuclei disintegrated between the time t_1 and t_2 ?

(A) $N_0(e^{-\lambda t_2} - e^{-\lambda t_1})$ (B) $N_0(e^{-\lambda t_1} - e^{-\lambda t_2})$

(C) $N_0(e^{\lambda t_2} - e^{\lambda t_1})$ (D) $N_0(e^{\lambda t_1} - e^{\lambda t_2})$

14. If the half-lives of a radioactive element for α-decay and β-decay are 4 yr and 12 yr respectively, what percent would its total activity be of its initial activity after 12 yrs ?

(A) 50 (B) 25 (C) 12.5 (D) 6.25

15. Out of C*d*, molten N*a*-metal and graphite which can be used respectively as moderator, coolant and the material for control rods in a reactor?

(A) Molten N*a*-metal, graphite, C*d* (B) graphite, molten N*a*-metal, C*d*

(C) C*d*, molten N*a*-metal, graphite (D) graphite, C*d*, molten N*a*-metal

16. If $^{27}_{13}Al$ is a stable nucleus, what could be emitted from $^{32}_{13}Al$ nucleus ?

(A) α-particle (B) β^--particle (C) proton (D) β^+-particle

17. The half-life of a radioactive element is 2 hr and that of the other is 4 hr. Their initial activities are equal. After 4 hr what will be the ratio of their activities ?

(A) 1 : 4 (B) 1 : 3 (C) 1 : 2 (D) 1 : 1

18. 1 mole of an element emitting α-particles is placed in a vessel which stores them. Half-life of that element is 5 hr. How long would it take for 4.515×10^{23} α-Particles to be stored in that vessel?

(A) 4.515 hr (B) 9.030 hr (C) 10 hr (D) 20 hr

19. In the radioactive transformation $^{A}_{Z}X \rightarrow {}^{A}_{Z+1}X_1 \rightarrow {}^{A-4}_{Z-1}X_2 \rightarrow {}^{A-4}_{Z}X_3$, which are the successively emitted radioactive radiations?

(A) α, β^-, β^- (B) β^-, α, β^- (C) β^-, β^-, α (D) α, α, β^-

20. In the radioactive transformation $X \xrightarrow{\alpha} X_1 \xrightarrow{\beta^-} X_2 \xrightarrow{\beta^-} X_3$ which two are the isotopes ?

(A) X and X_1 (B) X and X_3 (C) X_1 and X_2 (D) X_2 and X_3

21. Half life of a radioactive element X is 3 hr. It transforms to form a stable element Y. After the birth of X; at time *t*, the ratio of the nuclei of X and Y is 1 : 15, what is the value of *t* ?

(A) 12 hr (B) 6 hr (C) 24 hr (D) 45 hr

22. In the process of forming helium from hydrogen, the mass defect is 0.5%. What is the energy produced when helium is formed from 1 kg hydrogen ? [1 *k*WH = 36×10^5 J]

(A) 1.25 *k*WH (B) 1.25×10^6 *k*WH (C) 1.25×10^8 *k*WH (D) 1.25×10^4 *k*WH

23. A radioactive element X disintegrates successively as under :

$$X \xrightarrow{\alpha} X_1 \xrightarrow{\beta^-} X_2 \xrightarrow{\alpha} X_3 \xrightarrow{\gamma} X_4.$$

If the atomic number and the atomic mass number of X are respectively 72 and 180, what are the corresponding values for X_4 ?

(A) 69, 176 (B) 69, 172 (C) 71, 176 (D) 71, 172

24. The half life of a radioactive element X is equal to the average life of other element Y. Initially number of atoms in both of them is same. Then,

(A) initially rates of disintegration of X and Y would be equal

(B) X and Y both disintegrate at the same rate, always.

(C) initially rate of disintegration of Y would be greater than that of X

(D) initially rate of disintegration of X would be greater than that of Y.

25. The elements X_1 and X_2 have decay constants 10λ and λ respectively. If initially they have equal number of nuclei, then after what time would the ratio of numbers of nuclei of X_1 and X_2 be $\frac{1}{e}$.

(A) $\frac{1}{10\lambda}$ (B) $\frac{1}{11\lambda}$ (C) $\frac{11}{10\lambda}$ (D) $\frac{1}{9\lambda}$

ANSWERS

1. (C)	**2.** (A)	**3.** (B)	**4.** (B)	**5.** (D)	**6.** (A)
7. (C)	**8.** (D)	**9.** (D)	**10.** (A)	**11.** (A)	**12.** (C)
13. (B)	**14.** (D)	**15.** (B)	**16.** (B)	**17.** (C)	**18.** (C)
19. (B)	**20.** (B)	**21.** (A)	**22.** (C)	**23.** (B)	**24.** (C)
25. (D)					

Answer the following questions in brief :

1. What is mass defect ?

2. How can we say that radioactivity is a nuclear phenomenon ?

3. What is the meaning of rate of disintegration ?

4. 5 *m*C*i* = ...B*q*. (fill in the blank)

5. What is the slope of the ln I $-$ t graph? (I = activity)

6. In a specimen the number of nuclei of a radioactive element at $t = 0$ time is 2048. If its half life is 5 hr, how many nuclei would have been disintegrated in 25 hr ?

7. "The half life of a radioactive element shows the half of its total life-time."
Is this true ?

8. What is artificial nuclear disintegration ?

9. What is meant by Q-value of a nuclear reaction ?

10. What is nuclear fission ?

11. "Neutron is a good projectile." Why ?

12. How much energy is produced by the fission of U^{235} nucleus ?

13. What is nuclear chain reaction ?

14. What is the function of moderator ?

15. What is meant by multiplication factor (K) in a nuclear chain reaction ?

16. What is the function of the control rods in a nuclear reactor ?

17. What is nuclear fusion ?

18. Define the SI unit of radioactivity.

19. Define the unit 'curie' for the activity.

20. Electrons do not stay in a nucleus, then how do electrons come from the nucleus in the process of β^--decay ?

Answer the following questions :

1. Give a brief account of nuclear forces.

2. Explain the stability of a nucleus.

3. Explain the binding energy of the nucleus.

4. Show the nature of the graph of average binding energy per nucleon against atomic mass number and explain its notable points.
5. Explain natural radioactivity.
6. Which are the radioactive radiations ? Mention their properties.
7. Explain the rate of disintegration of a radioactive element and the decay constant.
8. Obtain the exponential law of radioactive disintegration.
9. Define half–life of a radioactive element and obtain its formula.
10. What is meant by the average life of a radioactive element? Obtain its formula.
11. Explain the phenomenon of β–decay.
12. Explain the Q-value of a nuclear reaction.
13. Explain the process of nuclear fission in detail.
14. What is a nuclear chain reaction ? Explain the difficulties and their removal in its success.
15. Explain nuclear reactor with its working.
16. Explain the thermonuclear fusion in the Sun and other stars.
17. Discuss nuclear hazards.

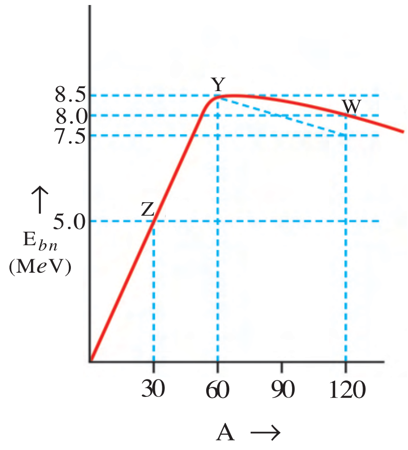
Solve the following examples :

1. In the figure, a graph of average binding energy per nucleon against atomic mass number is shown. In which one of the following reactions will energy be produced ?

   (*a*) Y $\rightarrow$ 2Z

   (*b*) W $\rightarrow$ 2Y

   [**Ans. :** Reaction (*b*)]



2. A radioactive element emits both α and β particles. The average life corresponding to α-emission is 1600 yr and that corresponding to β-emission is 400 yr. If both these emissions simultaneously take place, find the time for 75% of the specimen to decay.

   [**Ans. :** 443.52 yr]

3. Two protons in a star are involved in a head on collision. If the kinetic energy of each of these protons is 18 *ke*V, what would be the distance of closest approach between them ? ($k = 9 \times 10^9$ Nm^{2}C^{-2}) [**Ans. :** 4×10^{-14} m]

4. When a counter is brought near a patient injected with a radioactive dose, it records 16000 counts per minute. In equal circumstances, after 4 hours it records 500 counts per minute. Find the half life of the radioactive element in the given dose .

   [**Ans. :** 48 min]

5. Half life of Ra^{226} is 4.98×10^{10} s. Find the activity of its 1 g specimen. Take Avogadro number as 6.02×10^{23} mol^{-1}. [**Ans. :** 1 C*i*]

6. Mass of ${}^{35}_{17}\text{Cl}$ nucleus is 34.9800 u. Taking the mass of proton as 1.00783 u and that of neutron as 1.00866 u, find the binding energy per nucleon for ${}^{35}_{17}\text{Cl}$ nucleus.

[**Ans. :** $8.219 \frac{\text{MeV}}{\text{nucleon}}$]

7. The average radius of a nucleus is 6.6 fermi. If the average mass of the nucleon is 1.0088 u, find the average density of the nucleus. (R_0 = 1.1 *fermi*, 1 u = 1.66×10^{-27} kg)

[**Ans. :** 3×10^{17} kg m^{-3}]

8. In a given sample, at some instant, the rate of disintegration of radioactive element is 8000 disintegrations per second. At this instant, the number of undisintegrated nuclei of this element is 8×10^7. Find the decay constant and half life of this element.

[**Ans. :** $\lambda = 10^{-4}$ s^{-1}, $\tau_{\frac{1}{2}} = 6930$ s]

9. By the fusion of 1 kg deuterium (${}_1H^2$) according to the reaction, (${}_1H^2 + {}_1H^2 \rightarrow {}^3_2He + {}_0n^1$ + 3.27 MeV) how long can a bulb of 100 W give light ?

($N_A = 6.02 \times 10^{23}$, 1 yr = 3.16×10^7 s)

[**Ans. :** Nearly 24917 yr]

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7

SEMICONDUCTOR ELECTRONICS : MATERIALS, DEVICES AND SIMPLE CIRCUITS

7.1 Introduction

Electronics is a very familiar name in this modern age. The electron was discovered in the electric discharge experiment carried out in gases. It was later known that the electron is a very important particle in the constitution of matter. A detailed study on the various electronic property of the matter like the electrical conduction etc. has been done.

The electrical conductivity in the metals is due to its free electrons. Ohm's law is normally obeyed in the case of good conductors. This means that electric current is directly proportional to the electric potential difference.

We can establish different relation between the electric current I and the electric potential difference V, if we somehow control the number of electric charges which are responsible for the electric current. As a result, different types of application can be achieved by applying the electric potential difference and the current resulting due to it. We can think of making newer devices in which we obtain a specific I – V relationship by controlling the production of electrons, their numbers as well as its conduction. Such a device can be specially made or could be available in a natural form which can be modified as per our requirement. The branch of electronics deals with the study of such devices and its various applications.

(**Note :** The word electronics is coined from the word electron mechanics.)

There are many substances found in nature in which the conduction of electricity is different from the metals. By properly adding impurities in such a substance appropriate I-V relation can be established. The branch of solid state electronics has progressed due to such a substance.

Solid state devices have a very small dimension and they are lighter in weight. The electronic products made from such devices are small in dimension and are very efficient, at the same time there has been drastic reduction in their cost.

We shall study semiconductor device like the P-N junction diode, transistor, LED (Light Emitting Diode) and solar cell in the following section. We shall also study about logic circuits which are the pillars of the digital electronics.

7.2 Conductors, Insulators and Semiconductors (A Bond Picture)

The elements in the first three groups of the periodic table like the alkali metals, noble metals, aluminium etc. are good conductors. The electrical conduction is easily possible in such elements due to the presence of free electrons. The electrical resistivity of such elements is comparatively quite less. Non-metals (insulators) are almost bad conductors of electricity. There are no free electrons in such elements. These elements have a larger electrical resistivity.

The elements in the fourth group of the periodic table like the Si and Ge have greater electrical resistivity than the good conductors but have a lower resistivity than the bad conductors. Such elements are known as semiconductors.

The mechanism of flow of electric current is different in conductors and semiconductors. Pure semiconductors behave as insulator at 0K temperature.

The resistivity of conductors increases with temperature, while the resistivity of the semiconductors decreases with increase in temperature upto certain limit. The conductivity of the semiconductor is changed by incidenting radiation of suitable frequency.

Table 7.1

Resistivity of Conductors, Insulators and Semiconductor (At Room Temperature)
(Only for Information)

Substance	Resistivity (ρ) (Ω m)	Conductivity ($\sigma = \frac{1}{\rho}$) (S/m)	Classification
Silver	1.6×10^{-8}	6.25×10^{7}	Conductors
Copper	1.7×10^{-8}	5.88×10^{7}	
Aluminium	2.6×10^{-8}	3.85×10^{7}	
Germanium (pure)	6.5×10^{-1}	1.54	Semiconductors
Silicon (pure)	2×10^{8}	5×10^{-9}	
Glass	1.7×10^{11}	5.88×10^{-12}	Insulators
Hard rubber	1.0×10^{16}	1×10^{-16}	

Si and Ge are known as elemental semiconductors. PN junction diode, zener diode, transistor etc are fabricated from it.

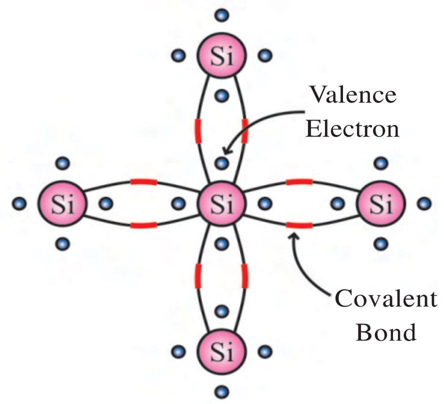
Many of the compounds apart from the elemental semiconductors also behave as semiconductors. Many of such compounds are carbonic, non-carbonic compounds as well as polymer carbonic substances. As for example CdS (Cadmium sulphide), GaAs (Gallium Arsenide), CdSe (Cadmium selenide) etc. are non-carbonic semiconductors. Solar cell, LED, LASER diode etc. are some of the devices which are made from such semiconductors.

After 1990, some of the electronic devices are made from carbonic and polymer carbonic substances. As result, branches of polymer electronics and molecular electronics have developed.

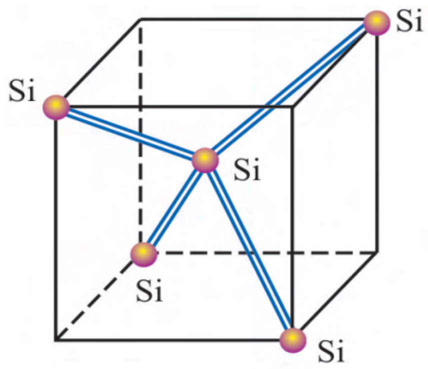
We will study only elemental semiconductors in this chapter. The Si is a very important semiconductor.

Atomic number of Si is 14. The electronic configuration of Si is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^2$. The electrons up to $1s^2\ 2s^2\ 2p^6$ completely occupy the K and L shells. $3s^2\ 3p^2$ electrons are the valence electrons. Hence Si (and Ge(z = 32)) is tetravalent. Here the two s orbital and two

p orbitals combine to form four (sp^3) complex orbitals. These orbitals combine with similar such orbitals of the neighbouring atoms and constitute covalent bond. There are two electrons for every covalent bond. In this way the four valance electrons of the Silicon makes a covalent bond with its four neighbouring atoms by sharing one-one electron. Figures 7.1 (*a*) and 7.1 (*b*) shows the above situation in two dimensions and three dimensions respectively. A crystal lattice of a diamond is obtained if one extends the above arrangement of atoms in three dimensional space. Thus Si and Ge have diamond structure.



(*a*)



(*b*)

Figure 7.1

Concept of Hole : At absolute zero temperature each of the valence electrons of Si (and Ge) is bound by the covalent bond. As a consequence Si (and Ge) behave as insulators at absolute zero temperature. The atoms of the crystal perform thermal oscillations at the room temperature. This results in the breaking of several covalent bonds and results in the electrons freeing from the covalent bond. These free electrons are responsible for electrical conduction.

Deficiency of electron is created at the place from where the electron became free. This deficiency has the ability of attracting the electrons. An electron which has become free from any other covalent bond can get trapped in this place. This deficiency of electron is known as hole. It behaves as if it has positive electric charge. (see figure 7.2) It has to be remembered that hole is not a real particle and it neither has any positive electric charge. The question then arises is that, what is the importance of hole.

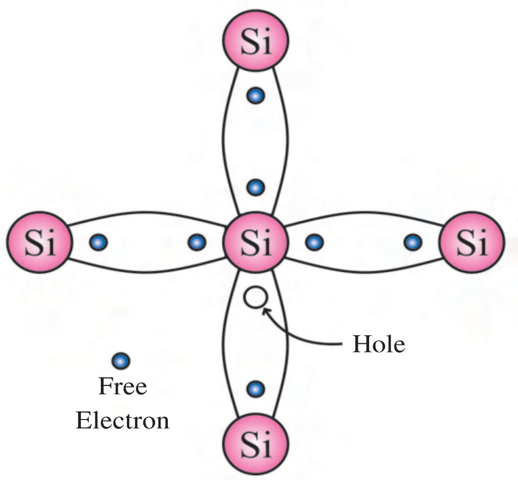


Figure 7.2

At room temperature in Si the required energy for electrons to escape from covalent bond is 1.1 eV and for Ge it is 0.72 eV.

Electrical Conduction in Semiconductors : We have seen how an electron leaves behind a hole on becoming free from the covalent bond. In this situation on applying a p.d. between two ends of a crystal, the free electrons move from negative end to positive end and constitute the electric current (see figure 7.3).

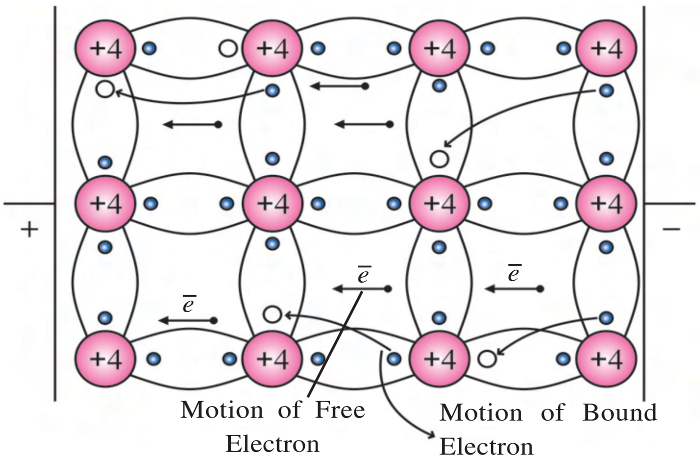


Figure 7.3 Electrical Conduction in Semiconductor

Apart from this, thermal oscillations and external electric field causes the bound electrons to be free from covalent bond and gets trapped in the nearest hole. And a new hole is created at the place where the electron escaped from the covalent bond. The motion of the bound electron is from the negative end towards the positive. Hence, it is understood that motion of hole is from positive end towards the negative end.

Thus, we get two types of currents in a semiconductor, (1) Due to motion of free electron (I_e) (2) Due to motion of bound electron or hole (I_h).

Such an electron becomes free from a bound state and again gets bound in the nearest hole. Such electrons cannot be considered as free electrons. To differentiate the motion of the bound and the free electron, one can consider the motion of the hole in a direction opposite to the direction of the motion of the bound electron.

The hole behaves as a particle having positive charge. Since its motion is in the opposite direction to that of the electron i.e. from the positive end towards the negative end. We will have to remember that the conduction of holes means that the conduction is due to bound electrons. In the case of a pure semiconductor like Si and Ge the electrical conduction is due to both electrons as well as holes.

The number density of free electron and hole in a intrinsic (pure) semiconductor n_e and n_h respectively are equal due to pair production. Here, electron and hole are also known as intrinsic electric charge carriers, hence it's number density is indicated as n_i. For an intrinsic semiconductor, $n_e = n_h = n_i$

In Si (or Ge) more number of bonds get broken with the increase in temperature. This results in increase in the number of electrons and hole. Due to increases in the number density of charge carriers the conductivity also increases.

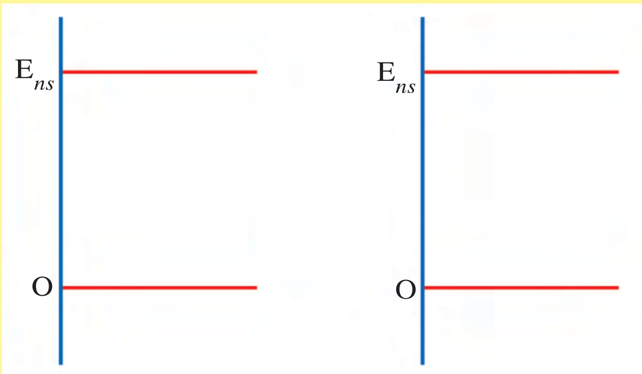
7.3 Conductor, Insulator and Semiconductors (A Band Picture)

Only for Information : The X-ray and other studies show that some solids have a crystalline structure. This shows that there is a systematic arrangement of atoms or molecules. In previous chapter, we have studied the energy level of an electron of a hydrogen atom, but it is not applicable to crystalline material. When atoms are arranged close to each other, the atom gets interact with neighbouring atoms and other atoms. As a result energy levels of electrons of atoms are changed. The energy level of inner shells electrons are not affected, hence they are strongly bound with the nucleus. But the energy levels of the outer shells electrons (valence electrons) are changed, since these electrons are shared, by more than one atom in the crystal. It is observed that electrons of the atoms in the crystal has closely spaced different energy levels instead of widely separated energy level of electrons of isolated atom. Such a band of close energy levels is called energy band. Now we shall learn, how energy bands are formed in solid materials.

We know that quantized energy levels of quantized system are drawn on the vertical axis with proper scale and with small horizontal line is called energy level diagram.

The study of quantum mechanics is inevitable in the understanding of the molecules containing more than one atom as well as the behaviour of the electron and their energy levels in solid substance.

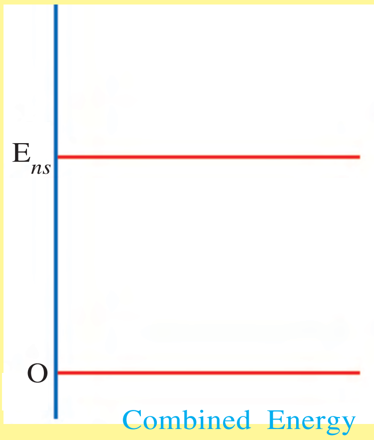
The valence electrons of the constituent atoms are only considered to determine the configuration of the electron of a solid substance. Let us consider a simple example in order to understand the energy levels of the electrons of a solid substance. Consider a simple molecule containing only two atoms. Let each of the atom contain one valence electron represented as *ns* (Here n = principal quantum number and s means $l = 0$ orbital quantum number). The energy of the atom be represented as E_{ns}. When these atoms are at infinite distance from each other, then no interaction takes place between them. Hence the two atoms can be thought as independent. Each of them has an *ns* electron whose energy is equal in E_{ns}. figure 7.4 shows the energy level diagrams of each of these atoms respectively.



Energy Level for Atom (1)

Energy Level for Atom (2)

(*a*)



Combined Energy Level of Two Atoms

(*b*)

Figure 7.4 Energy Level of Atom

If the above two diagrams are represented on a single figure then we obtain a single line E_{ns} as shown in the figure 7.4 (*b*) indicating the energy of the two atoms.

When the two atoms are brought closer to each other then the constituent particles of the two atoms will interact with each other and as a result the energy of the valance electron will be different from the situation when they were separated by an infinite distance. A new wave function and as a consequence a different value of energy is obtained. The value of the energy E_{ns} will be now different for the two electrons which

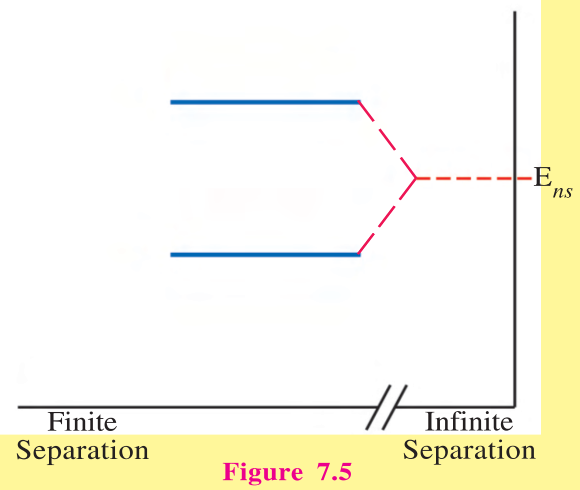


Figure 7.5

was earlier identical and represented as a single line. This results in the splitting of the energy E_{ns}. Such a representation of the splitting of the energy is given by an imaginary representation in the figure 7.5.

In the first case, we have two energy levels of energy E_{ns} and from which we obtained two energy levels having different energy.

In our imaginary example, if we had three atoms, E_{ns} then we would have obtained three different energy levels or in other words the original energy level will get split into three different energy levels.

If we extend our discussion to the energy E_{ns} of the N atoms, we shall obtain N energy levels. (It has to be noted that the total number of energy levels remain fixed.) For a two atom system the distance versus energy contains two graphs, while the N atom system will contain N graphs. For the sake of simplicity figure 7.6 shows the energy levels of the electron having the maximum and the minimum value, after the splitting has taken place. In between energy levels are merely indicated by horizontal lines.

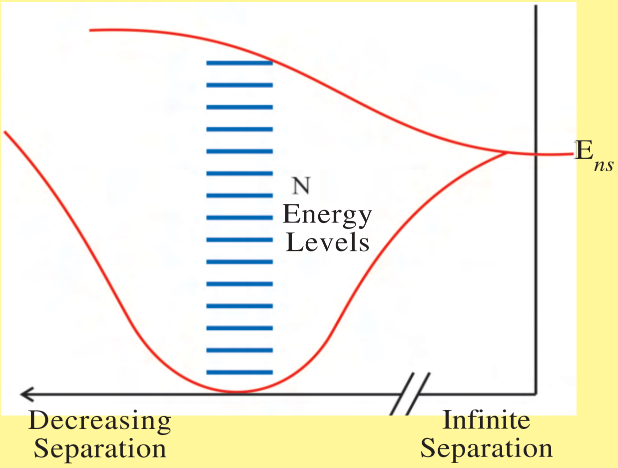


Figure 7.6 Energy Levels on N Atoms

The difference in the energy between two adjacent splitted energy levels is very less. One can say that the splitting of the N energy levels constitutes an energy band. In the example considered so far, we have been discussing the band structure due to the *ns* band. Since *ns* is considered as the state of valence electron, the band can also be referred to as valence band.

Let us try to understand the band structure obtained in case of silicon (Si).

The electronic configuration of silicon atom is given as $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^2$. Here the $n = 1$ (K-shell) and $n = 2$ (L-shell) are completely filled hence $3s^2\ 3p^2$ four electrons are valence electrons. Hence, we have a total of four valence electron. Let us consider that N number of atoms are present in solid silicon. If initially these atoms are separated by an infinite distance, we have 2 N electrons of the 3*s* type and 2N electrons of the 3*p* type. The total number of valence electrons would be equal to 4 N. Each atom has total number of 2 valency states in the 3s sub shell and 6 number of 3*p* valency states. Hence a total of 8 valence states are available for the 3*s* and 3*p* states. For a solid containing

N number of atoms, a total (2 N + 6 N) valence states are existing. All 2N states of E_{3s} as well as all the 2 N states of E_{3p} are identical when they are an infinite distance from each other. This is shown in the figure 7.7.

As the separation between the atoms is reduced, the splitting of the 3*s* state into 2N levels and the splitting of the 3*p* state into 6N levels takes place. For example, the splitting for distance R_1 of the 3s and 3*p* states into 2N and 6N levels respectively is shown in the figure. Here E_g is called the band gap.

Band gap is the difference between the maximum energy level of the E_{3s} state and the minimum energy level of the E_{3p} state for the given inter atomic separation.

On further reduction in the inter atomic separation, there is no gap between 3*s* and 3p types of bands. From this separation onwards, we get a combined wave function, by carrying out the integration of the 3*s* and 3*p* type wave function. On further reducing the separation the two bands containing 4N energy level each are obtained. The band gap between them is shown in figure 7.7. The two bands for the equilibrium position of silicon is shown in the figure. As per Pauli's exclusion principle, we can have one electron for each energy level. As a result, the lower band will be completely occupied by 4 N number electrons. (As per Pauli's exclusion principle, each level cannot accommodate more than one electrons.) Here lower band is called valence band.

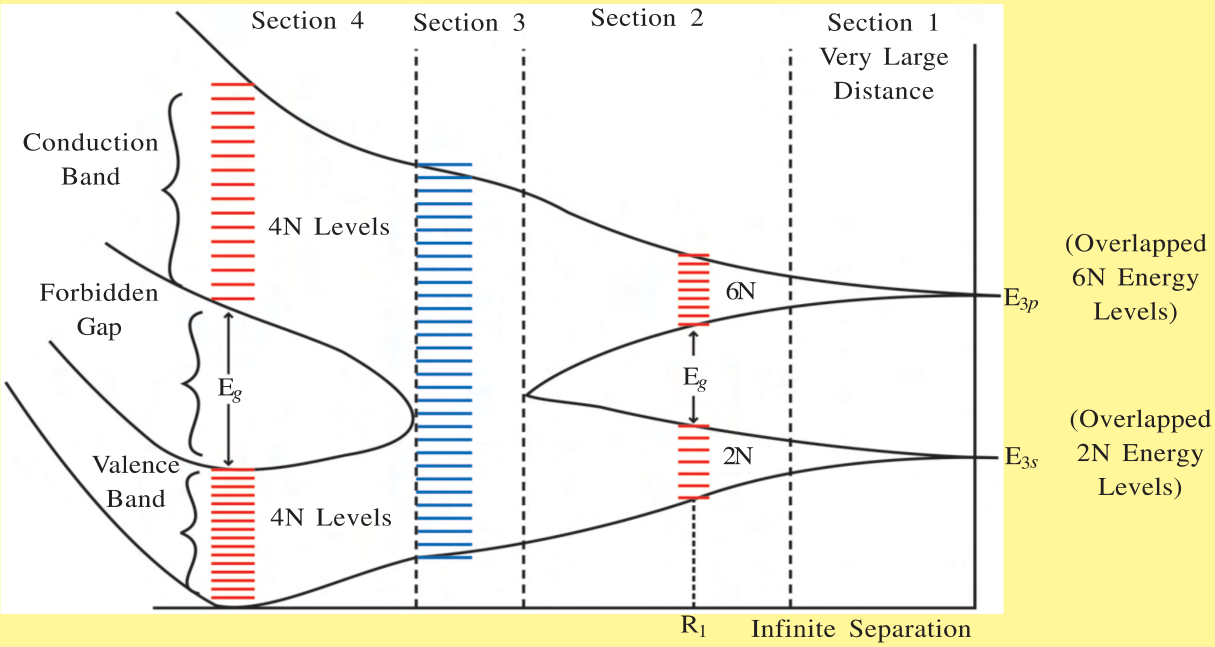


Figure 7.7 Band Diagram of Si

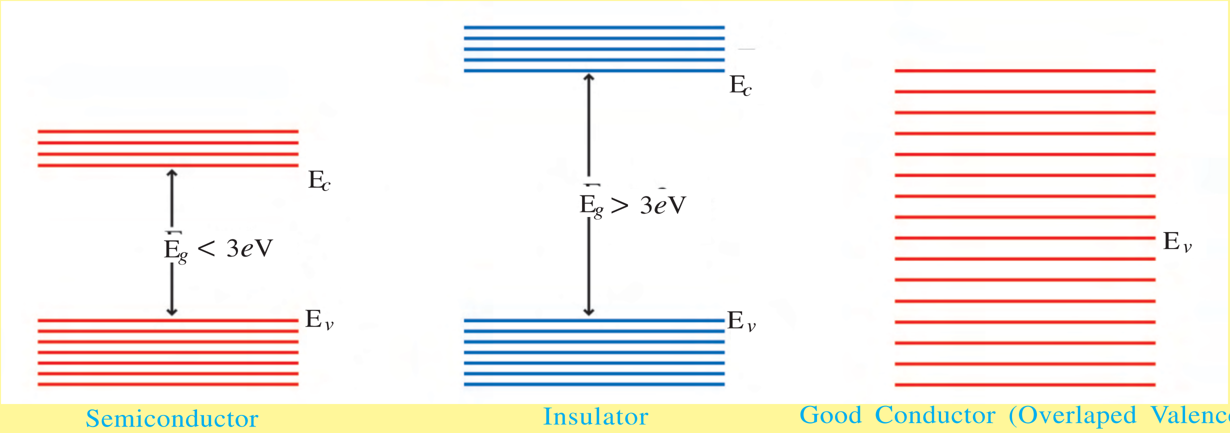
The upper band is completely empty. A minimum of E_g energy will have to be supplied to the electron to move from the valence band to the upper band. It can move in to any of the energy levels in the upper band, since it is completely empty. If it does so it becomes a free electron and is available for electrical conductions. It is for this reason the upper band is known as the conduction band.

The difference in the lowest energy of the conduction band and the maximum energy of the valence band is known as band gap (E_g). There are no available energy states in the band gap. It is for this reason that this gap is known as forbidden gap.

The situation of an insulator is similar to that of semiconductors in which the valence band is completely filled while the conduction band is completely empty. The only difference between the two is the value of the band gap. The value of the band gap is large in case of an insulator. It is more than 3eV while for a semiconductor it is less than 3eV.

Let us consider an example of the sodium metal in order to understand the energy levels of a conductor. The electronic configuration of Na is given as $1s^2\ 2s^2\ 2p^6\ 3s^1$. Thus there is one valence electron in Na atom. There are two quantum states if one considers the spin for the 3s state. If we consider the sodium atom to be made up of N atoms then there are total number of 2N available states or 2N number of energy levels. (Here we are discussing the energy levels arising from the 3 s state). If we start filling the electrons in the valence band, since there are N number of valence electrons, the valence band will be half filled according to the Pauli's exclusion principle. In such a situation, with very little energy, the electron can move into the available energy levels. These electrons behave as if they are free electrons and contribute towards electrical conduction. The study of the band structure of the metals containing more than one valence electrons suggests that the valence band and the conduction band overlap each other.

The insulators, semiconductors and the conductors have the valence band and the conduction band structure as shown in the symbolic diagram of figure 7.8.



Semiconductor Insulator Good Conductor (Overlaped Valence Band and Conduction Band)

Figure 7.8 Band Diagram for Conductor Insulator and Semiconductor

The explanation given in the box gives us an idea about of the band structure of the solid substance. Let us summarize the discussion as follows.

The atoms have energy levels similar to the energy states of all atoms. The classification of insulators, semiconductors and conductors is based on the basis of these energy levels.

Let us consider the example of silicon in order to understand the electrical conductivity of the insulators.

Semiconducter : Let there be N number of atoms in the diamond structured silicon crystal. There are two electrons in the $3s^2$ state and six electrons in $3p^2$ of the silicon atom which are the available valence states. Out of which four states are filled.

When the Si atom constitutes the crystal, we have a total of 8N valence state and the corresponding energy levels are indicated in the figure 7.9.

The closely spaced 4N levels constitute a band structure. As per the Pauli's exclusion principle, one electron occupies only one energy level. As a result with the 4N available electrons the lower band is completely filled.

This lower band is known as valence band. As the band is completely filled, the electrons cannot move to other energy levels. Hence, there is no electrical conductivity.

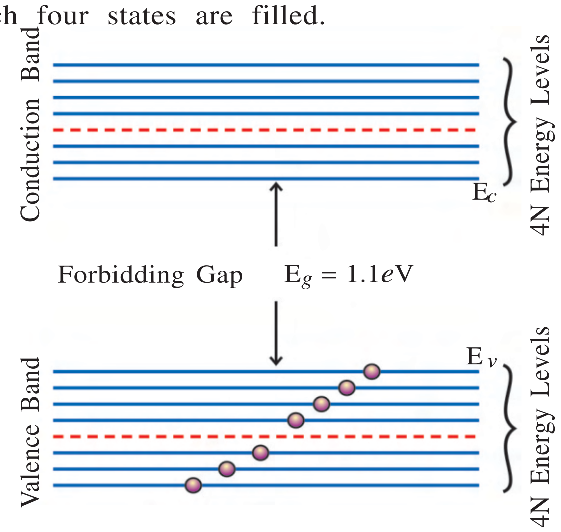


Figure 7.9 Band Diagram of S_i at (0 K Temperature)

Above the valence band, there is a region where there is no available energy levels. This region is known as the forbidden gap. The width of the forbidden gap is < 3 eV. The values of E_g for Si and Ge are 1.1 eV and 0.72 eV respectively.

The region of the energy above the forbidden gap is known as the conduction band. At 0 K temperature, the conduction band is completely empty. If the electron has enough energy to cross the forbidden gap then the electron can move from the valence band to the conduction band. These electrons will then contribute towards the electrical conduction.

A hole is created when an electron moves from valence band to the conduction band. The number of holes created is equal to the number of electrons present in the conduction band. Hence in an intrinsic semiconductor number of electrons and holes are same.

Insulator Substances : Such a substance has large forbidden gap (>3 eV). For diamond value of E_g is 5.4eV. As a result, the electrons are not able to move easily from the valence band to the conduction band and such a substances are bad conductors of electricity.

Conductors : Figure 7.10 shows the band structure of a sodium atom, which explains the reason why it is a good conductor of electricity. The electronic configuration of sodium atom is given as $1s^2\ 2s^2\ 2p^6\ 3s^1$. There are 2 N valence states for the 3s state. Out of which N states are filled due to the contribution of one electron from each of the sodium atom. The remaining N states are empty. As a result, the electrons can move easily into the empty available states and contribute towards electrical conductivity. In many of the metals, the conduction and the valence bands overlap each other. In such a situation too the electrons contribute in the electrical conduction.

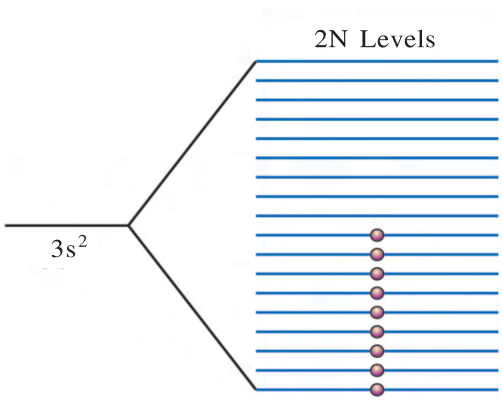


Figure 7.10 Band Diagram of Sodium

7.4 N and P Types Semiconductors (Extrinsic Semiconductors)

The conductivity of the pure semiconductor can be drastically changed by adding impurities in the right proportion. This process of adding impurities in the semiconductor is known as doping. As for example pentavalent impurities like Antimony and Arsenic or trivalent impurities like aluminium, Gallium or Indium can be used for doping, the doped atoms arrange themselves in place of the original atom in the host crystal.

Two dimensional symbolic representation of silicon crystal lattice structure is shown in figure 7.11. Here the Arsenic atoms have been added as impurity atoms. The figure shows Arsenic atoms replacing one silicon atom.

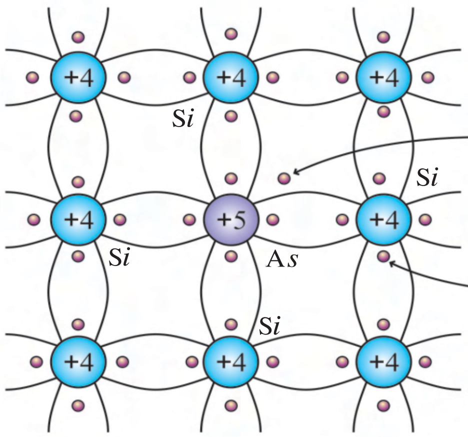


Figure 7.11 Si Crystal Lattice with as Impurity

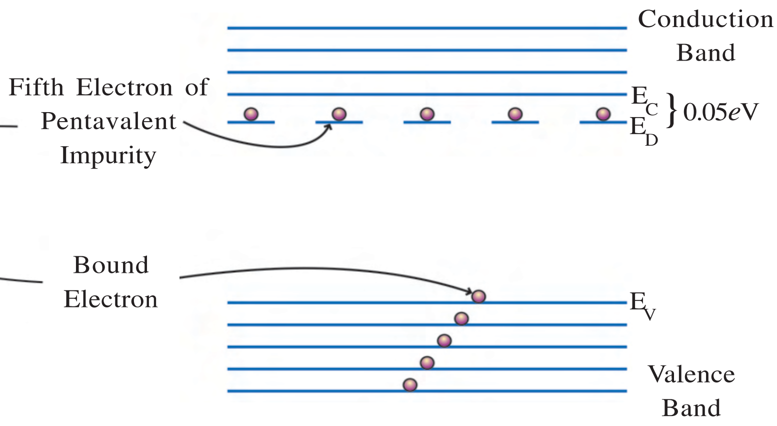


Figure 7.12 Band Diagram of N-type Semiconduction (At 0 K Temperature)

The four out of the five valence electrons of the Arsenic atom are involved in the formation of the covalent bonds with its four neighbouring silicon atoms. The fifth electron is available as an extra electron to the crystal. If 0.05 eV energy is available to this electron, it can act as a free electron. This energy is 0.01 eV in case of Germanium. This much energy is already available to the electron in the form of thermal energy at the room temperature. The impurity atoms donate electric charge carriers (electron) to the host crystal. Such an impurity atom is known as donor atom. Their proportion is kept as approximately as 1 in 10^6 pure atoms. Hence, 1 mole crystal contains approximately 10^{17} impure atoms. Each of these impurity atoms contribute one electron. 1 mole crystal contains approximately 10^{17} free electrons. A metal like copper which is a good conductor contains approximately 10^{23} free electron.

Apart from the number of free electrons mentioned above, some more free electrons are obtained due to the breaking of the bonds resulting in the formation of the holes. Their number is very small compared to the number of free electrons donated by the impurity atoms. We can thus say that the charge carriers available for electrical conduction is primarily obtained from the electrons donated by the impurity atoms. Thus electrons are known as majority charge carriers, in the case of the addition of pentavalent impurities. The electron carries negative charges and hence such a crystal is known as N-type semiconductor crystal, deriving it's name from the first letter of the word Negative. The electrical conduction due to holes in such a crystal is very less, so holes are known as minority charge carriers. It is very clear that $n_e > n_h$.

Figure 7.12 gives the energy levels in N-type semiconductors which helps in understanding the electrical conductivity in N-type semiconductor. The figure shows the completely filled valence band as well as the completely empty conduction band. Apart from these energy levels the valence energy levels of the impurity atoms is also indicated by the dashed lines. The above situation refers to 0 K temperature.

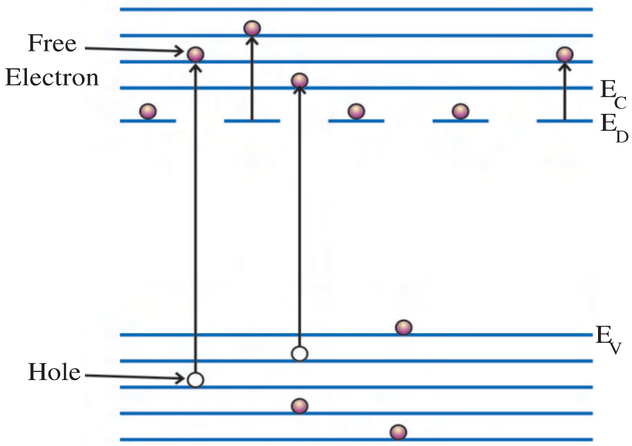


Figure 7.13 Band Diagram of N-type Semiconductor (At Room Temperature)

At 0 K temperature, one electron each of the impurity atoms occupies one of these energy levels. We are aware of the fact that the impurity atoms are scattered in the crystal structure of the semiconductor. The wave function of these valence states lie closer to the impurity atoms or in other words are not present in the entire crystal. Hence the symbolic representation is shown by dotted line.

The difference between (E_C) and (E_D) being very less, when the temperature increases, more and more electrons from the valence band of the semiconductor as well as the valence electrons of the impurity atoms would cross over to the conduction band and occupy empty energy levels in it. The number of charge carriers available for electrical conduction will be much more than the pure semiconductors. Hence, in N-type semiconductor $n_e > n_h$.

P-type semiconductors : If we add trivalent impurity like Aluminium in the Germanium or silicon then three free electrons of these impurity atoms form covalent bonds with its neighbouring three Germanium or Silicon atoms. As a result, there is a deficiency of one electron in the formation of the covalent bond in one of the four neighbouring Ge or Si atoms. This deficiency of electron can be considered as a hole. This hole is present in one of the bonds between the aluminium and silicon atoms. This hole attracts electron and hence in this sense the aluminium impurity is known as acceptor impurities. The electrical conduction in such a crystal is primarily due to holes. Holes behave as a positively charged particle. Hence, such a semiconductor is known as P-type semiconductor. Holes are the majority carriers while electrons are the minority charge carriers in a P-type semiconductor. In this case $n_h > n_e$.

Figure 7.14 shows symbolic representation of Aluminium impurity added to a Germanium crystal lattice.

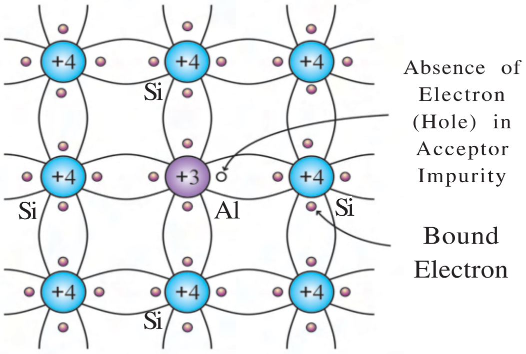


Figure 7.14 Si Crystal with Al Impurity

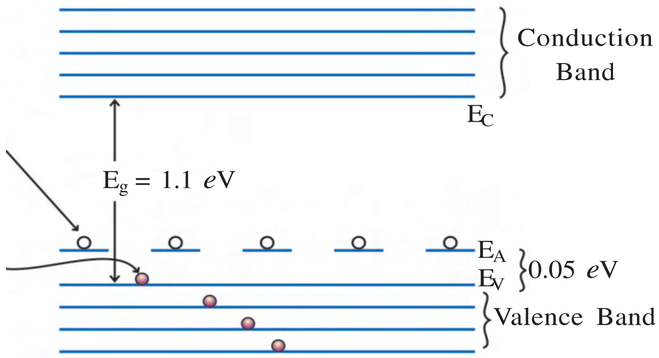


Figure 7.15 Band Diagram of P-type Semiconductor (At 0 K Temperature)

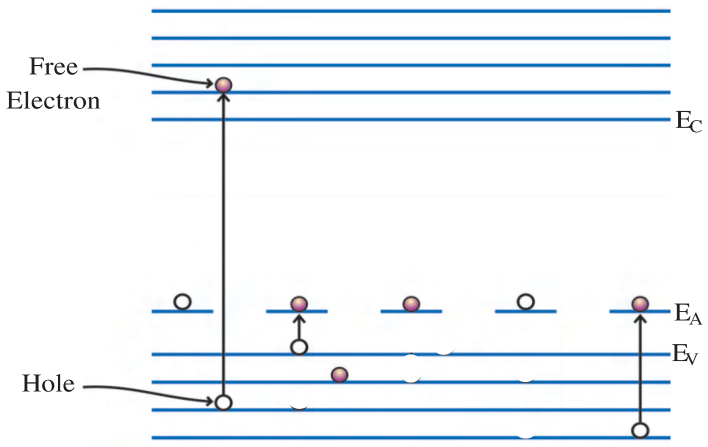


Figure 7.16 Band Diagram of P-type Semiconductor (At Room Temperature)

Figure 7.16 shows the energy levels of the semiconductor as well as the impurity atoms of a P-type semiconductor. Here the impurity atoms energy levels E_A lies very close to E_V. Since no electrons are present at these energy levels one can say that there is an existence of holes. The electrons present in the valence band of the semiconductors can easily occupy the empty energy levels of the impurity atoms on getting sufficient energy at the room temperature. Apart from these, some of the electrons occupy empty energy levels in the conduction band and as a result create holes in large numbers and the possibility of the motion of the electrons also increases. Hence in a P-type semiconductor the electrical conductivity is much more than the electrical conductivity of a pure semiconductor. Here $n_h > n_e$.

The next question arises in our mind as to why don't all the electrons from the valence band cross over to the conduction band. Actually the creation of electron hole pair due to the migration of the electron to the conduction band is not a very stable situation. The electrons and holes collide with each other as per the laws of thermodynamics and the temperature. The electrons once again occupy the hole. The creation of the electron hole pair and it's recombination process takes place at the same time. In the equilibrium position the rate of electron hole pair formation and their recombination is equal.

The recombination rate $\propto n_h \, n_e$

Recombination rate $= R \, n_h \, n_e$ (7.4.1)

Here R is known as the recombination coefficient.

For an intrinsic (or pure) semiconductor, $n_e = n_h = n_i$

Hence the recombination rate $= Rn_h n_e = Rn_i^2$ (7.4.2)

The recombination rate for an intrinsic semiconductor and it's extrinsic semiconductor as per the laws of thermodynamics are equal.

$\therefore \; Rn_e n_h = Rn_i^2$

$\therefore \; n_i^2 = n_e n_h$ (7.4.3)

7.5 P-N Junction Diode

When Si or Ge wafer is doped with donor impurity (As) at one end and accepter impurity (A*l*) at the other end, the silicon wafer contains N-type semiconductor region, P-type semiconductor region and junction between them. Figure 7.17 shows the situation of P region and N region before PN Junction is formed.

There are excess holes in the P-section. The holes are represented as a small circle (O). These holes exist in the site of the covalent bond between the impurity atoms Al and Si atoms. In the N section there are excess electrons and it is represented as small line (–). These electrons are obtained from the impurity atoms (As). To illustrate the impurity atoms three As atoms and three Al atoms are shown in figure 7.17.

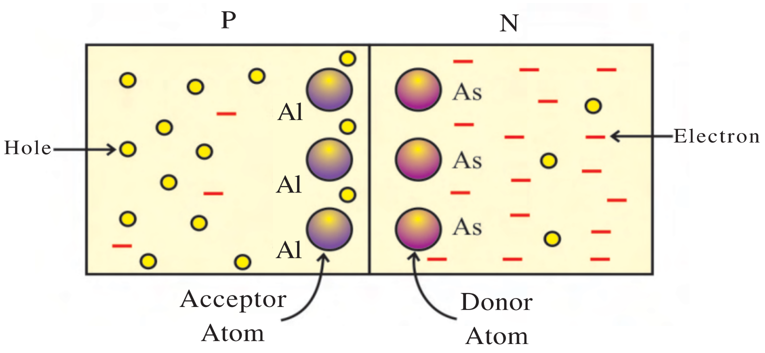


Figure 7.17 A Situation before Formation of PN Junction

In this situation both N and P are electrically neutral. In N section, Arsenic atom donates one electrons but its nucleus carries one excess positive charge. In a similar way in the P section, there is a deficiency of one electron due to aluminium atom but its nucleus also has one positive charge less.

N–section has excess of free electrons compared to the P section. Hence, the diffusion of electron takes place from the N-section towards the P-section. As a result, the electrons diffuse in the hole existing in the P section of the junction. Similarly, the holes also diffuse from P-section towards N-section in a small amount. i.e. near the junction small amount of valence electrons of N-section diffuse in the holes of P-section.

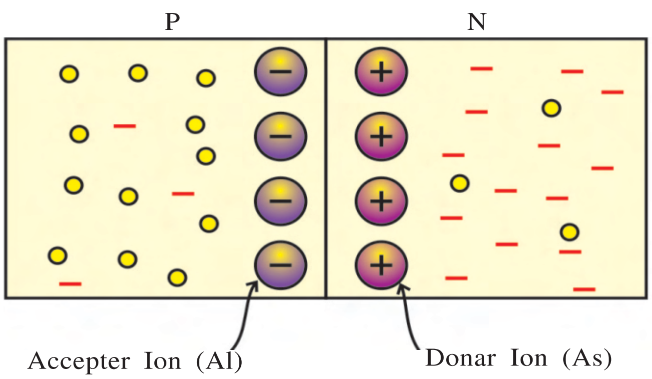


Figure 7.18 Situation of PN Junction After the Diffusion

Figure 7.18 shows the situation after some diffusion has taken place. As the electron from Arsenic atom of the N-section diffuses into the hole of Al atom in P-section. Arsenic atom becomes positive ion and Al atom becomes negative ion near the junction. As the diffusion process continues the number of the positive ion increases on the N side of the junction and negative ion increases on the P side of the junction. Thus the negative charges and positive charges are accumulated near the junction in the P-section and N-section respectively. These charges are steady since they are charges of the ions. Due to these charges, electric field is established at the junction from N region towards the P region. In other words, positive potential on the N side and negative potential on the P side is established. Now the electrons have to overcome the electric field in order to diffuse from N region to P region. The diffusion process of the electron and hole stops as the electric filed is sufficiently established.

This situation of PN Junction is shown in figure 7.19. A graph also shows the established electric potential near the junction region. Two points can be concluded from the figure 7.19.

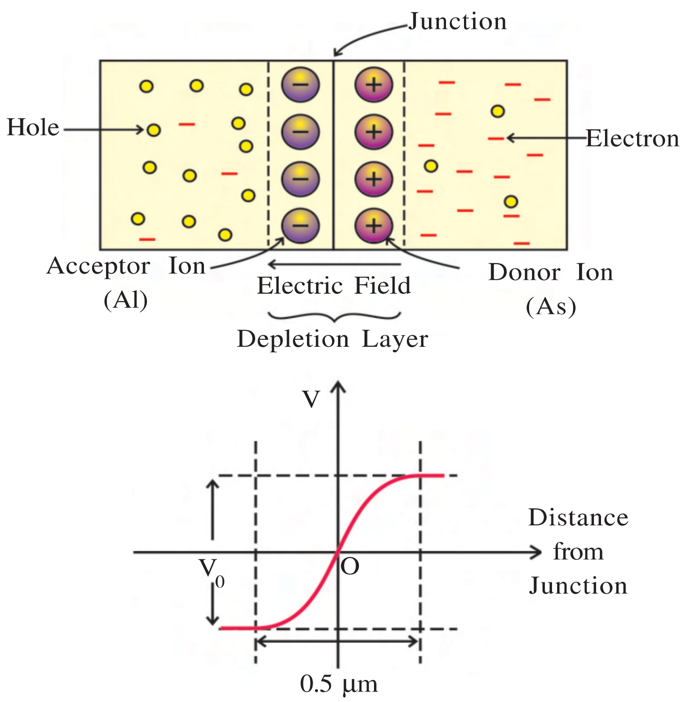


Figure 7.19 Depletion Layer of PN Junction

(1) Electrons are no longer the majority charge carriers in the small region of the N-type material near the junction. Similarly holes are not the majority charge carriers in the small region of the P-type semi-conductors near the junction. These regions are known as depletion region since they are depleted of the respective majority charge carriers. This region is known as depletion layer or space charge region. The width of depletion region is approximately $0.5\,\mu m$.

(2) The distribution of electric potential in the depletion layer is called the depletion barrier or potential barrier. This potential barrier is in order of 0.1 V. This value is about 0.7 V for Si and 0.3 V for Ge.

In PN Junction, the magnitude of depletion barrier and width of depletion region are dependent on the concentration of the impurity added to the P and N type semiconductor. The depletion region is wider if the amount of impurity atom added is less and the electric field becomes weaker near the junction. The width of the depletion region decreases with the increase in the impurity concentration. This increases the intensity of electric field near the junction. Thus, the characteristics of the junction can be changed by increasing or decreasing the impurity concentration. As a result we can fabricate different types of the semiconductor devices. The symbol of PN junction diode is shown in figure 7.20.

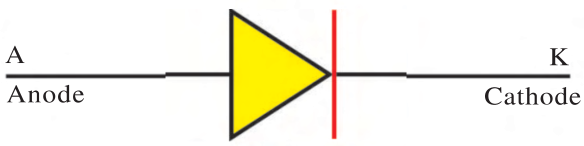


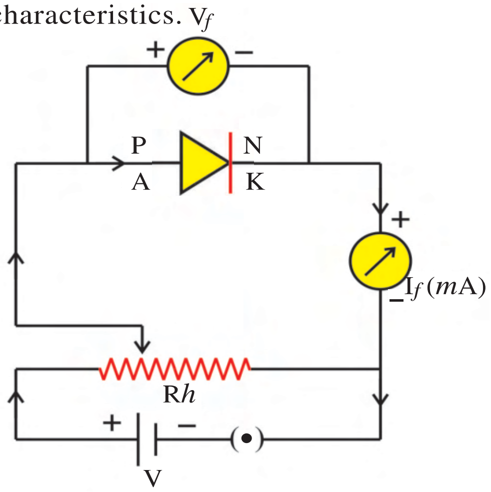
Figure 7.20 Symbol of PN Junction

Here, P region is referred to as anode (A) and N is referred to as cathode (K). Since, there are two electrodes, it is known as PN junction diode.The arrow shows the direction of conventional flow of current (when diode is in forward bias) in PN junction diode. Now we will discuss the characteristics of the diode.

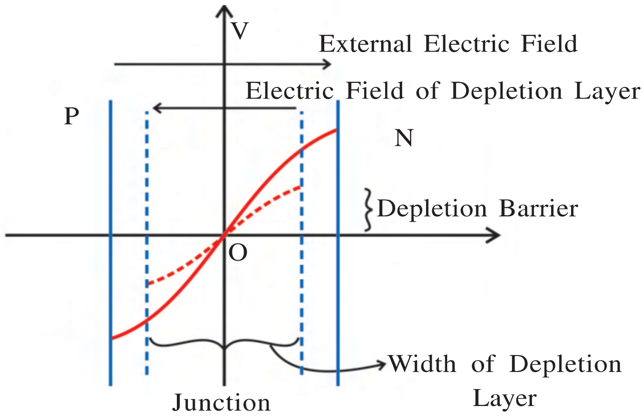
7.6 Static Characteristics of PN Junction Diode

We shall study the I-V curve of the PN junction diode, which is also known as the characteristics of the P-N junction diode.

The circuit diagram to study the characteristic of diode is shown in figure 7.21 (*a*) and 7.23 (*a*). Continuous voltage can be varied across the diode with the help of the rheostat connected in a parallel connection with the battery. The voltmeter measures this applied voltage. The milliammeter or the microammeter (depending on the value of the current)measures the current. There are two different types of the voltage applied across the diode to study its I-V characteristics.



(*a*) Electric Circuit



(*b*) Width of depletion region and reduction in barrier potential (dotted line) in forward bias mode

Figure 7.21 Forward Bias Connection of P-N Junction

Forward Bias : When the positive terminal of battery is connected to P side of junction and negative terminal connected to N side of junction such a connection is known as forward bias.

In such a connection the electric field due to external battery and electric field of depletion layer are in opposite direction. The emf (V) of the external battery and the potential difference (V_0) existing at the depletion region oppose each other. As a result the width and height of the depletion barrier reduces (See figure 7.21 (*b*)). Now, the work done by electron will be less to move from N-type to P-type and more and more electrons cross the junction easily. Similarly, hole can easily cross the junction from P-type to N-type. Thus, there is a flow of current in the junction due to both types of majority charge carriers. In forward bias the total current is sum of the hole diffusion current and electron current. This direction of the current in the junction is from P-type towards N-type. The magnitude of this current is in order of mA. If the battery voltage increase, the current in the junction also increase as shown in figure 7.22.

As shown in the figure 7.22, the initial increase in current is very less compared to the increase in the voltage. As the value of the voltage increases beyond a point, the current starts increasing rapidly (exponentially). This voltage is known as the threshold voltage or cut in voltage. The approximate value of threshold voltage for G*e* and S*i* is ≈ 0.3 V and ≈ 0.7 V respectively.

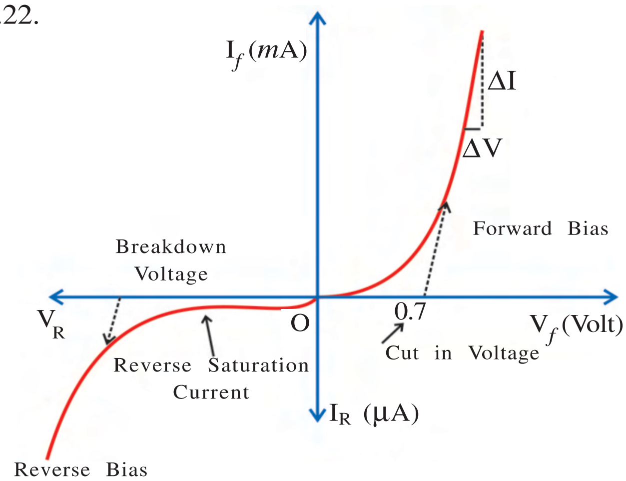


Figure 7.22 Characteristics of PN Junction

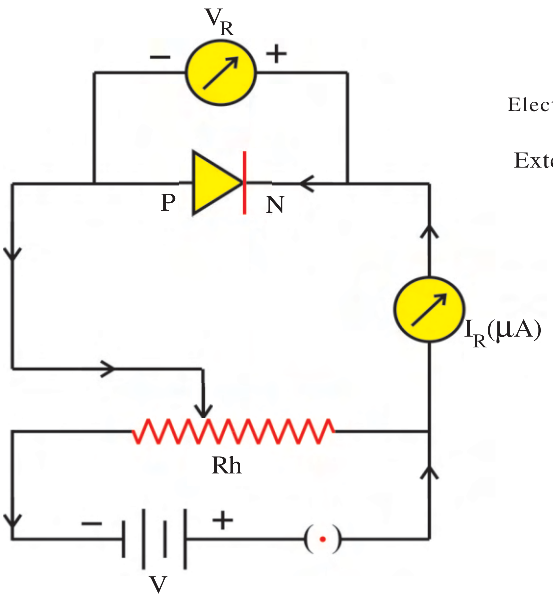
The current and the voltage do not have a linear relationship. Hence, the Ohm's law cannot be used to measure the resistance of the PN junction diode. But we can find its resistance by following method.

The dynamic resistance (r_f) of the diode can be found at any point on the characteristics curve by taking small changes in voltage (ΔV) and small changes in current (ΔI) and taking their ratio.

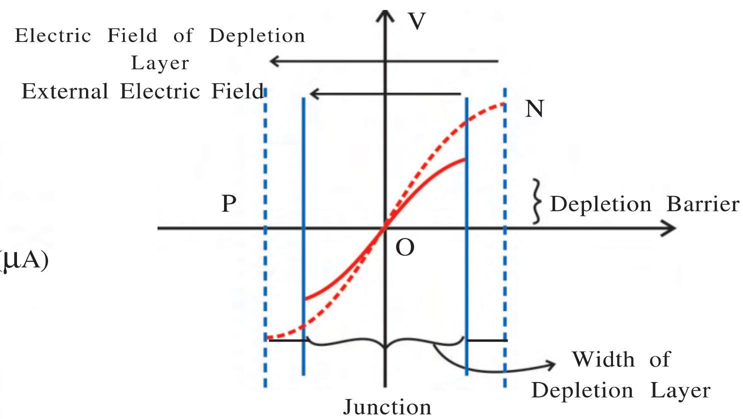
$$\therefore \quad r_f = \frac{\Delta V}{\Delta I}$$

The values of r_f will be different at different points on the curve. The resistance of the diode in forward bias mode is approximately between 10 Ω to 100 Ω.

Reverse bias : When the positive terminal of battery is connected to N side of junction and negative terminal connected to P side of junction such a connection is known as reverse bias.



(a) **Circuit Diagram**



(*b*) Width of the Depletion Layer and Increase in Depletion Layer (Doted Line)

Figure 7.23 Reverse Bias Connection of P-N-Junction

In such connection the electric field due to external battery and electic field of depletion layer are in same direction, means potential difference of external battery and potential of deplation layer are in helping position. As a result the width and height of the deplation barrier increases. (See figure 7.23 (*b*)). As a result electrons find it more difficult tomove from N to the P region of the dicide as well as holes to move from pregieon to N region.

But resultant electric field is in such a direction that minority charge carriers in P and N section cross the junction. In reverse bias, due to minority charge carrier, current produced in μA range. This current remains constant with battery voltage. So it is called reverse saturated current. There is sudden rise in current when the voltage is increased beyond point. This value at voltage is known as breakdown voltage (V_R). Voltage given to PN junction greater than breakdown voltage may damage it.

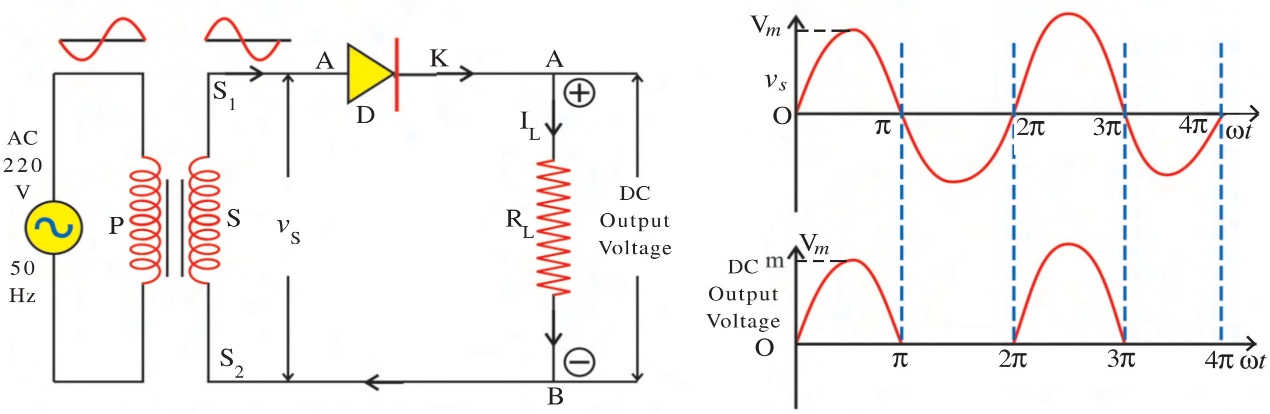
In reverse bias mode Dynamic resistance (r_{rb}) of PN junction is very of the order (≈ 10^6 Ω.). It is range of 10^6 Ω.

7.7 PN Junction Diode as a Rectifier

Most of the electronics devices and instruments require the DC energy for their operation. For example, radio, TV, cellphone etc. This DC energy we can be obtained from the different types of the batteries. But during their operation it gets discharged as well as being very costly. AC energy is easily available at our home as well as it is also very cheap. So that we required the circuit which can convert cheap AC energy into DC energy. The process of converting AC energy inot DC energy is called rectification. The circuit which performs this process is called the rectifier. We can use the PN junction diode for making rectifier circuit.

We have seen that the conventional current flows from P towards N when the P-N junction is forward biased. When the P-N junction is reverse biased the current flowing from N towards P is almost zero. This clearly tells us that when AC voltage is applied to the diode, current will flow in the circuit only during that half cycle for which the P-N diode is forward biased. During the next half cycle the diode becomes reverse biased since the P end is at a negative potential with respect to the N end. In this situation, if we place a resistor in the circuit we would obtain unidirectional current which will be varying with time and producing pulsating DC voltage. The circuit diagram which can realize the above process is shown in figure 7.24 and 7.25.

Half Wave Rectifier : The circuit diagram of half wave rectifier is shown in figure 7.24. The primary coil of transformer is connected to AC mains voltage (220 V, 50 Hz). The secondary coil of transformers is connected in series with the PN junction diode D and load resistance R_L. Figure shows how the AC voltage in secondary coil of transformer is changes when AC mains voltage is connected to primary coil.



(*a*) Electric Circuit

(*b*) Waveforms of HW Rectifier

Figure 7.24 Half Wave Rectifier

During the first positive half cycle of v_S $(0 \leq \omega t \leq \pi)$ the S_1 end of the secondary coil is positive with respect to S_2 as a result PN junction diode will be in forward bias. The conventional current flows through secondary coil of transformer, PN junction diode and load resistance R_L. In this situation, current flows from A towards B through the load resistor R_L. Here, A end becomes positive and B end becomes negative. The output voltage obtained during this half cycle is shown in figure 7.24 (*b*).

Now, during the second half cycle ($\pi \leq \omega t \leq 2\pi$) S_1 end of secondary coil becomes negative with respect to S_2. As a result PN junction will be in reverse bias and no current will flow in the circuit. The output voltage developed across R_L will be zero. [See figure 7.24 (*b*)].

Thus, the sequence of events taking place in the first two half-cycles is repeated. Now you can understand that at every half-alternate half cycle, the current flows through R_L only in one direction (i.e. from A towards B) which is a direct current (DC). As a result the voltage developed across R_L will be also DC voltage.

In this arrangement we get the output voltage during only one half-cycle, therefore, it is called half-wave ractified.

Full Wave Rectifier : In order to get the output voltage during both the half cycle, two PN junction diodes are used. The circuit diagram of full wave rectifier is as shown in figure 7.25.

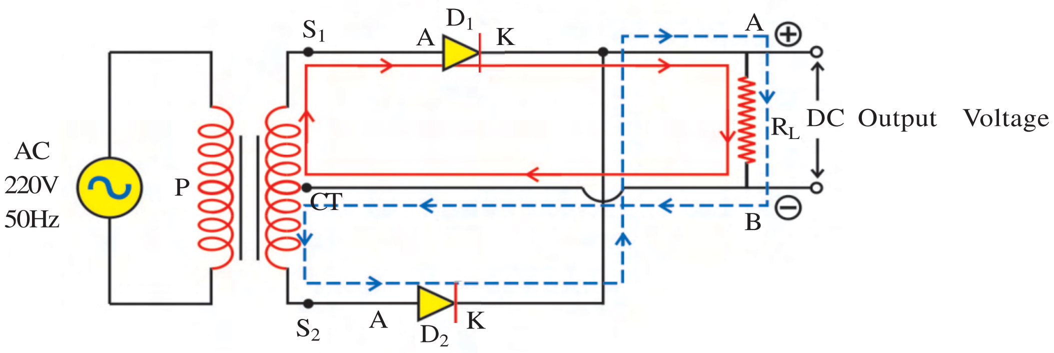


Figure 7.25 Full Wave Rectifier

As shown in the figure 7.25 the anode of diodes D_1 and D_2 are connected to S_1 and S_2 of secondary coil of centretaped transformer.

The load resistance R_L is connected between the two cathodes of diodes and centre tap of the transformer.

The applied voltage to both the diodes are same ($v_{s_1} = v_{s_2}$) but phase difference between them is 180°. Since the number of turns are equal on both the sides of centre tapped transformer. Let at any instant of the input voltage, S_1 end of the secondary coil becomes positive and S_2 end become negative with respect to centre tap ($0 \leq \omega t \leq \pi$). In this situation D_1 diode is forward biased and D_2 diode is reverse biased. Hence the conventional current flows in the $S_1 - D_1 - A - R_L - B - CT - S_1$ direction. The A end of the R_L becomes positive and B end becomes negative.

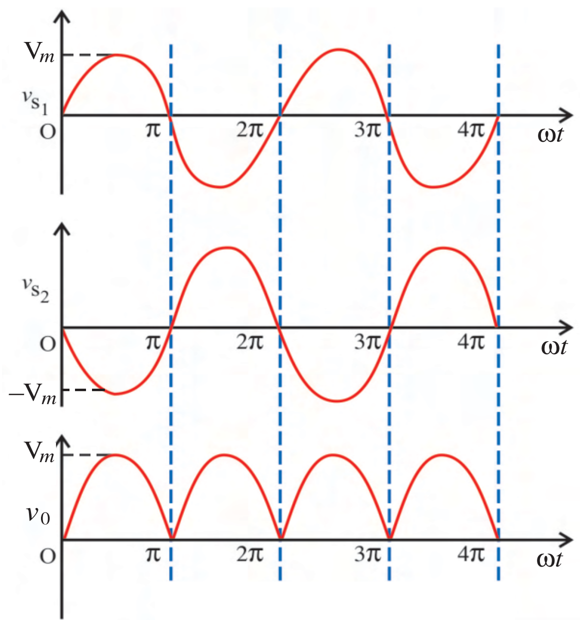


Figure 7.26 Wave forms of full Wave Rectifier

During the next half cycle S_2 end of secondary coil becomes positive and S_1 end becomes negative ($\pi \leq \omega t \leq 2\pi$) with respect to centre tap (CT). Now D_2 diode is forward biased and D_1 diode is reverse biased. The conversional current-during this half cycle flows through the $S_2 - D_2 - A - R_L - B - C_T - S_2$ direction. (The path of the current is shown with dotted line in the figure 7.26). Even during this half cycle current-flows from A towards B in the load

R_L.Thus, during both the cycles of the input signal DC current is flowing through load R_L and DC voltage is obtained across R_L. Hence, this circuit is called full wave rectifier.

The rectification efficiency of full wave rectifier is high compared to half wave rectifier. Therefore, it is widely used in the power supply.

Filter Circuit : The DC output voltage of the half wave rectifier and full wave rectifier does not-remain constant – with time. This DC voltage is known as pulsating DC voltage. This DC voltage contains DC component as well as AC components.

Different types of the filters are used to remove these AC components. Filter circuits consist of only capacitor, of only inductor or combination of capacitor and inductor.

A filter circuit containing only capacitor is shown in the figure 7.27. Here the value of capacitance of capacitor is large as a result its impedance $\left(\frac{1}{\omega C}\right)$ for AC mains frequency (50 Hz) is very low. In this condition, the AC component present on the output voltage of rectifier can be grounded through the capacitor. This leaves only the DC component-across the load R_L. (The filtering action of capacitor can be also explained by charging and discharging of the capacitor).

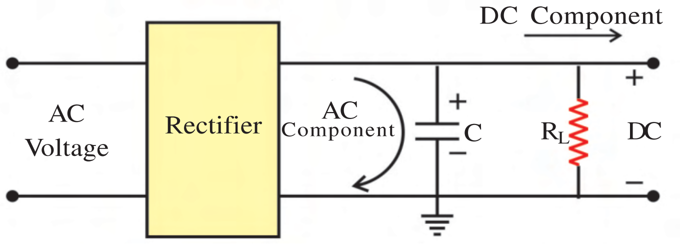
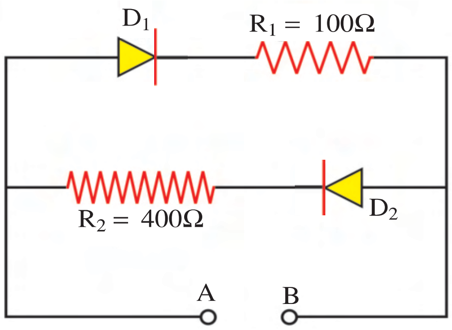


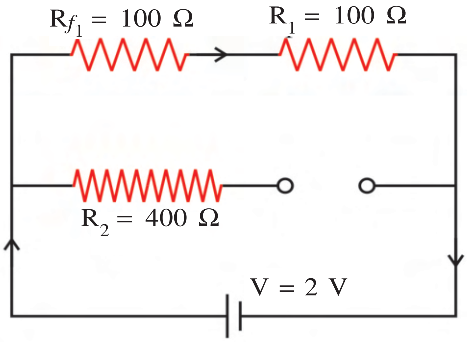
Figure 7.27 Filter Circuit

Illustration 1 : Calculate the current flowing through the diode D_1 and D_2 when the positive terminal the 2V battery is connected at point A and the negative terminal is connected at point B in figure (1). What will be the current flowing through the diodes if the battery terminals are interchanged? The resistance of the diodes D_1 and D_2 are 100 Ω in the forward and infinite in reverse bias mode.



(i)

Solution : (1) In the first part of the question, diode D_1 is forward biased and diode D_2 is reverse biased. The resistance of the diode D_1 is 100 Ω in the forward bias mode and the resistance of the diode D_2 in the reverse bias mode is infinite. The equivalent circuit is shown in figure (2) :



(ii)

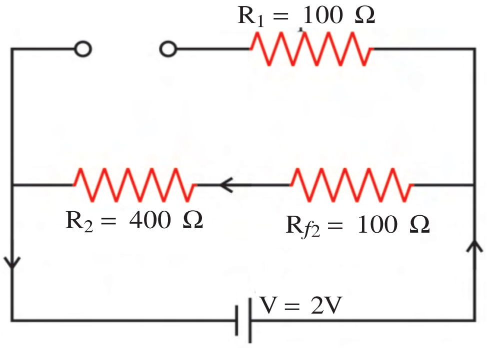
The current flowing through the diode D_1 will be equal to

$$I_1 = \frac{V}{R_{f_1} + R_1}$$

$$= \frac{2}{100+100} = 10 \text{ mA}$$

No current will flow through the diode D_2 since its resistance is infinite.

(2) When the terminal of the diodes are reversed, diode D_1 becomes reverse biased while diode D_2 becomes forward biased. The equivalent circuit in this situation will be as per figure (iii).



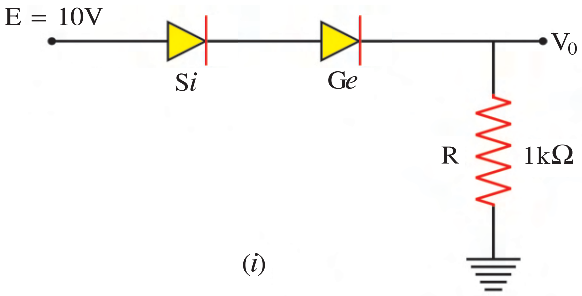
(iii)

The currnet through the diode D_2 will be,

$$I_2 = \frac{V}{R_{f2} + R_2}$$

$$= \frac{2}{100 + 400} = 4 \text{ mA}$$

Illustration 2 : As shown in figure diode made of *Si* and *Ge* are connected in series with resistor R. Calculate the current I_D flowing through the diode and also the output voltage V.



(*i*)

Solution : The potential drop across the diode is equal to the knee voltage when the diode is forward biased. This voltage for the *Si* diode is 0.7 V and for the *Ge* diode it is 0.3 V.

Now, E > (0.7 + 0.3 = 1V). Hence both the diodes will be forward biased. The equivalent circuit in this case can be as in figure (ii).

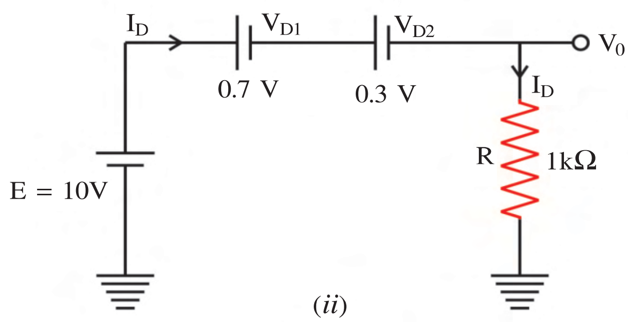
As per the Kirchhoff's law,

$$E - V_{D1} - V_{D2} - I_D R = 0$$

$$\therefore \ I_D = \frac{10 - 0.7 - 0.3}{10^3}$$

$$\therefore \ I_D = 9 \text{ mA}$$

$$\therefore \ \text{Output voltage } V_0 = I_D R$$



(*ii*)

$$= 9 \times 10^{-3} \times 10^{+3} = 9 \text{ V}$$

7.8 Special Types of PN Junction Diode

(a) Zener Diode : We have studied in the earlier section that very little current flows due to the minority charge carriers when the diode is reverse biased. This current is of the order of μA. As we go on increasing the reverse bias, at one particular voltage the current starts suddenly increasing. This voltage is called the breakdown voltage. The reverse current can be obtained in the order of milliampere near the breakdown voltage if the concentration of the impurity atoms is increased. Two types of effects are responsible for the current which is obtained in the breakdown region of the diode in the reverse mode. (1) Zener effect (2) Avalanche effect.

The width of the depletion region is very less when the impurity concentration is high. As a result at a very low reverse voltage we get a high intensity electric field at the depletion region. As for example if the reverse voltage is 2 V and the width of the depletion region is

200 Å, the electric field intensity will be $\frac{2}{200\times10^{-8}} = 10^6$ V/cm. This magnitude of electric field intensity is sufficient to break the covalent bonds and make the electrons free. A large number of covalent bonds are broken. This results in large number of electron and hole pair formation as well as the sudden increase in the reverse current (I_R). This explanation was given by a scientist known as C.E. Zener. Hence it is known as the Zener effect. Such diodes are known as Zener diodes.

The breakdown voltage can be obtained at a large value by decreasing the concentration of the impurity atoms. At the high value of the breakdown voltage the intensity of the electric field becomes high. When the charge carriers cross the depletion region (like the electron as for example) then it gets accelerated due to high electric field. This accelerated charge breaks many covalent bonds in the depletion region and creates electron hole pair. This newly created electron also gets accelerated and breaks many more covalent bonds to further create more electron hole pairs. This process keeps on repeating. This results in increase in the electric current flowing through the diode. We say that the diode has reached the breakdown point. This type of breakdown is called the Avalanche Effect. The diode in which the breakdown is due to the Avalanche effect is known as the Avalanche Diode.

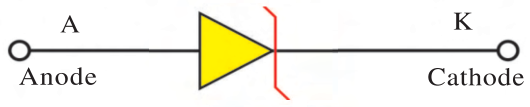


Figure 7.28 Symbol of Zener Diode

If the breakdown voltage is less than 4 V, then the breakdown is due to the Zener effect. If the breakdown is obtained at voltages greater than 6 V then the breakdown is due to the Avalanche effect. At voltages between 4 V and 6 V, the breakdown is due to both Zener and Avalanche effects. All these types of diodes are called Zener diodes.

The symbolic diagram of the zener diode is shown in figure 7.28. The symbolic representation of Zener diode is similar to that of normal diode. The only difference being that the cathode has been shaped in the form of the letter Z.

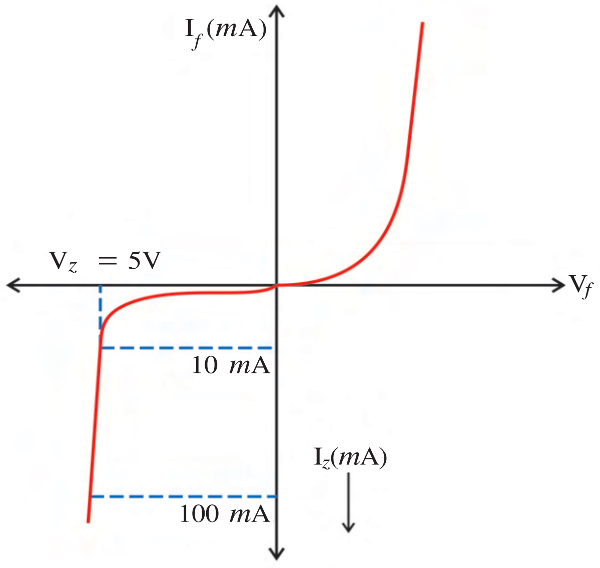


Figure 7.29 Characteristics of Zener Diode

Figure 7.29 shows the characteristic of the zener diode. The forward bias characteristic of the zener diode is very similar to that of the PN junction diode. In the voltage less than the breakdown voltage the magnitude of the current obtained is very low (of the order of μA). Near the breakdown voltage (V_Z) this current suddenly increases to the order of milliampere. This current is called the zener current (I_Z).

The breakdown obtained in this case is very sharp. It can be seen from the reverse bias characteristic that a small change in the voltage near the breakdown voltage produces a large change in the current. This means that in this situation the voltage across the zener diode remains constant over large changes in the current. Hence such a diode can be used in a voltage regulator citcuit.

Zener Diode as a Voltage Regulator : Rectifier circuits are used in power supply. When there is a change in the AC mains voltage, the secondary voltage v_s of the transformer also changes. As a result DC output voltage across R_L is also changes. This type of power supply is called unregulated power supply. If the output voltage remains constant with the change in the input voltage, such power supply is called regulated power supply.

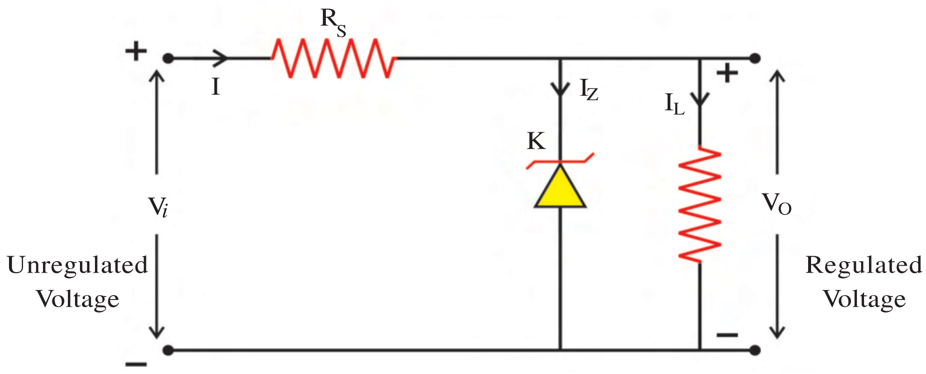


Figure 7.30 Regulator Circuit Using Zener Diode

Figure 7.30 shows the circuit-diagram of a regulated power supply using zener diode.

As shown in the circuit, zener diode is connected in a reverse bias mode. A resistance R_S connected in series with zener diode controlles the current. The output voltage is obtained across the load R_L, which is connected parallel to the zener diode. Applied input voltage (V_i) is always greater then regulated output voltage (V_O). Zener diode connected in the circuit must have zener break down voltage(V_Z) equal to required regulated output voltage.

When the input voltage(V_i) increases, the current (I) through R_S also increases so that voltage drop across R_S also increases. This change in the voltage is the same as change in the input voltage. This increases in the current increase in the zener current (I_Z), but the voltage (V_Z) across zener diode remains constant, hence the output voltage across R_L also remains constant. The decreases in the input voltage produces a reverse process. The voltage across R_S is reduced which will be equal to the decrease in the input voltage and voltage across zener diode remains constant. Thus, the output voltage across load R_L remains constant. In this way. using a zener diode, we can get regulated output voltage.

(b) LED (Light Emitting Diode) : Whenever electron in a Germanium of Silicon atom makes a transition from the conduction band to the valence band then the excess energy of the electron is obtained in the form of heat energy.

In some of the semiconductors like Gallium Arsenide the energy is obtained in the form of Light. The maximum wavelength of the electromagnetic waves have a wavelength

$\lambda = \dfrac{hc}{E_g}$. Here, (E_g) is the band gap energy.

In order to effectively obtain the intensity of the light, it is essential that the number of electron in the conduction band and the number of holes in the valence band have to be large. The above mentioned requirement is not satisfied adding impurity atoms in a pure semiconductor.

In order to achieve the above mentioned objective, two different types of doped semiconductors are taken and their junction is formed. As shown in the figure N and P type of semiconductors with large concentration of impurities is taken and a P-N junction is formed.

The PN junction diode is kept in fairly large forward bias condition. This results in a large current as shown in the figure 7.31, due to large concentration of electron in the N region and large concentration of holes in the P region of the diode. As explained earlier, the width of the depletion region is extremely small (of the order of few μm). As a result the electrons are easily able to cross the junction and recombine with the holes.

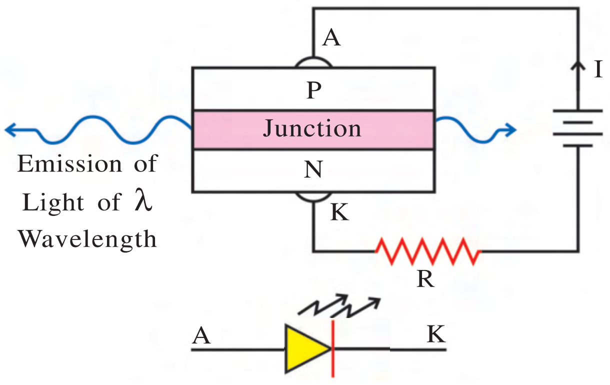


Figure 7.31 LED and Its Symbol

In order to obtain emission in the visible region, Arsenic and Phosphrous impurities are added in Gallium semiconductor.

LED that can emit red, yellow, orange, green and blue lights are commercially available. The semiconductor used for fabrication of visible LEDs must at least have a band gap of 1.8 *e*V. The compound semiconductor Gallium Arsenide – phosphide ($GaAs_{1-x}P_x$) is used for making LEDs of different colours.

These LEDs are widely used in remote control, ON/OFF indicator, optical communication, display board and decorative lighting.

(c) Photo Diode : The construction of photo diode is similar to the normal diode. The only difference between the two is that there is window in a photo diode through which the light enters and incident on the junction. This diode is always connected in a reverse bias mode. (see figure 7.32)

Reverse saturation current flows through the PN junction diode on connecting it in a reverse bias mode. The reverse saturation current can be increased either by increasing the temperature of the diode or by making more light incident over it. When the energy of the light incident on the junction $\frac{hc}{\lambda} > E_g$, large number of

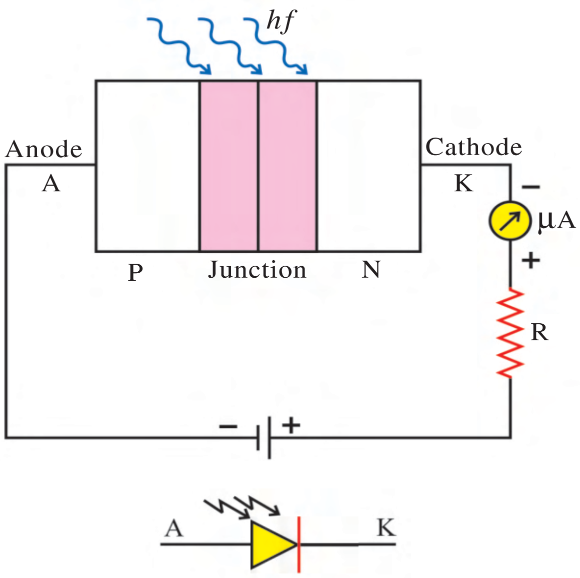


Figure 7.32 Photodiode and its Symbol

covalent bonds are broken near the junction. This further produces large number of electron hole pair. (or due to the incident light many electrons from the valence band move over to the conduction band). Thus the increase in the minority charge carriers contribute towards increase in the reverse current. This reverse current is of the order of μA.

The reverse current flowing through the diode in the absence of the incident light is known as dark current. The electrone hole pair increases on increasing the intensity of the incident light. This results in proportionate increase in current.

Photo diode converts the light into electrical signal. Hence it is widely used in optical communication. It is also used in CD player, computer as well as in security systems.

(d) Solar Cell : Solar cell is a semiconductor device. There is an emf induced at the two ends of a solar cell when the light is incident on the PN junction. This device converts light energy into electrical energy. The principle of the working of the photo diode and solar cell are same. The only difference is that there is no need of an external battery in case of a solar cell.

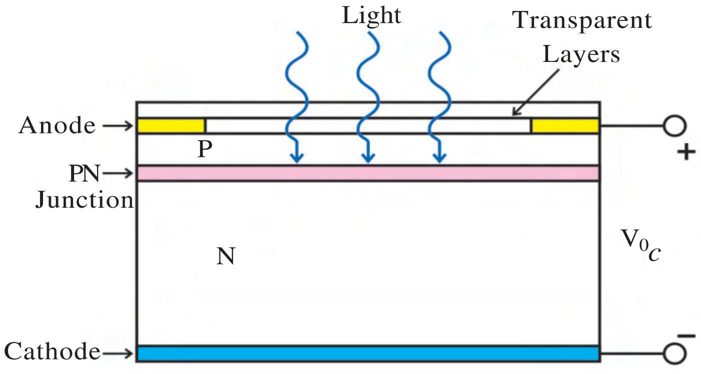


Figure 7.33 Construction of a Solar Cell

The figure 7.33 shows the construction of a solar cell. A thin layer of N-type semiconductor layer is combined with a thin layer of P-type semiconductor and a PN junction is constructed. The metal lead connected with the N-section of the PN junction is called the cathode and the metal lead taken from the P-section of the PN junction is the anode. Which is made in the shape of finger so that anode is not covered by it, as shown in the figure 7.34.

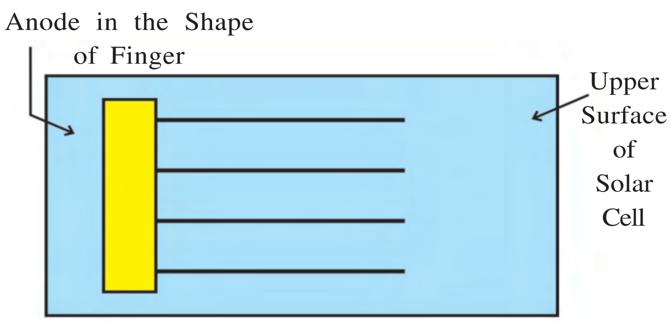


Figure 7.34

The thin layer of P-type semiconductor is called the emitter and the N-type semiconductor is known as the base. The incident light is directly incident on the PN junction since the P type material is made up of a very thin layer.

The region of the PN junction is kept large in order to obtain large amount of power. Electron-hole pair is produced when the incident photon energy $hf > E_g$. These electron and hole will move in mutually opposite direction due to the junction potential. The electron produced due to the incident photon will move towards the N-type material while the hole will move towards the P-type material if it is not connected to the external circuit with a resistor. This produces emf whose value is of the order of 0.5 V to 0.6 V.

As shown in figure 7.35 the current I_L known as photo current flows in the external circuit when it is connected with a resistor R_L. The value of this current and photo voltage both depend on the intensity of the light.

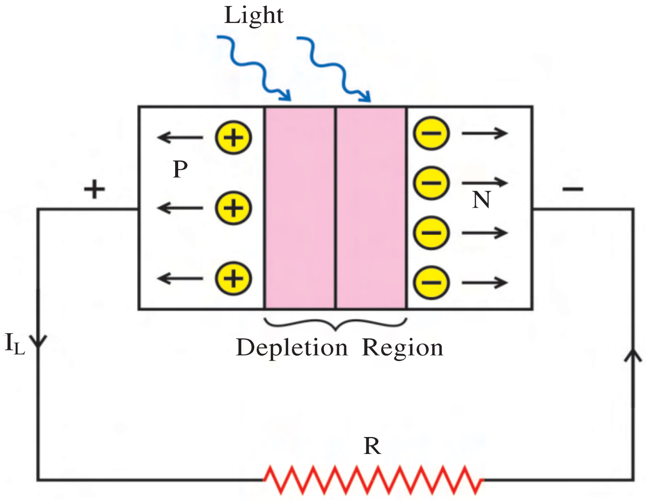


Figure 7.35

Si, GaAs, Cadmium Sulphide (CdS), Cadmium Selenide (CdSe) are some of the semiconductors used in the manufacturing of the solar cell.

The important criteria for the selection of a material for a solar cell fabrication are (i) Band gap energy (~1.0eV to 1.8eV), (ii) high optical absorption (iii) electrical conductivity and (iv) availability of the raw material.

Note that sunlight is not always required for a solar cell. Any light-with photon energy greater than the band gap will also produce photo voltage.

The solar cells can be connected in series or parallel connection to obtain the desired value of the voltage and current. Such an arrangement is called a solar panel. Such panels are used in converting light energy into electricity in the satellites. Such panels can be used as a storage battery, which can be charged during the day time and can be utilized during night time. Solar cells are used in calculators, electronic watch and camera.

Circuit Symbols of Different Types of P-N Junction Diodes

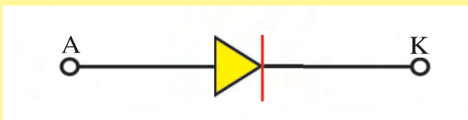
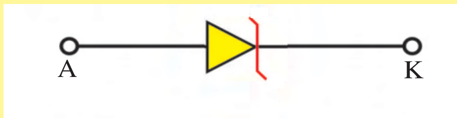
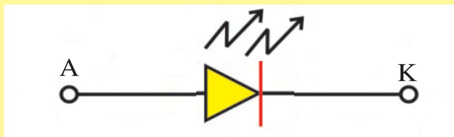
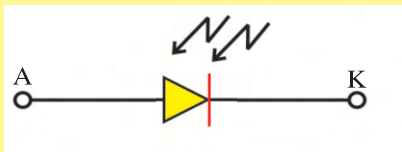
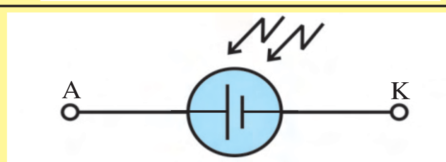
(1)	P-N junction diode	
(2)	Zener diode	
(3)	LED	
(4)	Photo diode	
(5)	Solar cell	

Illustration 3 : A photodiode is made from a semiconductor having 2.8eV band gap. Will it be able to detect a 6620 nm wavelength radiation ? (h = 6.62 × 10^{-34} Js)

Solution : $E_g = 2.8eV = 2.8 \times 1.6 \times 10^{-19} = 4.48 \times 10^{-19}$ J

The wavelength of the radiation $\lambda = 6620 \text{ nm} = 6.620 \times 10^{-6}$ m

The energy of the radiation, $E = hf = \frac{hc}{\lambda} = \frac{6.62 \times 10^{-34} \times 3 \times 10^{8}}{6.620 \times 10^{-6}} = 3 \times 10^{-20}$ J

Here, $E < E_g$ hence the diode will not detect 6620 nm wavelength radiation.

7.9 Transistor

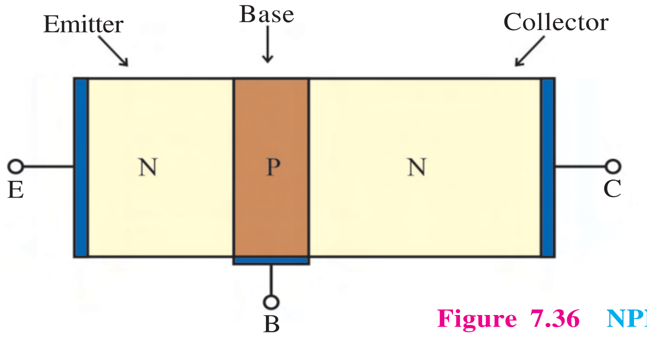
In 1948, transistor was invented in the Bell laboratory by three scientists namely John Bardeen, Walter Barten and William Schotky. They were awarded the Nobel prize for their invention. The size of the transistor is equal to the size of a groundnut but it is still able to perform various tasks performed by the vacuum tube. A revolution was brought about after this discovery in the electronic industry. The dimension of the electronic appliances have reduced due to the small size of the transistor. The weight has also reduced due to the lighter weight of the transistor.

Transistor is a device made up of two PN junction diodes. It is of two types.

(1) PNP Transistor : In this type of transistor, a thin N-type semiconductor wafer is sandwiched between two P-type semiconductors.

(2) NPN Transistor : In this type of transistor, a P-type of thin semiconductor wafer is sandwiched between two N-type semiconductors.

Figure 7.36 shows the construction and the symbol of the NPN transistor. Figure 7.37 shows the construction and the symbol of the PNP transistor.



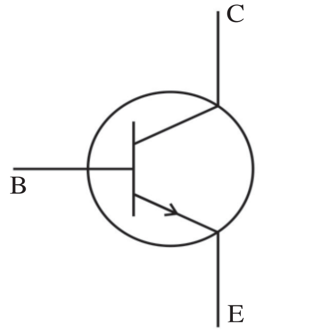
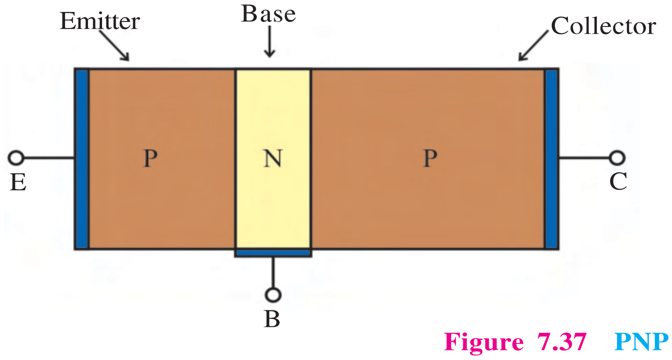


Figure 7.36 NPN Transistor

(Circuit Symbol)



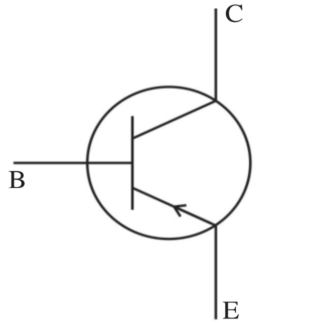


Figure 7.37 PNP Transistor

(Circuit Symbol)

The chip in the centre of the transistor is called the base. The region on one side of the base is the emitter and the other side is called the collector. The volume of the emitter and collector and their electrical properties are different.

The volume of the collector is more than that of the emitter. The concentration of impurities in the base is less, while its resistivity is higher. The impurity concentration in the emitter is high while its resistivity is very less. Impurity concentration in collector is relatively more than that of the base, but is less then that of the emitter.

The junction between the emitter and base is known as the emitter base junction. The junction between the base and the collector is known as the collector base junction. A transistor is operated with its emitter base junction is forward biased, while collector base junction is reverse biased. The direction of the conventional current when the NPN and PNP transistors are biased as mentioned above is along the direction of the arrow indicated in the emitter of the symbolic diagram 7.36 and 7.37.

The current in the transistor is due to both electrons and the holes. Hence, it is called Bipolar Junction transistor or BJT.

7.9.1 The Working of a Transistor : In practice NPN transistors are more widely used. In this section, we shall discuss the working of a NPN transistor.

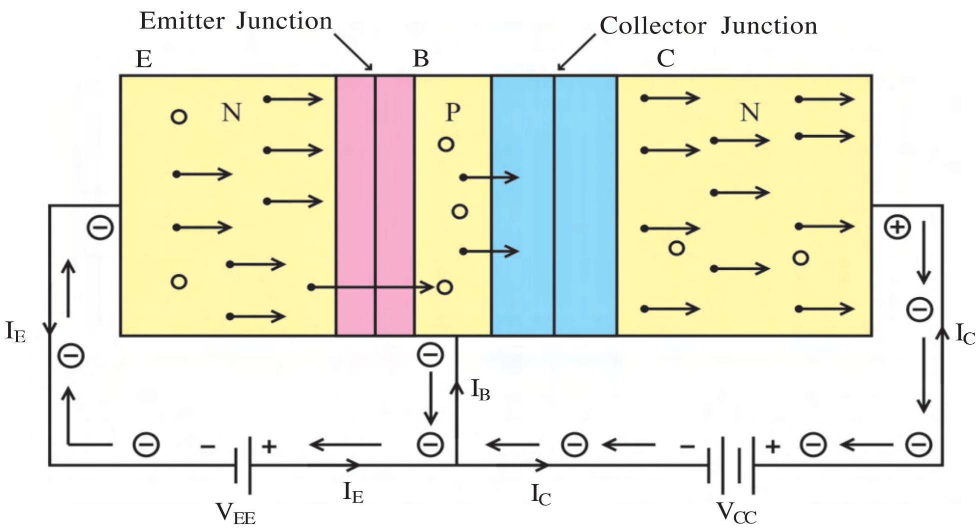


Figure 7.38 Working of NPN Transistor

Let us consider the circuit shown in figure 7.38 to understand the working of a NPN transistor. The emitter junction is forward biased with the help of a battery V_{EE} and the collector junction is reverse biased with the help of the battery V_{CC}. The value of the forward bias voltage is approximately between 0.5 V to 1 V and the reverse voltage V_{CC} is between 5 V to 10 V. The width of the emitter junction is reduced since it is forward biased while the collector junction's width is more since it is reverse biased. In a NPN transistor, electrons are the majority charge carriers in the emitter while holes are the majority charge carriers in the base.

The electrons from the emitter are easily able to go into the base, since the emitter junction is forward biased under the effect of the battery V_{EE}. The current constituted due to this is known as the emitter current (I_E). The base width is small and has fewer concentration of the impurity. As a result only 5% of the electrons entering the base recombine with the holes. The rest of the electron enter the collector region due to the influence of the battery V_{CC}. For every electron entering the collector one electron flows in the external circuit and constitutes the collector current I_C. Similarly for every electron recombining with the hole in the base section, there is one electron which gets attracted by V_{EE} and flows as base current I_B in the external circuit.

Applying the Kirchhoff's first law near the junction point.

$$I_E = I_B + I_C$$

The magnitude of I_E and I_C is normally in the mA range, while I_B is of the order of μA.

Similarly we can understand the working of a PNP transistor. The majority carriers leaving the emitter are holes in case of a PNP transistor. It enters the narrow region of the base and goes to the collector and constitutes the collector current I_C.

In any normal electronic circuit, there are two inputs and two output terminals. There is a total of 4 terminals in all. In a transistor there are only 3 terminals: Base (B), Emitter (E) and Collector (C). Hence, any one terminal is kept common to the both input and output. Hence, in this way three different types of circuits are possible in case of a transistor: (1) Common Base circuit (CB), (2) Common Emitter circuit (CE), (3) Common Collector circuit (CC).

In a CB circuit, I_C is the output current and I_E is the input current. The ratio of I_C and I_E is called the current gain α_{dc}.

$$\alpha_{dc} = \frac{I_C}{I_E}$$

Here, $I_E > I_C$ hence $\alpha_{dc} < 1$

For a CE circuit I_C is the output current and I_B is the input current. Hence for a CE circuit, the current gain $\beta_{dc} = \frac{I_C}{I_B}$.

Here, $I_C >> I_B$ hence $\beta_{dc} >> 1$

All the three circuits CB, CE and CC have different characteristics. Hence all the three circuits have different applications.

Most of the electronic circuits employ CE circuit. Hence we shall study the characteristics of CE circuit in detail.

7.9.2 Characteristics of a Transistor : We need to understand the relationship between the applied voltage and the current flowing through the transistor in order to understand the working of a transistor. The curve showing the relationship between the voltage and the corresponding current for a transistor is known as it's static characteristic curves. The curve showing the relationship between the input voltage and the input current for any given value of the output voltage is known as the input characteristics curve. The curve indicating the relationship between the output voltage and the output current for any given value of the input current is known as the output characteristics.

The figure 7.39 shows the circuit to study the static characteristics of a CE transistor circuit.

The emitter junction is forward biased due to the battery V_{BB} and the collector junction is reverse biased with the help of battery V_{CC}. Rheostat R_1 is used to vary the base voltage V_{BE} and rheostat R_2 is used to vary the collector voltage V_{CE}.

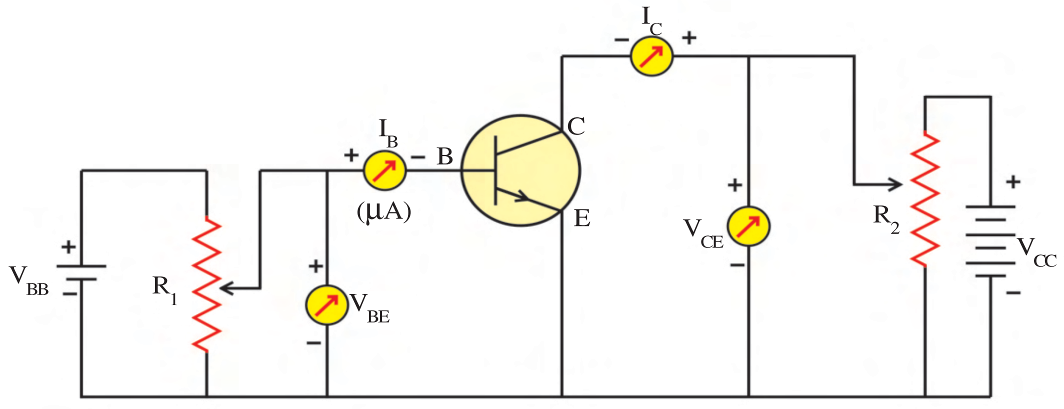


Figure 7.39 Circuit Diagram to Obtain Characteristics of a Transistor

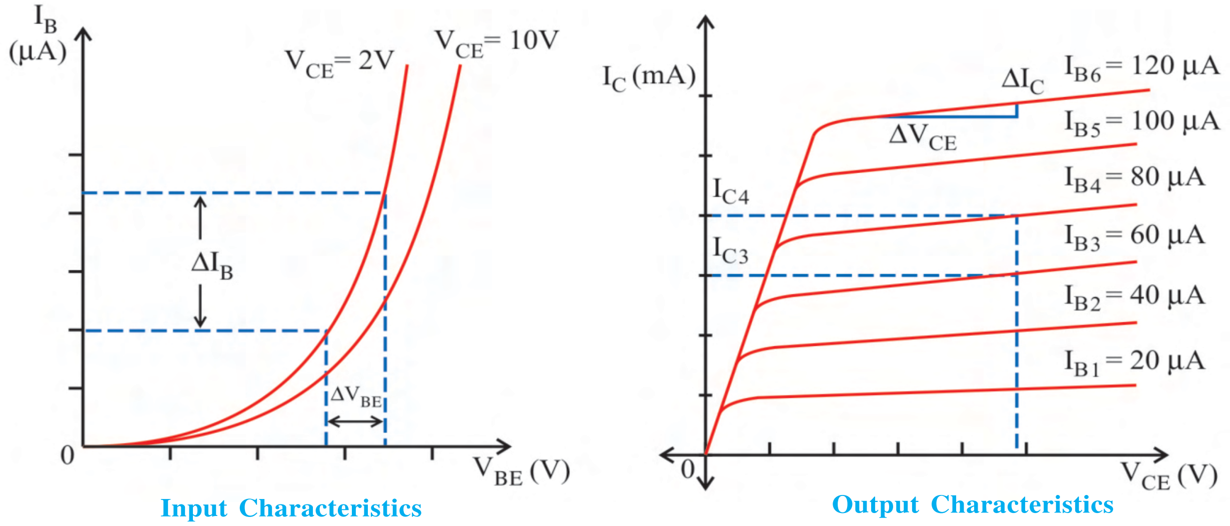


Figure 7.40 Characteristics of a Transistor

For studying the characteristics the collector voltage V_{CE} is set to any one value (for example, $V_{CE} = 2$ V) and with the help of the rheostat R_1, base current I_B is noted for different values of the voltage V_{BE}. Next keep the voltage V_{CE} to some high value (as for example, $V_{CE} = 10$ V) and obtain the relationship between I_B and V_{BE} and plot the graph $I_B - V_{BE}$. The above procedure gives us the input characteristics. Such an input characteristics curve is shown in figure 7.40. Such a characteristic curve is similar to the one for a PN junction diode.

In order to study the output characteristics the base current I_B is kept constant (as for example $I_B = 20$ μA), the collector voltage V_{CE} is varied and the corresponding changes in the collector current I_C is noted down. Repeat the above experiment for three to four different values of the base current I_B (as for example 40 μA, 60 μA and 80 μA). Plot the graph between $I_C - V_{CE}$

for different values of the base current. This graph gives the output characteristic curve (refer to figure 7.40). The central position of the curve is known as the active region. In this region the collector current is not dependent on the value of V_{CE} and it is almost constant. The transistor is operated in this region if it has to be used as an amplifier.

The transistor parameters can be found from the characteristic curves as follows :

(1) Input Resistance : The ratio of the change in the input base voltage (ΔV_{BE}) to the change in the input base current (ΔI_B) at a constant collector voltage (V_{CE}) is known as the input resistance (r_i).

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE}} = \text{constant}$$

This parameter can be found from the input characteristic curve. Normally r_i is of the order of k Ω.

(2) Output Resistance : The ratio of the change in the collector voltage (ΔV_{CE})to the change in the collector current for a constant base current (I_B) is known as the output resistance r_0.

$$r_0 = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_B} = \text{constant}$$

This parameter can be found from the output characteristic curve. Normally it's value is found between 50k Ω to 100k Ω.

(3) Current Gain : The ratio of the change in the collector current, (ΔI_C) to the corresponding change in the base current (ΔI_B) at constant value of the collector voltage (V_{CE}) is known as the current gain β.

$$\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}} = \text{constant}$$

This parameter can be found from the active region of the output characteristic curve. Normally the value of β is between 10 and 1000.

Taking the ratio of β and r_i for a CE circuit,

$$\frac{\beta}{r_i} = \frac{\Delta I_C / \Delta I_B}{\Delta V_{BE} / \Delta I_B} = \frac{\Delta I_C}{\Delta V_{BE}}$$

This ratio of the change in the current in the output circuit (ΔI_C) to the change in the input voltage (ΔV_{BE}) is known as the transconductance (g_m).

$$\therefore \; g_m = \frac{\Delta I_C}{\Delta V_{BE}} \quad \text{or} \quad g_m = \frac{\beta}{r_i}$$

Its unit is *mho*.

(*c*) Transistor as an Amplifier : Amplifier circuits are one of the most important circuits in electronics. Most of the electronic appliances use amplifier circuits. Amplifier circuits are used in amplifying small signals. As for example the output voltage of a microphone, the signal received from a TV antenna or a radio antenna etc. are of the order of microvolt. Such type of signals can be amplified by means of an amplifier.

Let us understand the working of the widely used CE transistor amplifier. The circuit diagram of a CE transistor amplifier using a NPN transistor is shown in the figure 7.41. The emitter junction is forward biased with the help of the battery V_{BB} and the collector junction is reverse biased with the help of battery V_{CC}. The A.C. signal which has to be amplified is connected between the base and emitter of the transistor. The amplified signal is obtained between the collector and emitter terminals or in other words across the two ends of the resistor R_L.

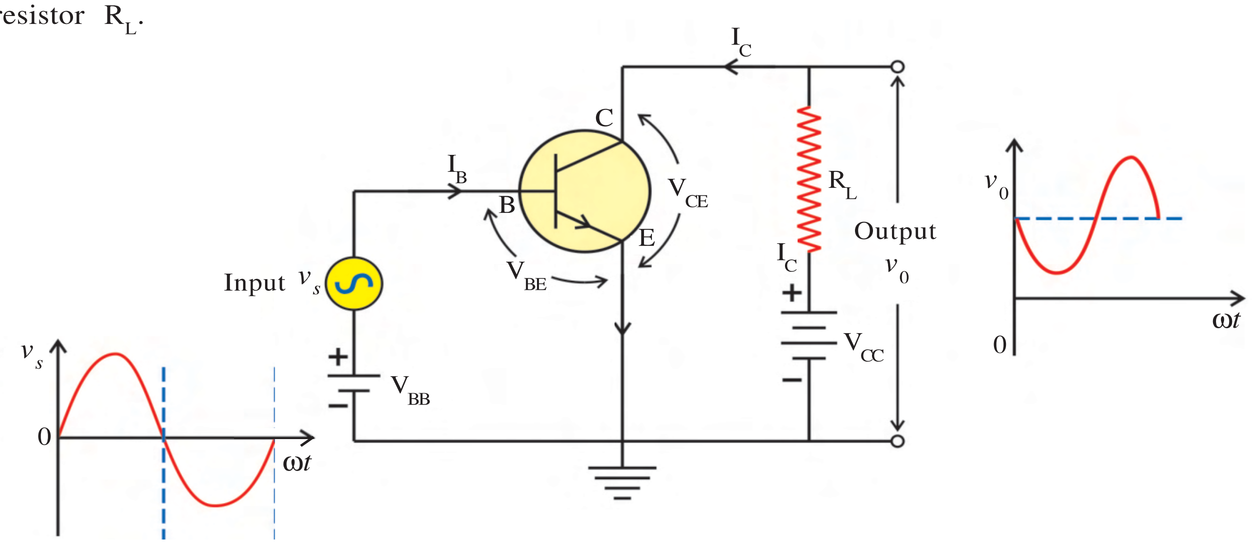


Figure 7.41 CE Transistor Amplifier

The A.C. signal (V_S) causes the change in the base emitter voltage V_{BE}. This results in the change in the base current I_B. The changes in the base current (ΔI_B) is of the order of microampere. This results in the change in the collector current equal to $\beta \Delta I_B$, which is of the order of milliampere. A large voltage is obtained by connecting a large value of (R_L) in the output circuit and taking the voltage drop across it. This is the output voltage of the circuit. This is how a transistor works as an amplifier. The ratio of the output voltage (v_0) and the input voltage (v_S) is known as the voltage gain (A_V).

The Working of the Circuit :

(1) Input Circuit : v_S is the input voltage of the amplifier which has to be amplified. In the absence of the signal v_S (i.e. $v_S = 0$) as per the Kirchhoff's second law,

$$V_{BB} = V_{BE} \qquad (7.9.1)$$

In the presence of the signal v_S, the change in the base emitter voltage is $\Delta V_{BE.}$

$\therefore\ V_{BB} + v_S = V_{BE} + \Delta V_{BE}$ (7.9.2)

Substituting the equation (7.9.2) in the equation (7.9.1), we get

$v_S = \Delta V_{BE}$ (7.9.3)

The change in the base current ΔI_B is due the voltage change ΔV_{BE}. As per the definition of the input resistance r_i we have,

$$r_i = \frac{\Delta V_{BE}}{\Delta I_B}$$

or

$\Delta V_{BE} = v_S = r_i \Delta I_B$ (7.9.4)

(2) Output circuit : The collector current changes by an amount ΔI_C due to the change in the base current ΔI_B. As a result the voltage change by an amount $R_L \Delta I_C$ across the resistor R_L.

As per the Kirchhoff's second law,

$V_{CC} = I_C R_L + V_{CE}$ (7.9.5)

$\therefore\ \Delta V_{CC} = R_L \Delta I_C + \Delta V_{CE}$

But the battery voltage V_{CC} remains constant. $\therefore\ \Delta V_{CC} = 0$

$\therefore\ 0 = R_L\ \Delta I_C + \Delta V_{CE}$

$\therefore\ \Delta V_{CE} = -R_L \Delta I_C$

$\therefore\ v_0 = -R_L \Delta I_C$ (7.9.6)

Here, ΔV_{CE} is obtained across the two ends of the load resistor and is known as the output voltage v_O.

Voltage gain (A_V) : As per the definition of the voltage gain,

$$\text{Voltage gain, } A_V = \frac{\text{Output Voltage}}{\text{Input Voltage}} = \frac{v_O}{v_S}$$

Substituting equation (7.9.6) and equation (7.9.4), we have,

$$A_V = -\frac{R_L \Delta I_C}{r_i \Delta I_B} \quad (7.9.7)$$

$$A_V = -\beta \frac{R_L}{r_i} \quad (7.9.8)$$

Where, $\beta = A_i = \dfrac{\Delta I_C}{\Delta I_B}$ and is known as the current gain of the transistor. $\dfrac{\beta}{r_i}$ is known as the transcondutance (g_m) of the transistor.

$\therefore\ A_V = -g_m R_L$ (7.9.9)

Here, the negative sign indicates that the phase difference between the input (v_S) and the output voltage (v_O) is 180°. Whenever the input voltage increases the output voltage decreases and vice versa.

Power Gain (A_P) : As per the definition of the gain A_P,

$$A_P = \frac{\text{Output AC Power}}{\text{Input AC Power}}$$

$$A_P = \frac{\Delta V_{CE}\,\Delta I_C}{\Delta V_{BE}\,\Delta I_B}$$

$$A_P = A_V A_i$$

$$= \left(-\beta\frac{R_L}{r_i}\right)(\beta)$$

$$\therefore\ |A_P| = \beta^2\frac{R_L}{r_i} \qquad (7.9.10)$$

The question which can arise in our mind is that from where did the extra power come from ? Is conservation law of the energy violated ? The answer to the above question is that the DC energy from the battery gets converted into the AC energy.

Illustration 4 : The current gain of a common base (CB) circuit is α and the current gain of a common emitter (CE) circuit is β. Find the relationship between α and β.

Solution : For a common base circuit, $\alpha = \frac{\Delta I_C}{\Delta I_E}$

For a CE circuit, $\beta = \frac{\Delta I_C}{\Delta I_B}$

Now for any configuration of a transistor circuit,

$\Delta I_E = \Delta I_C + \Delta I_B \quad (\because\ I_E = I_B + I_C)$

$$\therefore\ \frac{\Delta I_E}{\Delta I_C} = 1 + \frac{\Delta I_B}{\Delta I_C} \qquad \text{(Dividing both sides by } \Delta I_C\text{)}$$

$$\therefore\ \frac{1}{\alpha} = 1 + \frac{1}{\beta} \qquad \text{(As per the definition of } \alpha \text{ and } \beta\text{)}$$

$$\therefore\ \frac{1}{\beta} = \frac{1}{\alpha} - 1 = \frac{1-\alpha}{\alpha}$$

$$\therefore\ \beta = \frac{\alpha}{1-\alpha}$$

Similarly you can try to derive the relationship, $\alpha = \frac{\beta}{1+\beta}$

Illustration 5 : In a NPN transistor about 10^{10} electrons enter the emitter in 1 μs when it is connected to a battery. About 2 % electrons recombine with the holes in the base. Calculate the values of I_E, I_B, I_C, α_{dc} and β_{dc} (e = 1.6 × 10^{-19} C).

Solution : As per the definition of the current,

$$\text{Emitter current } I_E = \frac{Q}{t} = \frac{ne}{t} = \frac{10^{10} \times 1.6 \times 10^{-19}}{10^{-6}} = 1600 \ \mu A$$

2 % of the total electrons entering the base from the emitter recombine with the holes which constitutes the base current I_B. The rest of the 98 % electrons reach the collector and constitute the collector current.

$\therefore \ I_B = 0.02 \ I_E = 0.02 \times 1600 = 32 \ \mu A$

$\therefore \ I_C = 0.98 \ I_E = 0.98 \times 1600 = 1568 \ \mu A$

$$\text{Now, } \alpha_{dc} = \frac{I_C}{I_E} = \frac{1568 \times 10^{-6}}{1600 \times 10^{-6}} = 0.98 \qquad \left(\text{or } \alpha_{dc} = \frac{I_C}{I_E} = \frac{98\% I_E}{I_E} = 0.98\right)$$

$$\beta_{dc} = \frac{I_C}{I_B} = \frac{1568 \times 10^{-6}}{32 \times 10^{-6}} = 49 \qquad \left(\text{or using the equation } \beta_{dc} = \frac{\alpha_{dc}}{1 - \alpha_{dc}}\right)$$

Illustration 6 : A change of 0.02 V takes place between the base and emitter when an input signal is connected to the CE transistor amplifier. As a result, 20 μA change takes place in the base current and a change of 2 mA takes place in the collector current. Calculate the following quantities : (1) Input resistance (2) A.C. current gain (3) Transconductance (4) If the load resistance is 5 kΩ, what will be the voltage gain and power gain.

Solution : Here, $\Delta I_B = 20 \ \mu A = 20 \times 10^{-6}$ A, $\Delta V_{BE} = 0.02$ V $= 2 \times 10^{-2}$ V

$\Delta I_C = 2$ mA $= 2 \times 10^{-3}$ A, $R_L = 5$ kΩ $= 5000$ Ω

$$(1) \text{ Input resistance, } r_i = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{2 \times 10^{-2}}{20 \times 10^{-6}} = 1 \ k\Omega$$

$$(2) \text{ AC current gain } A_i = \beta = \frac{\Delta I_C}{\Delta I_B} = \frac{2 \times 10^{-3}}{20 \times 10^{-6}} = 100$$

$$(3) \text{ Transconductance } g_m = \frac{\beta}{r_i} = \frac{100}{1000} = 0.1 \text{ mho}$$

$$(4) \text{ Voltage gain, } |A_V| = g_m R_L$$

$$= (0.1)\ (5000)$$

$$= 500$$

(5) Power gain, $A_P = A_V A_i$

$= (500)\ (100)$

$= 5 \times 10^4$

Illustration 7 : The collector supply voltage in a CE transistor amplifier is 10 V. The base current is 10 μA in the absence of the signal voltage and the voltage between the collector and the emitter is 4 V. The current gain (β) of the transistor is 300. Calculate the value of the load resistance R_L.

Solution : Here, $V_{CC} = 10$ V, $I_B = 10\ \mu A = 10 \times 10^{-6}$ A, $V_{CE} = 4V$, $\beta = 300$, $R_L = ?$

Now, $I_C = \beta I_B = (300)(10 \times 10^{-6}) = 3$ mA

Applying Kirchhoff's second law to the output circuit of amplifier we have,

$V_{CC} = V_{CE} + I_C R_L$

$$\therefore\ R_L = \frac{V_{CC} - V_{CE}}{I_C} = \frac{10-4}{3\times10^{-3}} = 2000 = 2\ k\Omega$$

7.9.4 Transistor as a Switch : In an ideal ON/OFF switch, when it is OFF the current is not flowing in the circuit because switch offers infinite resistance. When switch is in ON condition, maximum current flows because its resistance is zero. We can prepare such an electronic switch by using the transistor.

Figure 7.42 shows the circuit diagram for the transistor as a switch. Apply the Kirchoff's law to the input section of the transistor.

$$V_i = I_B R_B + V_{BE} \qquad (7.9.11)$$

Apply the Kirchhoff's law to the output section of the transistor.

$$V_{CC} = I_C R_L + V_O$$

$$\therefore\ V_O = V_{CC} - I_C R_L \qquad (7.9.12)$$

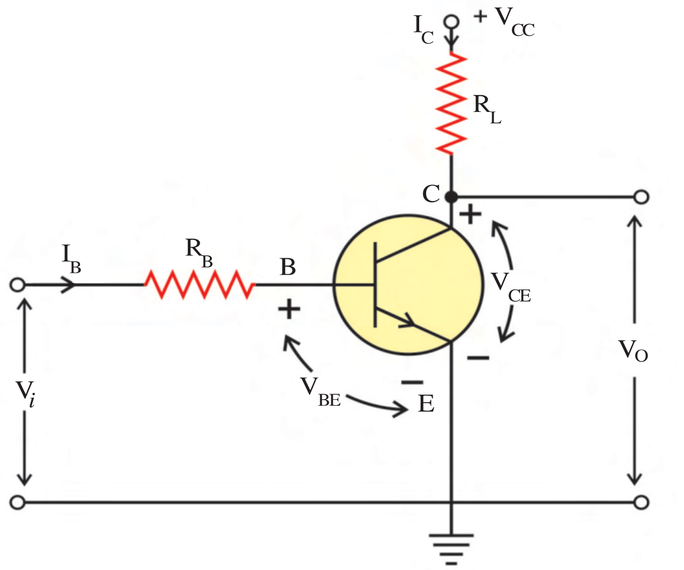


Figure 7.42 Transistor as a Switch

(i) When input voltage $V_i = 0$ or it is less than transistor's cut in voltage, the base current I_B will be zero ($I_B = 0$). Hence the collector current will also be zero ($I_C = 0$).

From equation 7.9.12.

$V_O \approx V_{CC}$

In this situation resistance of output circuit of the transistor is very large. Hence the current is not flowing through it. This shows the 'OFF' or 'cut off' condition of the transistor.

(ii) When the input voltage will be $V_i \approx V_{CC}$, the base current will be maximum, hence the collector current will also be maximum. The voltage drop ($I_C R_L$) across the load resistance R_L will be approximately V_{ec}. According to equation (7.9.12),

$$V_0 \approx 0$$

In this condition resistance of the output circuit of the transistor is very small to the effect that maximum current is flowing through it. This is called the 'ON' condition or saturation condition of the transistor.

This circuit is used to make a 'NOT' gate in the digital electronics.

7.9.5 Transistor as an Oscillator : We have studied the electrical oscillation in LC circuit in the chapter of the AC circuits. These oscillations get damp with time. If such an oscillation has to be sustained, then necessary energy has to be supplied to the circuit. This can be achieved with the help of oscillator circuit.

We know that in amplifier circuit input signal has to be applied to obtain the output signal while in oscillator circuit without input signal, we can obtain the output signal. The oscillator circuit can generate a signal of desired frequency with desired amplitude.

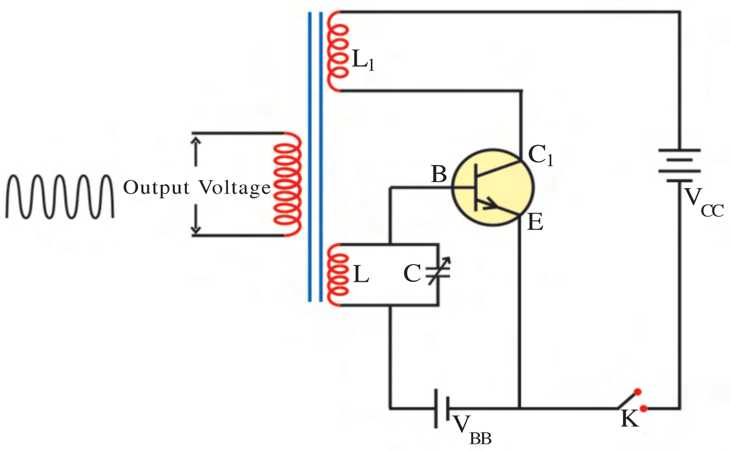


Figure 7.43 Transistor Oscillator

As shown in the figure 7.43, V_{BB} battery provides the forward bias to BE junction and V_{CC} battery provides the reverse bias to BC junctions of a transistor. LC network is introduced between input and output sections of the circuit. Inductors L_1 and L are associated with each other by means of a mutual inductance.

When the key is closed, the current starts flowing in the collector circuit through inductor L_1. This collector current increases with time. Hence, the magnetic flux linked with coil L_1 and as a result the magnetic flux linked with coil L starts increasing and positively charged the upper plate of the capacitor C. This increases the forward bias voltage of the transistor. As a result collector current is also increases. This process continues till the collector current reaches saturation.

When the collector current reaches saturation, the flux linked with the coil L_1 stops changing. As a result induced e.m.f. across the inductor L becomes zero. Now capacitor gets discharged through inductor L, which reduces the forward bias voltage of a transistor. As a result the magnitude of a collector current-decreses. Since the flux through the coil L decreases the other plate of the capacitor becomes more positive. This process continues till the collector current does not become zero. In this situation the capacitor is completely discharged and there is no opposition to the forward bias. The collector current again starts to increase. The above mentioned process keeps repeating. Here, the collector current oscillates between the maximum and zero value.

The frequency of oscillation is,

$$f = \frac{1}{2\pi\sqrt{LC}}$$

Here, LC network gets the necessary energy from the battery V_{CC} for the sustained oscillation. Thus, oscillator converts the DC energy into the AC energy.

Oscillators are used in the communication, radio, TV and transmitter to generate the high frequency. Such an oscillator circuit can generate the signal of low frequency upto 10^9 Hz.

Illustration 8 : In Transistor oscillator circuit an output signal of 1 MHz frequency is obtained. The value of capacitance C = 100 *p*F. What should be the value of the capacitor if a signal of 2 MHz frequency is to be obtained?

Solution : $C_1 = 100\ pF = 100 \times 10^{-12}$ F, $f_1 = 1$ MHz $= 10^6$ Hz,

$f_2 = 2$ MHz $= 2 \times 10^6$ Hz, $C_2 = ?$

$$f_1 = \frac{1}{2\pi\sqrt{LC_1}} \text{ and } f_2 = \frac{1}{2\pi\sqrt{LC_2}}$$

$$\therefore \frac{f_1}{f_2} = \frac{2\pi\sqrt{LC_2}}{2\pi\sqrt{LC_1}} = \sqrt{\frac{C_2}{C_1}}$$

$$\therefore C_2 = \left(\frac{f_1}{f_2}\right)^2 \times C_1 = \left(\frac{1}{2}\right)^2 \times 100 \times 10^{-12}$$

$C_2 = 25\ pF$

7.10 Digital Electronics and Logic Circuits

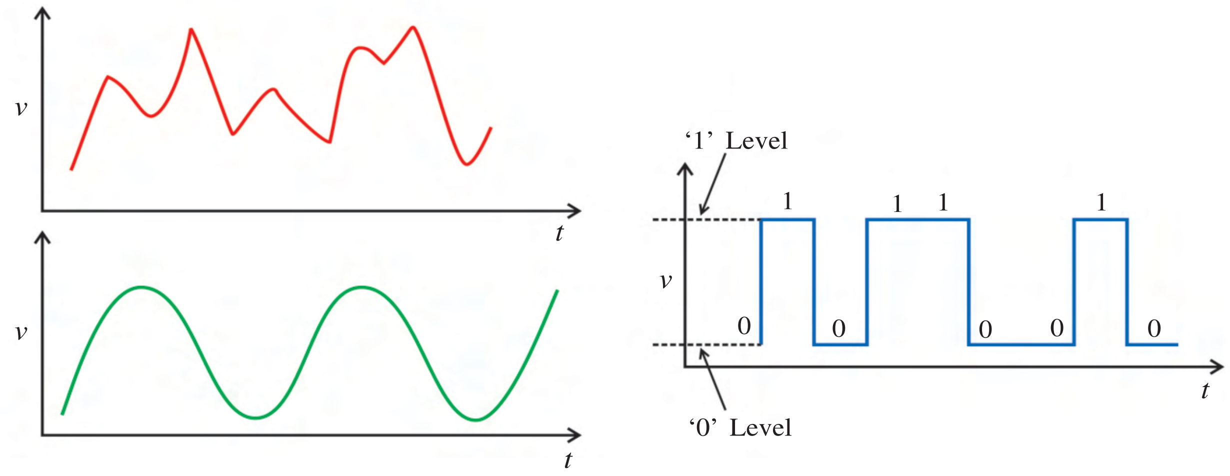
There are problems which have answer in either yes or no. As for example the answer to the question, 'whether it will rain tomorrow or not ?' 'Whether Indian cricket team will win the world cup ?' is either yes or no.

George Boole, mathematician developed, Boolean algebra based on the science of logic. In 1938 a scientist Shannon developed electrical circuit based on the Boolean algebra which are known as logic circuits. As a result branch of digital electronics is developed. Switching action takes place in such a logic circuit. If there is a presence of output voltage, then it is said to be in the ON position or state '1'. If the voltage at the output is zero then it is said to be in the OFF state or '0' state. In such a circuit the output voltage has only two states hence such a circuit used the binary number system.

In the present age, digital electronics is widely used in computers, microprocessors communications. T.V., CD player as well as in many medical appliances.

In amplifier or oscillator circuits the current or the voltage are constantly changing with time. These signals can take any value between the minimum and the maximum value of voltage or current. Such a signal is called Analog signal. Figure 7.44 (*a*) shows two different types of Analog signals.

Refer to the signal shown in figure 7.44 (*b*). Here the voltage or the current has only two values. [The maximum value of the voltage is indicated by '1' and the minimum value of the voltage is indicated by '0'. Such a signal is known as digital signal.]



(*a*) **Analog Signal** (*b*) **Digital Signal**

Figure 7.44 Analog and Digital Signal

There are two types of systems used in logic circuit.

(1) Positive Logic System : In this type of system the higher positive voltage is taken as high level of '1' and the less positive voltage is taken as low level or '0'.

(2) Negative Logic System : In this type of system, the more negative voltage is taken as '1' and the less negative voltage is taken as '0'.

We shall be employing positive logic system in our subsequent discussions. We shall consider +5V as '1' state and 0 V as '0'state.

Let us get familiarized with some of the terms used in digital electronics.

Logic Gate : The logic circuit in which there is one or more than one input but only one output is called a logic gate. It's output has only two states, '0' or '1', which depends on condition of input signal.

Logic gates are the most important components of the digital electronics circuit. OR gate, AND gate and NOT gates are the basic logic gates. The other gates like the NAND and NOR gates can be obtained from these basic gates.

Boolean Equation : The Boolean equation represents the special type of algebraic representation, which describes the working of the logic gates.

Truth Table : The table which indicates the output for different combinations of the input voltage is known as the truth table.

7.10.1 OR gate : The figure 7.45 shows the circuit containing the bulb and the two switches A and B to illustrate the working of an OR gate.

Table 1

A	B	Bulb
Open	Open	OFF
Open	Close	ON
Close	Open	ON
Close	Close	ON

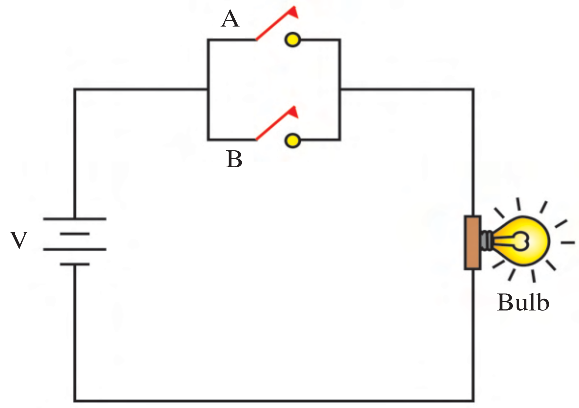


Figure 7.45

When any one of the switches or both the switches will be ON, then bulb will be ON. When both switches will be OFF then only bulb will remain OFF.

The status of the bulb with respect to the switch positions is shown in table 1. In this table if the switch A is taken as input A the switch B is taken as input B and the status of the bulb is taken as output Y, we get the truth table of an OR gate.

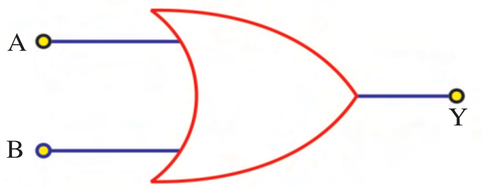


Figure 7.46 Symbol of OR Gate

Table 2

A	B	Y = A + B
0	0	0
0	1	1
1	0	1
1	1	1

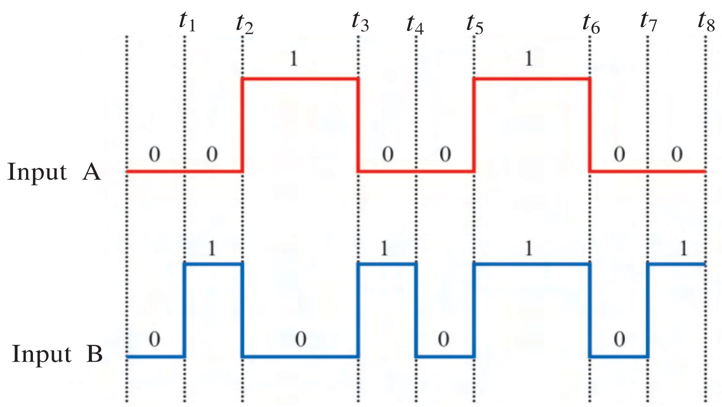
In this table the ON state is taken as '1' and off state is indicated as '0'. We can describe the characteristics of the OR gate based on the above truth table.

Whenever any one input or both the inputs are '1', then we get output '1'.

Boolean equation : Y = A + B : Here, the '+' sign does not indicate the sign of addition but it indicates the OR operator. The above equation is read as follows : "Y is equal to A or B"

The symbol of the OR gate is shown in figure 7.46.

Illustration 9 : Figure shows the digital signals for the two input OR gate. Draw the shape of the output signal of the output of the OR gate.



Solution : First obtain the output (Y) for different states of the input A and B from the truth table then draw the output signal.

When $t < t_1$; A = 0, B = 0, Y = 0

$t_1 < t < t_2$; A = 0, B = 1, Y = 1

$t_2 < t < t_3$; A = 1, B = 0, Y = 1

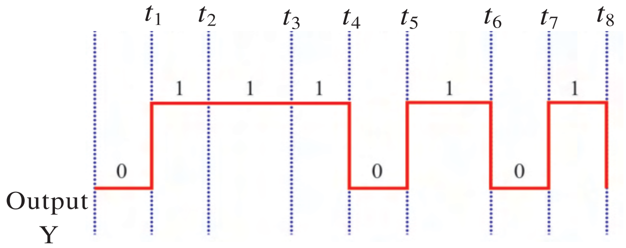
$t_3 < t < t_4$; A = 0, B = 1, Y = 1

$t_4 < t < t_5$; A = 0, B = 0, Y = 0

$t_5 < t < t_6$; A = 1, B = 1, Y = 1

$t_6 < t < t_7$; A = 0, B = 0, Y = 0

$t_7 < t < t_8$; A = 0, B = 1, Y = 1



7.10.2 AND Gate : Let us consider the circuit shown in figure 7.47 in order to understand the working of an AND gate. Here, the two switches A and B are connected in series with the bulb. Therefore, when any one of the switches will be OFF, current will not flow in the circuit and bulb will remain OFF. When both switches will be ON then only bulb will be ON.

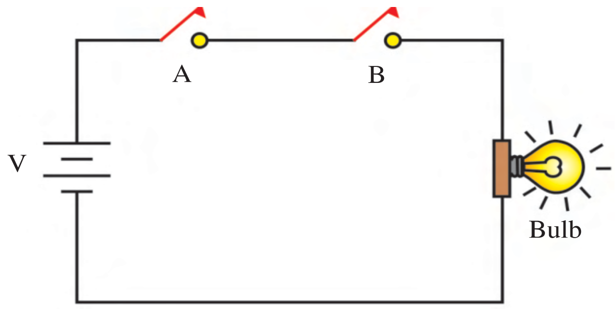


Figure 7.47

Table 3

A	B	Bulb
Open	Open	OFF
Open	Close	OFF
Close	Open	OFF
Close	Close	ON

Table 3 indicates the position of the switch and its corresponding state of the bulb. The truth table of the AND circuit can be prepared from this truth table as follows.

Figure 7.48 Symbol of AND Gate

Table 4

A	B	Y = A . B
0	0	0
0	1	0
1	0	0
1	1	1

From the truth table, function of the AND gate can be defined as follows.

The output of the AND gate is '1'only if all the inputs are '1'. For all other conditions of the input, it is '0'.

The Boolean equation is given as : Y = A·B.

Here '**.**' is known as AND operator. The equation is read as : "Y is equal to A AND B".

7.10.3 NOT Gate : NOT gate has one input and one output terminal. This gate inverts the input voltage. To understand the operation of the NOT gate, refer to figure 7.49.

Table 5

A	Bulb
Open	ON
Close	OFF

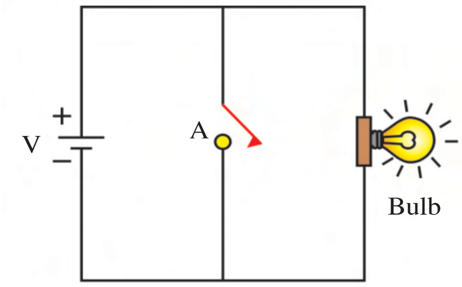


Figure 7.49

When the switch A is open, the current flows through the bulb and the bulb is in ON state. When the switch A is closed no current flows through the bulb and the bulb is in the OFF state. The working of this circuit is summarized in table 5. It is evident from the table 5 that the output reverses the input state. The truth table of the NOT gate is as per the table 6.

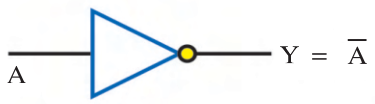


Figure 7.50 Symbol of NOT Gate

Table 6

A	Y = $\overline{A}$
1	0
0	1

From the truth table, function of the gate can be defined as follows.

"Whenever input is '1' the output is '0' and when the input is '0' the output is '1'."

Hence this gate is also called the inverter.

Boolean Equation : Y = $\overline{A}$: The NOT operator is indicated by the '–' (bar) symbol. The above equation is read as follows : "Y is equal to NOT A."

The AND, OR and NOT logic gates are called the basic logic gates in digital electronics. These gates can be combined in different ways to construct newer logic gates. We shall now study two such logic gates.

7.10.4 NOR Gate : The NOR gate is constructed by combining the OR gate and the NOT gate. (OR + NOT = NOR). Here, the output of the OR gate is given as input to the NOT gate. The Boolean expression for the NOR gate is given as follows :

$$Y = \overline{A+B}$$

This equation is read as : "Y is equal to NOT A or B."

The circuit diagram of the NOR gate and its symbol is shown in figure 7.51. The bubble on the OR gate indicates that the output of the OR gate gets inverted. The truthtable of the NOR gate is shown in table 7.

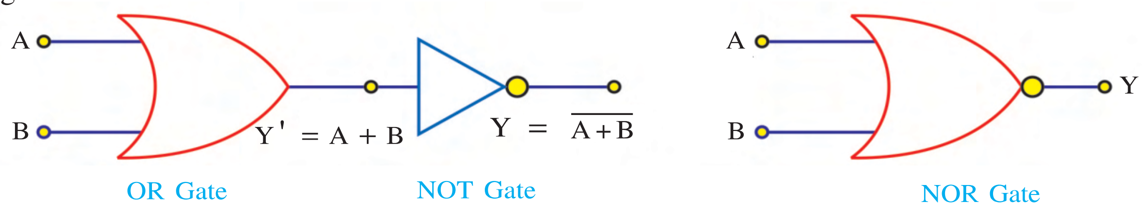


Figure 7.51 Logic Circuit and Symbol of NOR Gate

Table 7

A	B	A + B	$Y = \overline{A + B}$
0	0	0	1
0	1	1	0
1	0	1	0
1	1	1	0

The characteristic of the NOR gate can be given as follows :

The output is '0' whenever any one input is '1'. Whenever all the inputs are '0', the output is equal to '1'.

7.10.5 NAND Gate : NAND gate is constructed using the combination of AND gate and the NOT gate. (AND + NOT = NAND). Here, the output of the AND gate is given as input to the NOT gate. The Boolean expression is given as follows :

$Y = \overline{A \cdot B}$

This equation is read as : "Y is equal to NOT A and B".

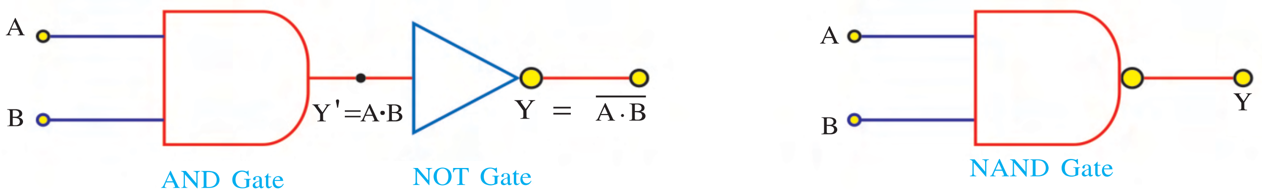


Figure 7.52 Logic Circuit and Symbol of NAND Gate

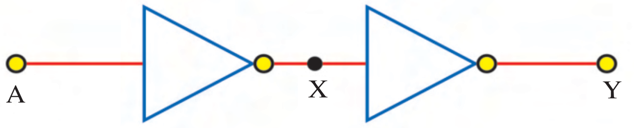
Table 8

A	B	$Y' = A \cdot B$	$Y = \overline{A . B}$
0	0	0	1
0	1	0	1
1	0	0	1
1	1	1	0

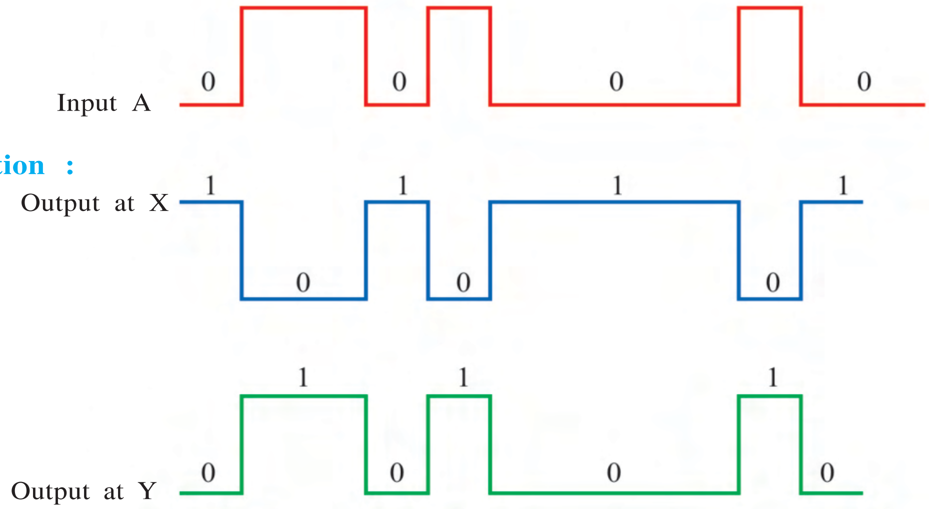
The circuit diagram and the symbol of the NAND gate is given by figure 7.52. The table gives its truth table. The truth table can be summarized as follows :

"The output is equal to '1' when any one input is equal to '0' and the output is equal to '0', when all the inputs are equal to '0'.

Illustration 10 : A logic circuit is shown in the diagram. Draw the output signal at the point X and Y for the input signal shown in the figure at point A.

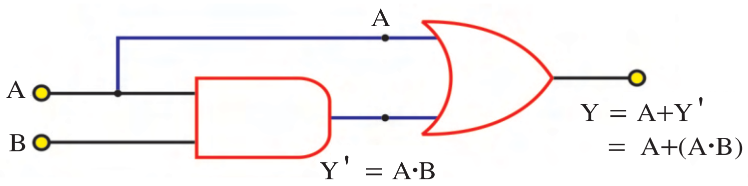


Solution :



Here, the output of the NOT gate will be equal to $X = \overline{A}$. Hence the output obtained will be equal to opposite of A. This signal is given as input to another NOT gate. The signal again gets inverted. As a result we again get back the original signal A. ($Y = \overline{X} = \overline{\overline{A}} = A$)

Illustration 11 : Prepare the truth table for the logic circuit given below.



Solution : Here Y' is the output of the AND gate having A and B as the input. The input to the OR gate is A and Y' (= A · B). Hence the output Y = A + Y' = A + (A · B). The truth table of the circuit can be given as under :

Table 9

A	B	Y'= A · B	Y = A + Y'
0	0	0	0
0	1	0	0
1	0	0	1
1	1	1	1

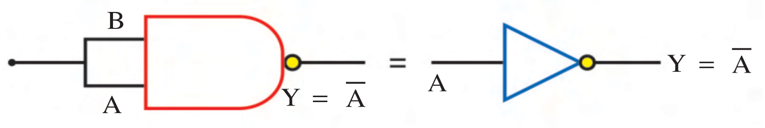
Illustration 12 : Which gate will be obtained by joining the two inputs of the NAND gate ?

Solution : When the two inputs are joined both the inputs will be identical. i.e. A = B. As per the characteristics of the NAND gate,

Y = 1 when A = 0 and B = 0 and

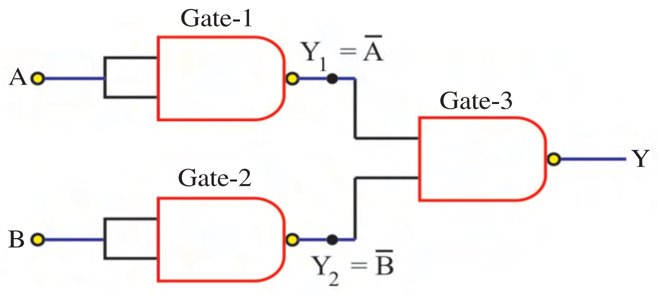
Y = 0 when A = 1 and B = 1

Here, the output Y will be opposite to that of the inputs A or B. Hence, we have a relation $Y = \overline{A}$. Hence the above logic circuit will behave as a NOT gate.



(**Note :** We can obtain a NOT gate by even joining the input terminals of a NOR gate.)

Illustration 13 : Show that the circuit drawn in the figure comprising of 3 NAND gates behaves as an OR gate.



Solution : As explained in the earlier illustration, the gates 1 and 2 will behave as a NOT gate. Hence $Y_1 = \overline{A}$ and $Y_2 = \overline{B}$. $\overline{A}$ and $\overline{B}$ are the inputs to the gate 3. The output Y of the gate 3 can be prepared using the truth table of the NAND gate.

Input A	Input B	$Y_1 = \overline{A}$	$Y_2 = \overline{B}$	Output $Y = \overline{Y_1 Y_2}$
0	0	1	1	0
0	1	1	0	1
1	0	0	1	1
1	1	0	0	1

It is clear from the truth table that the above truth table resembles the truth table of an OR gate. Hence we have shown that the above circuit behaves as an OR gate.

Note : Since the basic gates can be constructed from the NAND and NOR gates they are known as universal logic gates.

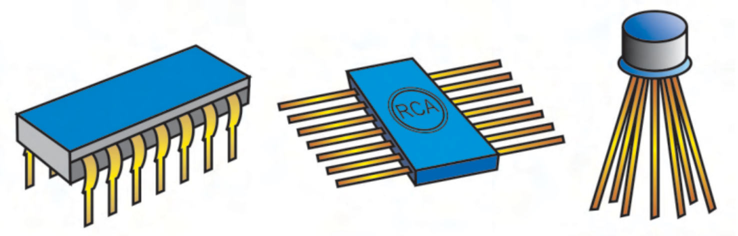
7.11 Primary Concept of IC

Present day electronic gadgets are smaller in size and at the same time are very efficient too. As for e.g. the size of the computers in 1960s was equal to the size of a big room. The present day laptop computers as well as the computers called palm top computers which can be kept in a pocket are commercially available. The Integrated circuits have a big role to play in miniaturization of these electronic gadgets.

About 50 years back the electronic circuits prepared from transistors, diodes and resistors which were joined using conducting wires were not reliable at the same time were huge in size. In the next generations the printed circuit board (PCB) came into existence. The components in a PCB are all arranged on a board and are connected with the help of metal strips.

The PCB helped in reducing the size of the electronic circuits. The circuits in a PCB are in three dimensions (3D). Later on attempt was made to make it two dimensional (2D) to further reduce the size. This gave rise to integrated circuits (IC). The size of the IC is about 1 mm $\times$ 1mm. In an IC, a small sized crystal (or chip) is taken and transistors, diodes resistors and capacitors are constructed within it and all these components are internally connected. This has brought about a revolution in the electronics industry. This has resulted in the size of the electronic gadgets to be reduced as well as succeeded in reducing the cost considerably.

The classification of IC is based on the number of logic gates present in it. S.S.I (Small Scale Integration) IC has about 10 or less than 10 number of logic gates. MSI (Medium Scale Integrotion) chips (or ICs) has about less than 100 number of logic gates. Chips having 100 to 1000 number of gates are known as LSI (Large Scale Integration). VLSI (Very Large Scale Integration) chips contain more than 1000 logic gates. VLSI chips are used in computers.



Complete IC in standard package

Figure 7.53 Different Types of IC

IC is basically of three types :

(1) Film Circuit : In this type of IC, thin film technique is used to fabricate components like resistance and capacitance.

(2) Monolithic Integrated Circuit : This IC has components like transistor, diode, resistance and capacitors.

(3) Hybrid Integrated Circuit : This type of IC is combination of film circuit and Monolithic type. This contains more than one chip. We shall obtain information about the Monolithic type IC.

The word Monolithic in Greek language mean one (monos) stone. It is made from only one type of semiconductor (Si or Ge). Hence it is called Monolithic IC.

SUMMARY

1. **Conductor, Insulator and Semiconductor :** The electrical conduction is easily possible in conductors due to the presence of free electrons. The electrical resistance is very low. In insulators there are no free electrons. Hence their electrical resistivity is very large. Semiconductors have more resistivity than conductors but less resistivity than insulators. At 0 K temperature semiconductors behave as insulators.

2. **Electrical Conduction in Intrinsic Semiconductors :** There are two types of electric current in a semiconductor. (i) Due to motion of free electrons. (ii) Due to motion of bound electrons or holes ($I = I_h + I_e$).

3. **N-type and P-type Semiconductors :** N-type semiconductors can be prepared by doping pentavalent impurity in the intrinsic semiconductors. This type of impurity is called donor impurity. In N-type semiconductor, electrons are majority charge carriers and holes are minority charge carriers. For N-type semiconductor, $n_e > n_h$.

   P-type semi-conductor can be prepared by doping trivalent impurity in the intrinsic semiconductors. This type of impurity is called acceptor impurity. In P-type semiconductor, holes are majority charge carriers and electrons are minority charge carriers. For P-type semiconductors, $n_h > n_e$.

4. **Band Diagram for Semiconductors :** There are 8N valence state and corresponding energy levels in a silicon crystal of N atoms. According to electronic configuration of Si, 4 N energy levels are filled. As per Pauli's principle, 4 N electrons occupy these 4 N energy levels of band and this band is completely filled. This band is called valence band.

   Above the valence band there is a region where no energy levels are available. This region is known as forbidden gap.

   The region above the forbidden gap is known as the conduction band. The conduction band is completely empty.

   The difference of minimum energy level of conduction band (E_c) and maximum energy level of valence band (E_v) is called band gap (E_g).

   If the energy supplied to the valence electron is E_g or greater than E_g, the electron can jump from the valence band to conduction band. These electron will then contribute towards the current.

   For insulator, band gap is $E_g > 3$ eV.

   For conductor, band gap is $E_g = 0$.

   For semiconductor band gap is $E_g < 3$ eV.

5. **Forward Bias :** When the positive terminal of the battery is connected to P and negative terminal is connected to N of the PN junction diode, this connection is called forward bias.

   In forward bias mode, height of the depletion barrier and width of the depletion layer is decreasing.

   Resistance of the PN Junction is approximately 10 Ω to 100 Ω in forward bias.

6. **Reverse Bias :** When the negative terminal of battery is connected to P-type and positive terminal is connected to N-type of PN junction, this connection is called reverse bias.

   In reverse bias resistance of PN junction is in order of M Ω.

7. Rectifier : The process of converting AC energy into DC energy is called rectification. The circuit which performs this process is called rectifier. There are two types of rectifiers. (i) Half wave rectifier (ii) Full wave rectifier.

8. Zener Effect : The electric field in the depletion region of PN junction diode is strong when small reverse bias voltage is applied because of the thin width of the depletion region. This strong electric field is sufficient to break covalent bonds and make the electron free. This results in large number of electron and hole pair formation as well as sudden increase in the current. This effect is known as zener effect.

9. Maximum Wavelength of the Light-emitted from the LED :

$$\lambda = \frac{hc}{E_g}$$

where, E_g = Bandgap energy

h = Planck's constant

c = Velocity of light

10. The condition for detecting light-incident on the photo diode is,

$$\frac{hc}{\lambda} > E_g$$

11. Transistor : A device with two PN junction is called a transistor. Transistor has three terminals. (i) Emitter (E) (ii) Base (B) and (iii) Collector (C).

The junction between emitter and base is called emitter junction and junction between base and collector is called collector junction.

To operate the transistor, emitter junction should be in forward bias and collector junction should be in reverse bias.

The relationship between different currents of the transistor.

$$I_E = I_B + I_C$$

12. Transistor's parameters :

(a) Input Resistance :

$$r_i = \left(\frac{\Delta V_{BE}}{\Delta I_B}\right) V_{CE} = \text{constant}$$

(b) Output Resistance :

$$r_o = \left(\frac{\Delta V_{CE}}{\Delta I_C}\right) I_B = \text{constant}$$

(c) Current Gain :

$$\beta = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE}} = \text{constant}$$

(d) Transconductance :

$$g_m = \frac{\Delta I_C}{\Delta V_{BE}} = \frac{\beta}{r_i}$$

The unit of g_m is mho.

13. Transistor Amplifier : Voltage gain of CE amplifier,

$$A_V = -\beta \frac{R_L}{r_i} = -gm \ R_L$$

Power gain of CE simplifier, $|A_P| = A_V A_i = \beta^2 \frac{R_L}{r_i}$

14. Oscillator : The circuit can generate desired frequency of desired amplitude is called an oscillator.

The frequency of an LC oscillator circuit, $f = \dfrac{1}{2\pi\sqrt{LC}}$.

15. Logic Gate : The logic circuit which has more than one input but only one output is called a logic gate. It has only two output state '0' or '1'.

OR, AND and NOT are the basic logic gates.

OR Gate : Whenever any one input or both the inputs are '1', then only output is '1'.

Boolean equation: $Y = A + B$.

AND Gate : The output of AND gate is '1' only if all the inputs are '1' for all other condition of input it is '0'.

Boolean equation : $Y = A \cdot B$.

NOT Gate : Whenever input is '1' the output is '0' and when the input is '0' the output is '1'.

Boolean equation : $Y = \overline{A}$

NOR Gate : The output is '0' whenever any one input is '1'. Whenever all the inputs are '0' the output is '1'.

Boolean equation : $Y = \overline{A + B}$

NAND Gate : The output is '1' when any one input is '0' and the output is '0', when all the inputs are '1'

Boolean equation : $Y = \overline{A \cdot B}$

EXERCISE

For the following statements choose the correct option from the given options :

1. The density of electron and holes in an intrinsic semiconductor is n_e and n_h respectively. Which of the following options are true ?

(A) $n_h > n_e$ (B) $n_e > n_h$ (C) $n_e = n_h$ (D) $n_h >> n_e$

2. The energy band diagram of a Si semiconductor crystal at absolute zero temperature,

(A) has completely empty valence band and completely filled conduction band.

(B) has completely empty conduction band and completely filled valence band.

(C) has completely empty valence and conduction band and completely filled forbidden gap.

(D) the conduction band is partially filled.

3. The band gaps of a conductor, semiconductor and insulator are respectively E_{g_1}, E_{g_2} and E_{g_3}. The relationship between them can be given as....

(A) $E_{g_1} = E_{g_2} = E_{g_3}$ (B) $E_{g_1} > E_{g_2} > E_{g_3}$

(C) $E_{g_1} < E_{g_2} < E_{g_3}$ (D) $E_{g_1} < E_{g_2} > E_{g_3}$

4. When will the conductivity of a Ge semiconductor decrease ?

(A) On adding donor impurity (B) On adding acceptor impurity

(C) In making UV light incident (D) On decreasing the temperature

5. For detecting the light,

(A) The photodiode has to be forward biased.

(B) The photodiode has to be reverse biased.

(C) The LED has to be connected in forward bias mode.

(D) The LED has to be connected in a reverse bias mode.

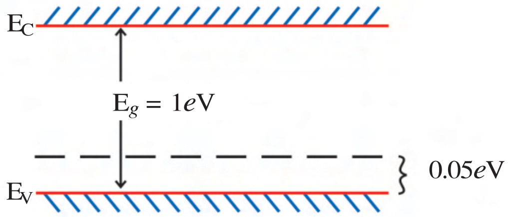
6. What is the type of the semiconductor, for the energy band diagram shown in the figure ?

(A) N-type semiconductor

(B) P-type semiconductor

(C) Intrinsic semiconductor

(D) Both N and P type semiconductors



7. In order to operate .... type of semiconductor, we have to apply the forward bais.

(A) photo diode (B) zener diode

(C) varactor diode (D) light emitting diode (LED)

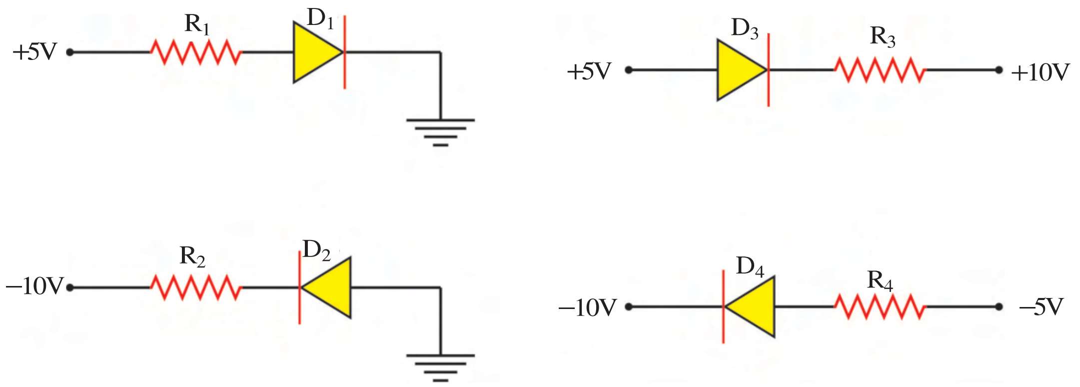
8. Which type of semiconductor device does not need any bias voltage ?

(A) photo diode (B) varactor diode (C) solar cell (D) transistor

9. A potential barrier of 0.50 V exists acrros of PN junction. If the depletion region is 5.0×10^{-7} m wide the intesisty of the electric field in this region is ....

(A) 1.0×10^9 V/m (B) 1.0×10^6 V/m (C) 2.0×10^5 V/m (D) 2.0×10^6 V/m

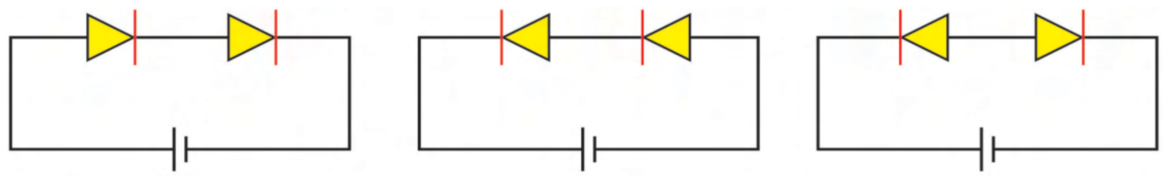
10. Which of the following P-N junction diode is reverse biased ?



(A) P-N junction diode D_1 (B) P-N junction diode D_2

(C) P-N junction diode D_3 (D) P-N junction diode D_4

11. Two identical P-N junction diodes are connected with the battery in three different ways (refer to the figure). For which circuit will the potential difference between the diodes be identical.



(A) For circuit (1) and (2) (B) For circuit (2) and (3)

(C) For circuit (3) and (1) (D) None of the above circuits

12. V_m is the maximum voltage between the ends of the secondary terminal of a transformer used in a half wave rectifier. When the PN junction diode is reverse biased, what will be the potential difference between the two ends of the diode ?

(A) Zero (B) $\frac{V_m}{2}$ (C) V_m (D) $2\ V_m$

13. In LC oscillator, the angular frequency of oscillation of current is obtained from ....

(A) $f = \frac{1}{2\pi LC}$ (B) $\omega^2 = \frac{1}{LC}$ (C) $\omega = \frac{1}{2\pi\sqrt{LC}}$ (D) $\sqrt{f} = \frac{1}{2\pi LC}$

14. The frequency of the output signal becomes .... times by doubling the value of the capacitance in the LC oscillator circuit.

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{1}{2}$ (D) 2

15. The emitter junction of the CE transistor amplifier is .... biased while the collector junction is .... biased.

(A) Reverse, forward (B) Forward, forward

(C) Reverse, reverse (D) Forward, reverse

16. The amplifier has voltage gain equal to 200 and its input signal is 0.5 cos(313 t) V. The output signal will be equal to .... volt.

(A) 100 cos(313 t + 90°) (B) 100 cos(313 t + 180°)

(C) 100 cos(493 t) (D) 0.5 cos(313 t + 180)

17. The collector has current of the NPN transistor is equal to 10 mA. If 90% of the electron from the emitter reaches collector then……

(A) $I_E \approx 9$ mA, $I_B \approx 1$ mA (B) $I_E \approx 11$ mA, $I_B \approx 9$ mA

(C) $I_E \approx 11$ mA, $I_B \approx 1$ mA (D) $I_E \approx 10$ mA, $I_B \approx 1$ mA

18. $\alpha = 0.99$ for a CE transistor amplifier circuit. The input resistance is equal to 1 kΩ and the load resistance is equal to 10 kΩ. The voltage gain of the circuit is……

(A) 99 (B) 990 (C) 9900 (D) 99000

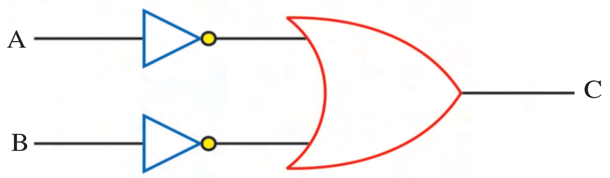
19. The logic circuit shown in the figure represents characteristic of which logic gate ?

(A) OR gate

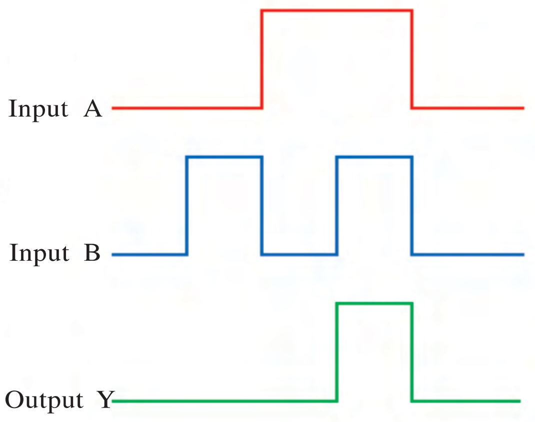
(B) AND gate

(C) NOR gate

(D) NAND gate



20. The figure shows the input signal A, input signal B and the output Y. Which logic gate does it represent ?



(A) OR gate

(B) AND gate

(C) NAND gate

(D) NOR gate

21. Which logic gate characteristic is represented by the truth table shown below :

(A) NAND gate

(B) NOR gate

(C) AND gate

(D) OR gate

A	B	Y
1	1	0
1	0	0
0	1	0
0	0	1

ANSWERS

1. (C)	**2.** (B)	**3.** (C)	**4.** (D)	**5.** (B)	**6.** (B)
7. (D)	**8.** (C)	**9.** (B)	**10.** (C)	**11.** (A)	**12.** (C)
13. (B)	**14.** (A)	**15.** (D)	**16.** (B)	**17.** (C)	**18.** (B)
19. (D)	**20.** (B)	**21.** (B)			

Answer the following questions in brief :

1. Give the electronic configuration of silicon.
2. What is a hole ?
3. Can we consider a bound electron as a free electron? Why ?
4. What is intrinsic semiconductor ?
5. What is forbidden gap ?
6. Draw a band diagram of a N-type semiconductor at room temperature.
7. What is depletion barrier ?
8. Draw a circuit symbol of a PN junction diode. What does the arrow indicate ?
9. What is rectification ?
10. What is called filter circuits ?
11. Write the equation for maximum wavelength of a light emitted from LED.
12. What is the value of photovoltage produced in a photo cell ?
13. Give the relation between I_E, I_C and I_B. Also give the order of their magnitude.
14. What is the phase difference between input and output signal of a CE amplifier ?
15. What is a logic gate ?
16. Write the Boolean equation of a NOR gate.
17. Which are the basic logic gates and universal logic gates ?
18. Write full form of VLSI.

Answer the following questions :

1. Explain the electrical conduction in intrinsic semiconductor with the help of a diagram.
2. Write a short note on P-type semiconductor.
3. Explain the valence band, conduction band and forbidden gap of Si semiconductor.
4. Draw and explain the band diagram of N-type semiconductor at 0 K temperature and room temperature.
5. Explain the depletion layer and depletion barrier of a PN junction diode.
6. Draw the circuit diagram to obtain forward bias characteristics of a PN junction and discuss the forward bias characteristics of it.
7. Draw the circuit diagram of a half wave rectifier and explain the working of the circuit.
8. Write short note on LED.
9. Draw the circuit of a CE amplifier using NPN transistor. Obtain the expression for the voltage gain and power gain of CE amplifier.

10. Explain the working of a OR gate. Give the symbol, Boolean expression and truth table of a OR gate.

11. Draw the logic diagram of a NAND gate. Give the symbol, Boolean expression and truth table of it.

Solve the following examples :

1. There are 6×10^{19} electrons per unit cubic metre of pure semiconductor. What will be the number of holes for this semiconductor of dimension 1 cm × 1 cm × 2 cm ?

   [**Ans. :** 12×10^{13}]

2. The density of electron hole pair in a pure semiconductor at 300 K temperature is 1.5×10^{16} m^{-3}. The number density of the majority charge carriers becomes equal to 4.5×10^{22} m^{-3} on adding trivalent impurity atoms. What will be the number density of the minority charge carriers in the above sample ? [**Ans. :** 5×10^{9} m^{-3}]

3. A electron hole pairs are formed when maximum 6000 Å wavelength of light is incident on the semiconductor. What will be the band gap energy of the semiconductor ?

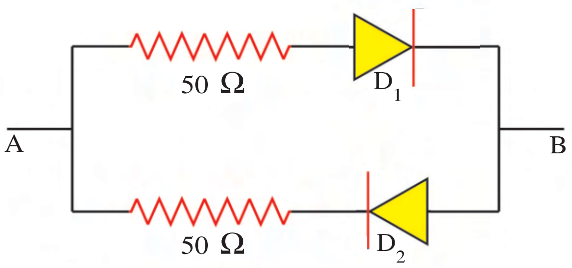
   [$h = 6.62 \times 10^{-34}$ J s] [**Ans. :** 2.07 eV]

4. If an LED has to emit 662 nm wavelength of light then what should be the band gap energy of its semiconductor ? [$h = 6.62 \times 10^{-34}$ J s] [**Ans. :** 1.875 eV]

5. The width of a depletion region is 400 nm. The intensity of the electric field at the depletion region is 5×10^{5} V/m. Then calculate the following quantities : (1) The value of the potential barrier. (2) The minimum energy required by an electron to move from the N-type to the P-type region of the diode. [**Ans. :** 0.2 V, 0.2 eV]

6. For the circuit shown in the figure, calculate the equivalent resistance for the two cases given as : (1) $V_A > V_B$ and (2) $V_B > V_A$. Here consider D_1 and D_2 to be ideal diodes.

   [**Ans. :** For both the cases $R_{AB} = 50$ Ω]



7. The voltage gains of a N-P-N common emitter amplifier is 200. Its load resistance is 10 kΩ. Calculate the value of the transconductance. If the input resistance is 1 kΩ, what will be the value of the A.C. current gain. [**Ans. :** $g_m = 0.02$ mho, $A_i = 20$]

8. For an NPN transistor about 7 % of the electron entering the base from the emitter recombines with the hole. This results in the collector current being 18.6 mA. Calculate the emitter current and the current gain.

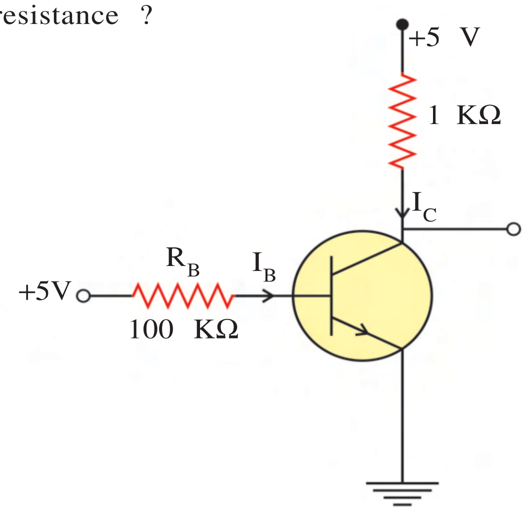
   [**Ans. :** $I_{EV} = 20$ mA, $\alpha = 0.93$]

9. The base current changes by 200 μA when a 200 mV signal is applied at the input of a CE amplifier. Find input resistance. If the output voltage is 2 volt, what is the voltage gain?

   [**Ans. :** $r_i = 1$ kΩ, $A_V = 10$]

10. The collector current changes by 10 mA when the input voltage of the NPN common emitter amplifier changes by 100 mV. The A.C current gain of this circuit is equal to 150. If we have to obtain a power gain of 4500 then what should be the value of the load resistance ? [**Ans. :** $R_L = 300\ \Omega$]

11.

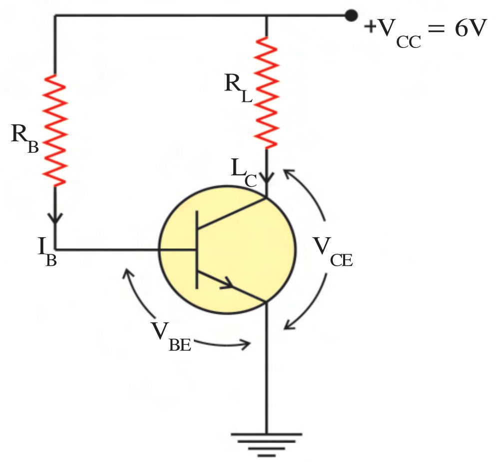


In the circuit shown in the figure, on applying +5V at the base resistor R_B, both the V_{BE} and V_{CE} voltages become zero. Then calculate the values of I_C, I_B and β.

[**Ans. :** $I_B = 50\ \mu A$, $I_C = 5$ mA, $\beta = 100$]

12. For the circuit shown in the figure, $I_B = 5\ \mu A$, $R_B = 1\ M\Omega$, $R_L = 1.1\ k\Omega$, $I_C = 5$ mA and $V_{CC} = 6V$. Calculate the values of V_{BE} and V_{CE}.

[**Ans. :** $V_{BE} = + 1.0V$, $V_{CE} = + 0.5V$]



13. The A.C current gain of a PNP common emitter circuit is 100. The value of the input resistance is 1 kΩ. What should be the value of the load resistor R_L in order to obtain power gain of 2000 ?

[**Ans. :** $R_L = 200\ \Omega$]

•

8 COMMUNICATION SYSTEMS

8.1 Introduction

Communication system means a system for information exchange. In ancient times, the king used to communicate with other states by sending the message through Pigeons or their trustful person. But it took long time in receiving messages.

In 1887, a scientist name Hertz produced electromagnetic waves in his laboratory. Thereafter scientists such as Jagdish chandra Bose, Morse, Marconi and Graham Bell developed the basics of modern communication system.

In our daily life, we use communication systems in various ways. For example, Telephone, Radio, TV, Cell phone etc are used for information exchange. How this information is received and transmitted from one place to another, that we shall study in this chapter.

8.2 Communication System

As shown in figure 8.1, the block diagram of communication system gives the preliminary information about the communication.

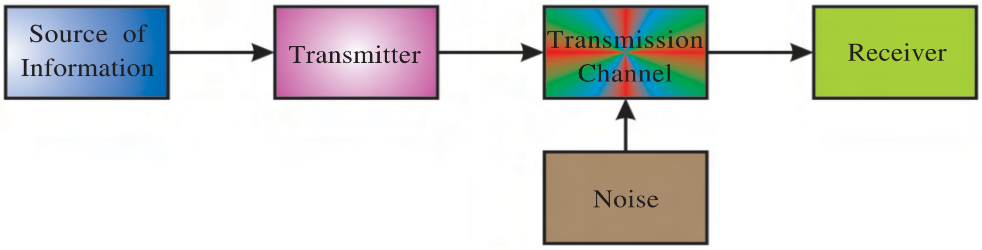


Figure 8.1 Block Diagram of Communication System

A communication system has three main components (1) Transmitter, (2) Transmission channel and (3) Receiver.

In communication system, the transmitter is located at one place, the receiver is located at some other place (far or near). A transmission channel or physical medium connects the transmitter and receiver.

(1) Transmitter : The information received from the source of information can be in various forms. For example, the speech of a person is in the form of sound waves, while its picture information is in the form of light waves. Such information cannot be directly communicated to a long distance.

For this purpose, the information or messages have to be converted into electrical signals. The devices which can perform this are generally known as transducer.

The device which transforms one form of energy into another form of energy, is generally called a transducer. For example, microphone converts sound energy into electrical energy. Hence, it is called a transducer.

The transmitter initially converts the received information into electrical signals. Then it amplifies the signal, performs the necessary process of modulation and passes it to the transmission channel.

(2) Transmission Channel : Transmission channel is a link (medium) between the transmitter and receiver. Transmission channel can be a coaxial cable or two wire line or free space or an optical fibre.

Free space is used as the transmission channel for radio and TV transmission. In this case, the transmitter transmits the signal in the form of electromagnetic radiation. There is no conducting wire between the transmitter and the receiver, hence, such a system is called a wireless communication.

In telephone system, two wire line is used as a transmission channel.

(3) Receiver : The signal transmitted in the transmission channel are intercepted and amplified by the receiver section. The signal passes through the demodulation process in the receiver and are converted into the original information using proper electronic device. For example, loudspeaker converts electrical signals into sound, while picture tube converts electrical signals into a picture.

(4) Noise : Noise is an unwanted signal. The signals transmitted by the transmitter while propagating through the transmission channel get mixed with the noise signals. As a result, the original information gets distorted. The noise signals may be natural or it may be manmade. Flashes of lightning and radiations from the sun or stars are natural noise while the noise due to vehicles, electric motors and heavy machinery or due to flickering of tube light are part of man-made noise.

Filter circuits in the receiver section, are used to reduce the level of noise signal.

There are two basic modes of communication (1) point-to-point communication mode and (2) Broadcast mode.

In point to point communication mode, communication takes place over a link between single transmitter and a receiver, for example, communication through telephone system. In the broadcast mode there are large number of receivers corresponding to single transmitter. For example, transmission through radio and TV system.

8.3 Signals and Bandwidth

Information converted in electrical form and suitable for transmission is called a signal. Signals can be of two types. (1) Analog signals and (2) Digital signals.

An analog signal is a continuously varying signal with respect to time. At a given instant, the signal may acquire any value between its maximum and minimum value. For example, output signal of a microphone or a video camera. Analog communication system uses analog signals.

Digital signals have only two values the minimum value of signal or the maximum value of signal represented as '0 'and '1' respectively. Here the information from the analog signal is sampled periodically with definite interval of time. This is then transmitted in the digital form by assigning '0' and '1' according to sample voltage. Digital signals are used in the digital communication system.

In a communication system, the information signal can be voice, music, picture or computer data. Each of these signals has different ranges of frequencies. The communication system must be efficient to transmit this band of frequencies. The difference of maximum frequency and minimum frequency efficiently transmitted by the system is called the bandwidth of the system.

For example, in normal speech we use frequency band of 300 Hz to 3100 Hz. The bandwidth of this signal is 3100 Hz − 300 Hz = 2800 Hz. Telephone communication system has the same bandwidth. Audio frequency range is from 20 Hz to 20 kHz. So the bandwidth of the system should be approximately 20 kHz to transmit the music. Similarly bandwidth of the system should be approximately 4.2 MHz to transmit video signals. Television system transmits both the audio and video signals. So that each channel of television has alloted the band width of 6 Mhz for transmission.

Digital signals are used in the digital communication. Digital signals are in the form of rectangular waves. This wave is composed of sinusoidal waves of frequencies f_0, $3f_0$, $5f_0$,...

All these frequencies are superposed with proper amplitude to form a digital signal of rectangular wave (see figure 8.2) which implies that signal has infinite bandwidth. However, for practical purpose the contribution from higher harmonics can be neglected, since they have small amplitude. In absence of higher harmonics, the signal gets distorted and bandwidth of the communication system is reduced, i.e. there is a loss of information from the signal.

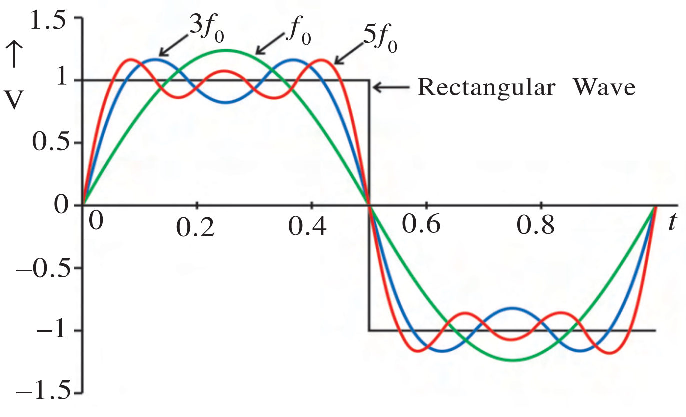


Figure 8.2 Wave of Rectangular Shape

Different types of communication channels offer different bandwidths. The commonly used transmission media are wire, Coaxial cable, free space and optical fiber. Coaxial cable offers a bandwidth of approximately 250 MHz. Communication through free space can takes place over a wide range of frequencies from few kHz to few GHz. The frequencies from 1 THz to 1000 THz can be propagated through the optical fiber. An optical fiber can offer a bandwidth of 100 GHz.

8.4 Modulation and its Necessity

In some communication system, an electrical signal of information is directly transmitted in to the transmission channel. For example, in telephone system the electrical signals of sound are sent through a conducting wire from one end to the other receiver end.

However, most of the signals of information have low frequency and they are not able to travel long distance directly in a free space. Let us see what factors prevent us from doing so and how we overcome these factors.

(1) Length of Antenna : In order to send message or information of sound waves, these are converted into electrical signals by using a microphone. Such electrical signals are called audio signals or base band signals. A transmitter converts these audio signals into electromagnetic radiation through an antenna and radiates in the space.

For effective transmission of electromagnetic radiation of audio signals, the minimum length of the antenna must be $\frac{\lambda}{4}$. Where λ is the wavelength of the audio signal.

Range of frequency of audio signal is from 20 Hz to 20 kHz. For example, if we want to transmit a signal of frequency of 1 kHz.

$$\lambda = \frac{c}{f} = \frac{3\times10^8}{1\times10^3} = 300 \text{ km}$$

$$\therefore \text{ Minimum length of antenna } = \frac{\lambda}{4} = \frac{300}{4} = 75 \text{ km}$$

Such length of antenna in reality is impractical as well as very costly. However, the length of the antenna for a signal of frequency 1 MHz, turns out to be only 75 m.

This shows that for effective transmission of high frequency signals, required antenna length is small and hence such an antenna can be easily constructed.

(2) Power Radiated from the Antenna : The study of electromagnetic radiation shows that the transmitted power by an antenna of a given length is inversely praportional to the square of the wavelength λ i.e. $P \propto \frac{1}{\lambda^2}$. This indicates that an antenna can transmit short wavelength or high frequency radiation with more efficiency. Hence, for this purpose, also, the use of high frequency signal is inevitable.

(3) Mixing up of Signals from Different Transmitters : If there is more than one transmitter in a region and if these transmit the information using frequency of audio signals, then all such signals get mixed. It is not possible to separate information of one transmitter from the information of other transmitter. Such a situation can be avoided if every transmitter is assigned different high frequencies for information transmission.

The conclusion of the above discussion is that if the transmission is done using high frequency instead of low frequency, then difficulties do not arise. Hence, the modulation process becomes necessary.

8.5 Modulation

The process of superposing low frequency audio signals on waves with high frequency is called modulation.

Here, the low frequency signal is called the modulating signal and the high frequency wave, since it carries the information is called a carrier wave.

Generally, the carrier wave is a sine wave, which is mathematically represented as,

$e_c = E_c \sin(\omega_c t + \phi)$

where, E_c = amplitude of carrier wave,

ω_c = angular frequency and ϕ is the initial phase of the wave. We have three types of modulations based on three parameters of the wave. (1) Amplitude modulation, (2) Frequency modulation and (3) Phase modulation.

Any one of the parameters can vary according to modulating wave by keeping other two parameter constant, we have three different types of the modulation. The wave form of different types of the modulation are shown in the figure 8.3.

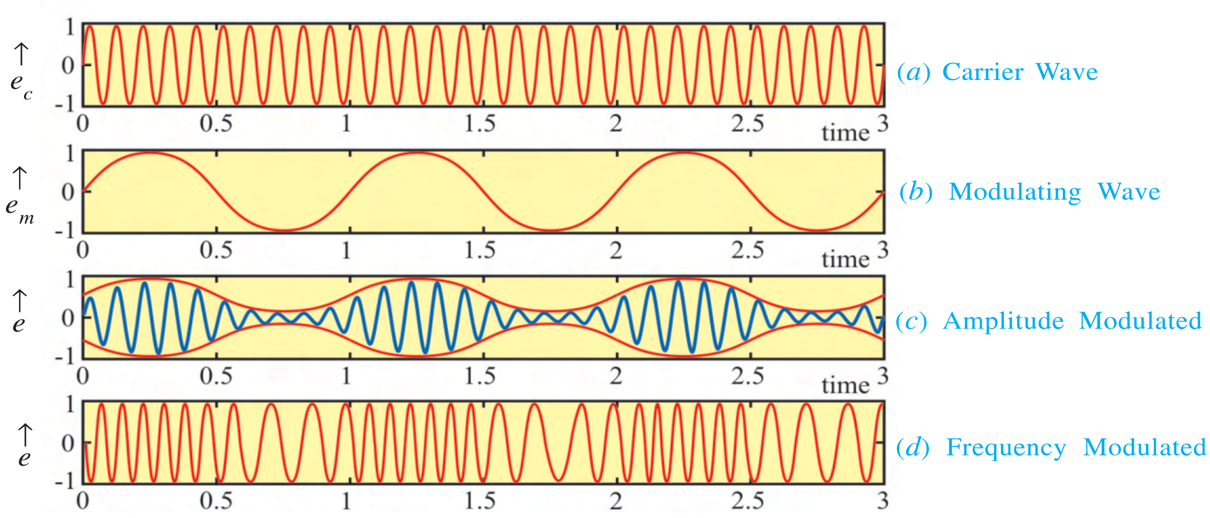


Figure 8.3 Different Types of the Modulator

In digital communication, pulse type wave is used as a carrier wave.

The pulse can be described with its three characteristics. (1) Pulse amplitude, (2) Pulse width and (3) Pulse position.

Hence, three different types of modulation are : (1) Pulse Amplitude Modulation (PAM), (2) Pulse Width Modulation (PWM) and (3) Pulse Position Modulation (PPM).

In this chapter, we shall confine to amplitude modulation only.

8.6 Amplitude Modulation

A modulation, in which the amplitude of the carrier wave (E_c), is varied in accordance with the instantaneous value of the modulating wave is called amplitude modulation (AM).

The frequency and initial phase of carrier wave remains constant.

The wave form of carrier wave, modulating signal and amplitude modulated waves are shown in figures 8.3. The figure clearly shows that as the instantaneous value of the modulating wave changes with time, the amplitude of the positive cycle and negative cycle of modulated wave vary accordingly. Thus the envelope of the modulated wave is of the same shape as that of the modulating signal associated with the information.

Radio broadcasting and video signal transmission in TV are of AM type.

Suppose, the carrier wave and modulating wave are as follow :

Carrier Wave : $e_c = E_c \sin(\omega_c t + \phi)$ (8.6.1)

Modulating Wave : $e_m = E_m \sin\omega_m t$ (8.6.2)

Where, ω_c and ω_m are angular frequency of carrier wave and modulating wave respectively.

The amplitude of carrier wave is varying according to instantaneous value of modulating signal, hence amplitude modulated wave can be represented as,

$e = (E_c + e_m) \sin\omega_c t$ (The frequency and phase of the carrier wave remain constant)

$= (E_c + E_m \sin\omega_m t) \sin\omega_c t$

$= E_c(1 + \frac{E_m}{E_C} \sin\omega_m t) \sin\omega_c t$

$e = E_c(1 + m_a \sin\omega_m t) \sin\omega_c t$ (8.6.3)

Equation (8.6.3) is a mathematical form of a AM wave.

In equation (8.6.3), $m_a = \frac{E_m}{E_C}$ is called a modulation index. Generally, value of m_a is between 0 and 1. If m_a is greater than 1, AM wave gets distorted.

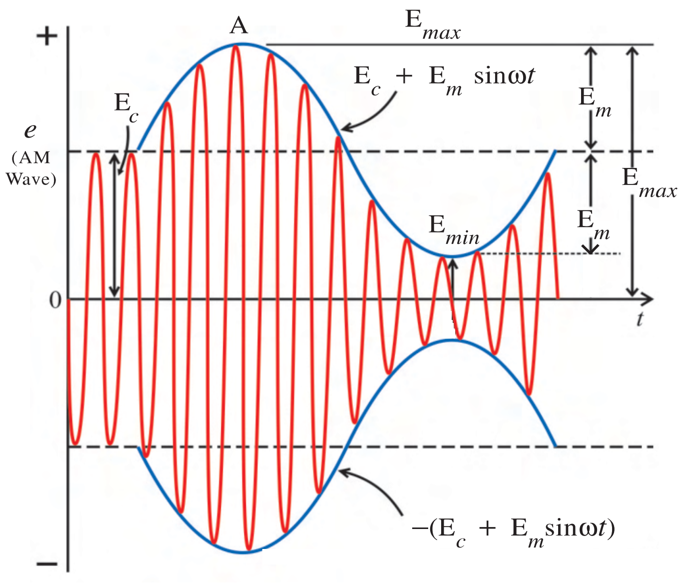


Figure 8.4 AM Wave

Figure 8.4 shows the AM wave. The upper envelope of AM wave varies according to $E_c + E_m \sin\omega_m t$.

At point A, $\sin\omega_m t = 1$, hence AM wave has maximum amplitude.

$E_{max} = E_c + E_m$ (8.6.4)

At point B, $\sin\omega_m t = -1$, hence AM wave have minimum amplitude. (8.6.5)

Taking addition of equation (8.6.4) and (8.6.5)

$$E_c = \frac{E_{max} + E_{min}}{2}$$

Subtracting equation (8.6.5) from equation (8.6.4),

$$E_m = \frac{E_{max} - E_{min}}{2}$$

According to definition of modulation index,

$$m_a = \frac{E_m}{E_c} = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \quad (8.6.6)$$

$$m_a(\%) = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \times 100$$

8.7 Frequency Spectrum of the AM Wave

According to equation (8.6.3)

$e = E_c \sin\omega_c t + m_a E_c \sin\omega_c t \sin\omega_m t$

Using trigonometric relation

$$\sin A \ \sin B = \frac{1}{2}[\cos\ (A - B) - \cos(A + B)]$$

$$e = E_c \sin\omega_c t + \frac{m_a}{2} E_c[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

$$= E_c \sin\omega_c t + \frac{m_a}{2} E_c \cos(\omega_c - \omega_m)t - \frac{m_a}{2} E_c \cos(\omega_c + \omega_m)t. \quad (8.7.1)$$

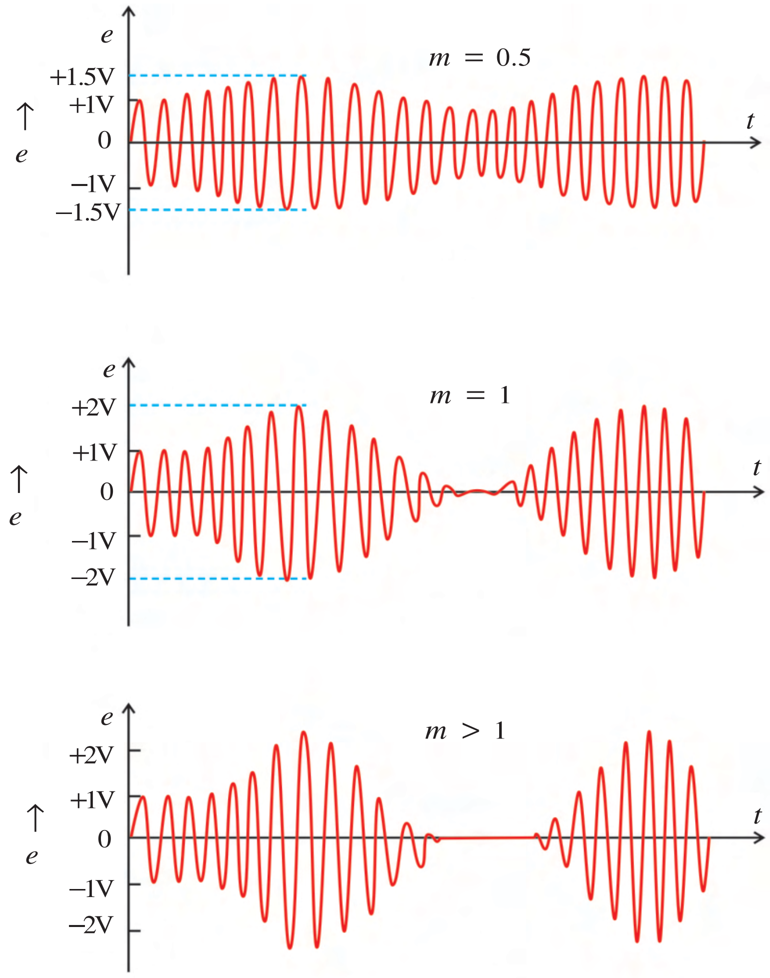


Figure 8.5 AM Waves with Different Modulation Indices

Equation (8.7.1) shows that AM wave contains three different frequencies.

(1) ω_c : which is an original carrier frequency with amplitude E_c.

(2) $\omega_c + \omega_m$: This frequency is greater than carrier frequency. It is called an upper side band frequency (USB).

(3) $\omega_c - \omega_m$: This frequency is lower than carrier frequency. It is called a lower side band frequency (LSB).

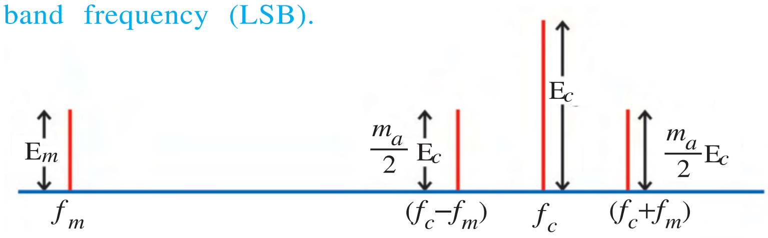


Figure 8.6 Frequency Spectrum of AM Wave

The amplitude of USB and LSB frequency is $\frac{m_a}{2}E_c$. These side band frequencies are very important to carry the information. You will learn about this in future.

Frequency spectrum containing modulating wave, carrier wave and side band frequencies are shown in figure 8.6.

Illustration 1 : A modulating signal of frequency 5 kHz and peak voltage of 6 V is used to modulate a carrier of frequency 10 MHz and peak voltage of 10 V. Determine

(1) Modulation index, (2) Frequency of LSB and USB and (3) Amplitude of LSB and USB.

Solution : $f_c = 10$ MHz, $f_m = 5$ kHz $= 0.005$ MHz

$E_c = 10$ V, $E_m = 6$ V

(1) Modulation index $m_a = \frac{E_m}{E_C} = \frac{6}{10} = 0.6$

(2) Frequency of LSB $= f_c - f_m = 10 - 0.005 = 9.995$ MHz

Frequency of USB $= f_c + f_m = 10 + 0.005 = 10.005$ MHz

(3) Amplitude of LSB $= \frac{m_a}{2}E_c = \frac{0.6}{2} \times 10 = 3$ V

Similarly amplitude of USB will be 3 V.

Illustration 2 : For an amplitude modulated wave, the maximum amplitude is found to be 10V while the minimum amplitude is found to be 6 V. Determine the modulation index (%) and amplitude of original carrier frequency.

Solution : $E_{max} = 10$ V, $E_{min} = 6$ V,

Modulation index $m_a(\%) = \frac{E_{max} - E_{min}}{E_{max} + E_{min}} \times 100 = \frac{10 - 6}{10 + 6} \times 100 = 25\%$

Amplitude of carrier wave, $E_c = \frac{E_{max} + E_{min}}{2} = \frac{10 + 6}{2} = 8$ V

8.8 Production of AM Wave

An electrical circuit which produces an AM wave is called amplitude modulator.

Amplitude modulation can be produced by a different method. A simple method is shown in the block diagram of figure 8.7.

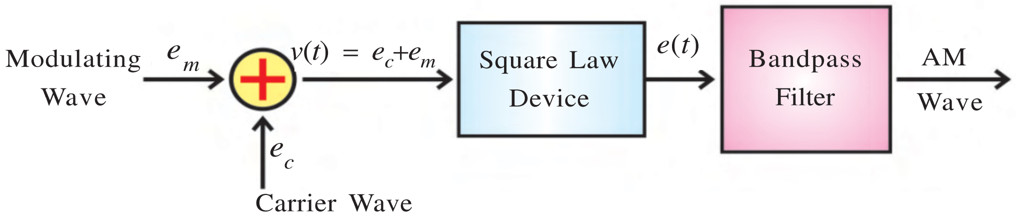


Figure 8.7 Production of AM Wave

As shown in the figure, the modulating signal e_m is added to the carrier signal e_c to produce the signal $v(t)$

$$v(t) = e_c + e_m \tag{8.8.1}$$

This $v(t)$ signal is given to a square law device which is a non-linear device. The device which does not follow Ohm's law, i.e. the relation between current and voltage is not linear is called non linear device. The output voltage across non linear device is,

$$e(t) = av(t) + bv^2(t) + cv^3(t) + \ldots$$

Where, a, b and c are constants. Neglecting higher order terms.

$$e(t) = av(t) + bv^2(t)$$

from equation (8.8)

$$e(t) = a(e_c + e_m) + b(e_c + e_m)^2$$

$$= a(e_c + e_m) + b(e_c^2 + e_m^2 + 2e_ce_m)$$

$$= a(E_c\sin\omega_ct + E_m\sin\omega_mt) + b(E_c^2\sin^2\omega_ct + E_m^2\sin^2\omega_mt + 2E_cE_m\sin\omega_ct\sin\omega_mt)$$

$$= a(E_c\sin\omega_ct + E_m\sin\omega_mt) + bE_c^2\left(\frac{1-\cos 2\omega_Ct}{2}\right)$$

$$+ bE_m^2\left(\frac{1-\cos 2\omega_mt}{2}\right) + bE_cE_m[\cos(\omega_c - \omega_m)t - \cos(\omega_c + \omega_m)t]$$

$$= aE_c\sin\omega_ct + aE_m\sin\omega_mt + \frac{bE_C^2}{2} - \frac{bE_C^2}{2}\cos 2\omega_ct$$

$$+ \frac{bE_m^2}{2} - \frac{bE_m^2}{2}\cos 2\omega_mt + bE_cE_m\cos(\omega_c - \omega_m)t - bE_cE_m\cos(\omega_c + \omega_m)t$$

In above equation there are DC terms like $\frac{bE_C^2}{2}$, $\frac{bE_m^2}{2}$ and sinusoids of frequencies ω_m, ω_c, $2\omega_m$, $\omega_c + \omega_m$ and $\omega_c - \omega_m$.

As shown in the block diagram this signal is passed through a band pass filter which rejects DC and sinusoidal frequencies ω_m, $2\omega_m$ and $2\omega_c$ and retains the frequencies, ω_c, $\omega_c + \omega_m$, $\omega_c - \omega_m$.

$$\therefore\ e(t) = aE_c \sin\omega_c t + bE_c E_m \cos(\omega_c - \omega_m)t - bE_c E_m \cos(\omega_c + \omega_m)t \quad (8.8.1)$$

This equation is the same as equation of AM wave. This signal cannot be directly transmitted through antenna. The modulator is to be followed by a power amplifier which provides necessary power and then signal is fed to an antenna of appropriate size for efficient radiation. This radiation can propagate to a long distance.

8.9 Demodulation

When the AM waves transmitted by a radio transmitter come in contact with the receiver antenna, they are converted into electrical signals, which are still in the form of an AM wave.

Generally, in receiver this signal is amplified by an amplifier, then AM wave gets mixed with another signal of higher frequency so that carrier wave changes to a lower frequency. This frequency is called intermediate frequency (IF). IF signal is also a AM wave which contains original informations.

The main function of a receiver is to separate the signals corresponding to the information from the carrier wave. This process is called demodulation. It is a reverse process of the modulation process.

A circuit, separating these waves is termed as demodulation circuit or a detector circuit. A simple detector circuit using a diode is shown in figure 8.8.

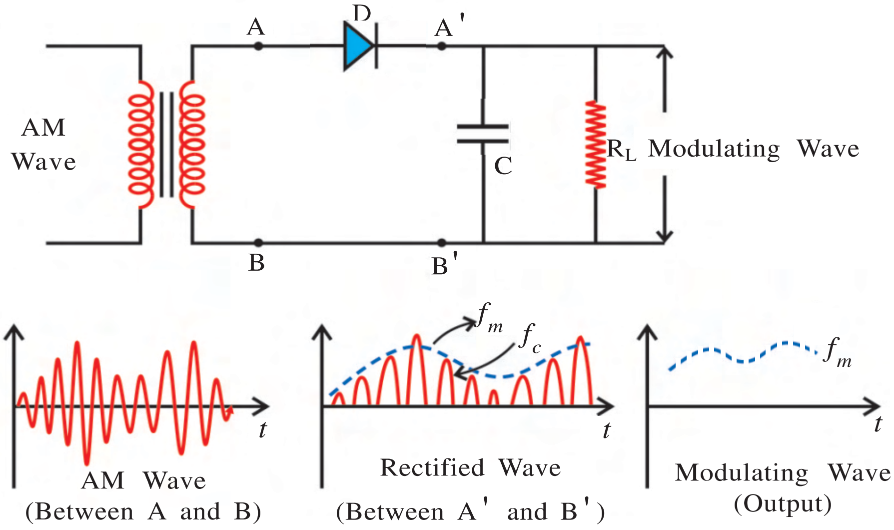


Figure 8.8 Diode Detector Circuit

The diode D acts as a rectifier in the circuit. R_L and C form a filter circuit. The current flows through the diode during the positive half cycle of the AM wave, while no current flows through during the negative half cycle of the AM wave. The rectified waves produced between points A' and B' are shown in figure 8.8.

The envelope of this rectified wave is the information signal (f_m). Using RC filter, the signal f_m is separated from carrier wave and it is given to an amplifier. The output voltage of amplifier is connected to a loud speaker which produce the original sound waves of the information.

8.10 Propagation of Electromagnetic Waves

Electromagnetic waves are used in the communication systems like radio, TV, cellphone etc. The electromagnetic waves radiated from transmitting antenna propagates in all the direction of the space with the speed of light. The earth's atmosphere plays a vital role in the propagation of waves.

There are different ways by which electromagnetic waves emitted by a transmitting antenna propagate through space and reach to the receiver.

(1) The waves propagating along the surface of the earth are called surface waves or ground waves.

(2) The waves from the transmitting antenna reaching a receiver in a straight line or after reflection from the ground are called space waves.

(3) The waves reaching the receiver at a large distance from transmitter after reflected from ionosphere at a height of about 60 km to 300 km from the surface of the earth is called sky waves.

Conductivity, permittivity, permeability and refractive index of the medium play an important role in all these types of propagations.

8.10.1 Ground Wave Propagation or Surface Wave Propagation : In the ground wave propagation, electromagnetic waves propagate along the surface of the earth. Due to different electrical properties of the earth and the atmosphere, these waves from the transmitter, follow the curved surface of the earth to reach the receiver. Ground wave propagation is possible if the transmitter and the receiver are close to the earth's surface.

Due to the electric field of the ground waves the electric charge is induced on the surface of the earth. During wave propagation. This charge is also moving, producing electrical current.

In fact, the conductivity of the earth is not infinite. The surface is sandy at some places, while it has boulders and stones etc. at other places. Thus the surface wave propagates due to mediums of different but finite conductivities. Hence the energy of the wave goes on decreasing due to finite conductivity.

In addition to this, the intensity of the electric field of the wave is inversely proportional to the distance from the sources. The absorption of the energy of the wave also depends on its frequency. Absorption of the energy is more for high frequency waves. Therefore, the waves with frequency 2 MH*z* or more do not propagate to long distance via surface waves.

The waves in MW band (frequency 550 kH*z* – 1600 kH*z*) of AM Radio propagate through ground waves.

8.10.2 Sky Wave Propagation : The propagation of radio waves (with frequency 2 MH*z* to 30 MHz) is due to sky waves. The electromagnetic waves emitted by the transmitter, return to the earth after getting reflected by the ionosphere at a height of about 80 - 300 km. A receiver at large distance can receive these reflected waves. (See figure 8.9) The ionosphere behaves as a mirror for these radio waves.

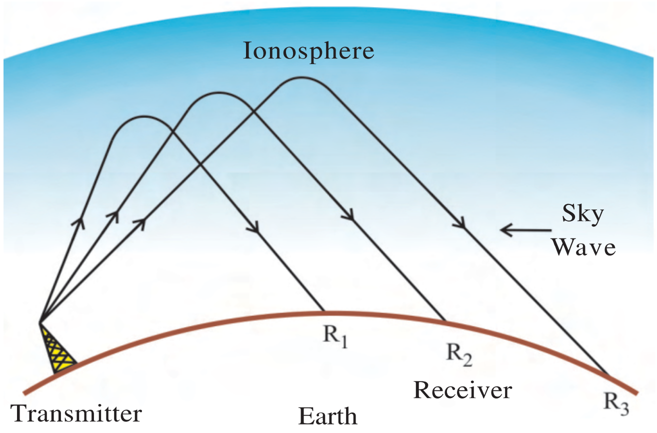


Figure 8.9 Ionosphere

The gases in the earth's atmosphere and their various compounds are present in the atmosphere at various heights, depending upon their physical properties. Whenever the energy of radiations like cosmic rays, UV rays, X-rays coming from the sun and stars is larger than the ionization potentials of the gases, the gases get ionized due to absorption of such rays. Due to this, some gas molecules get separated into electrons and positive ions. The layer of the atmosphere containing such electrons and ions is called the Ionosphere. There are different layers at various heights, depends on the density of gas, intensity of radiation and selective ionization of gases by various radiations. During daytime the ionosphere has four layers D, E, F_1 and F_2.

During day time D layer is at height of 65 - 75 km, E layer is approximately at height of 100 km, F_1 layer is at 170 - 190 km height and F_2 layer is at 250 - 400 km height.

At night time, D and E layers disappear and F_1 layer merges into F_2 layer because of the absence of the sun.

The electron density of all the four layers are different. So the radio waves with different frequencies get reflected from the ionosphere of different height. Due to total internal reflection phenomenon the frequency form 2 MH*z* to 30 MH*z* are reflected from the ionosphere and can be received at far distance from the transmitter on the earth. The frequencies above the 30 MH*z* penetrate the ionosphere and cannot be reflected from the ionosphere.

The waves with frequencies corresponding to SW (short wave) band of radio broadcasting can propagate to long distance due to the ionosphere.

8.10.3 Space Wave or Tropospheric Wave Propagation : The ionosphere cannot reflect the waves with frequency more than 30 MH*z*. The surface wave propagation is also not possible for such high frequency waves. The propagation of such high frequency waves occurs via space waves.

We can neglect the curvature of the earth if the distance between the transmitter and the receiver is small. In this case, the waves from the transmitter reach the receiver by two different paths as shown in figure 8.10.

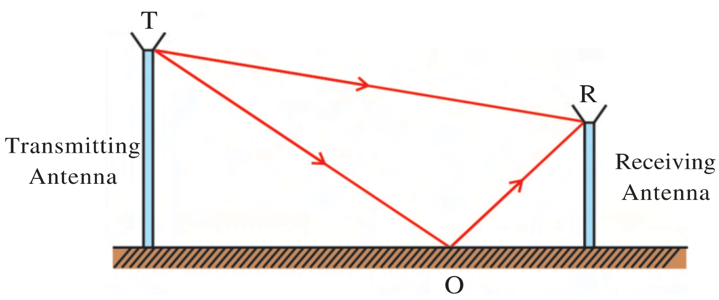


Figure 8.10

(1) The wave transmitted by antenna T, reaches receiver R, in a straight-line path (TR)

(2) The wave transmitted by antenna T, reaches receiver R after it's reflection by the earth at O (TOR).

The electric field intensity at the receiver antenna is the vector sum of the electric fields due to the two waves. The attenuation of both the waves due to space can be neglected. Their amplitudes, however, reduce and reduction is inversely proportional to the distance from the source.

If the distance between the transmitting antenna and receiver antenna is large, then the curvature of the earth comes in the way of space wave propagation. This is because such waves can be received on the earth, only up to the distance determined by the line of sight.

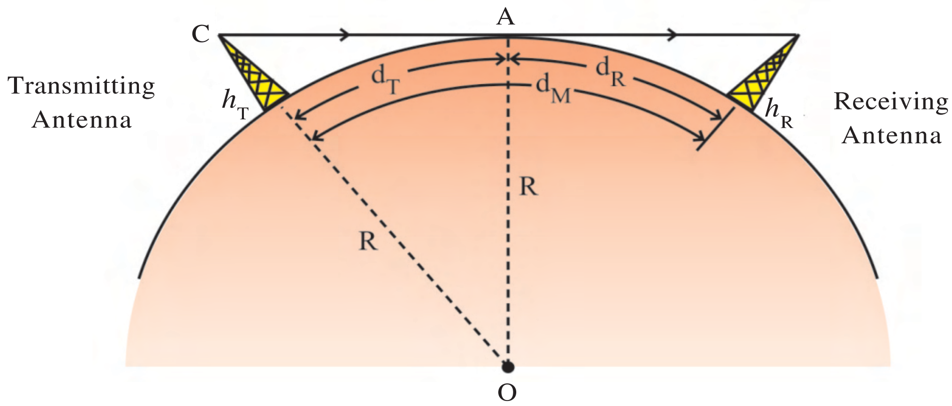


Figure 8.11 Line of Single Communication

As shown in the figure 8.11, the height of the transmitting antenna is h_T. The waves transmitted by it and travelling in a straight line can be received at a point A, a distance d_T from the antenna on the curved surface of the earth. Outside this region these waves cannot be received. This type of transmission is called line of sight (LOS) communication. d_T is called the line of sight distance or communication range or radio horizon.

Let the radius of the earth is R, then OA = R, OC = h_T + R

From the geometry of the figure,

$OC^2 = AC^2 + OA^2$

$(h_T + R)^2 = d_T^2 + R^2 \quad (\because \; h_T << d_T)$

$\therefore \; d_T^2 = h_T^2 + 2h_T R$

But $h_T << R$ hence h_T^2 can be neglected with respect to $2h_T R$.

$$d_T = \sqrt{2h_T R} \qquad (8.10.1)$$

If the receiving antenna is placed at height h_R and height of transmitting antenna is h_T, then maximum communication range is given by,

$$d_M = \sqrt{2h_T R} + \sqrt{2h_R R} \qquad (8.10.2)$$

It is clear from the equation that the line of sight distance increases if the antenna height is increased; correspondingly the region of wave transmission also increase. Now we can understand the necessity of keeping transmitting antenna at the highest possible position, for television broadcast and for microwave communication. If the transmission region has to be increased without increasing the height of the antenna, then the messages have to be relayed using repeater or booster transmitters located at the horizon corresponding to the transmitting antenna. By using many such relay stations, the transmission can be sent to far away places and the transmission region can be increased.

The waves with high frequency, like VHF band (30 MH*z* – 300 MH*z*), UHF band as well as microwaves, are propagated by space waves. The waves transmitted by a TV transmitter or by an FM radio station, propagate as space waves to reach our receiver.

Illustration 3 : Height of a TV tower is 100 m. If the average population density is $1000/km^2$. How many people can observe the programmes of this station ? (Radius of the earth $= 6.4 \times 10^6$ m)

Solution : $h_T = 100$ m, $R = 6.4 \times 10^6$ m

Population density $= 1000\ km^{-2} = 1000 \times (10^3)^{-2} = 10^{-3}\ m^{-2}$

The area of transmission region of the TV waves

$$\text{Area} = \pi(d_T)^2 = \pi(\sqrt{2h_T R})^2 = 2\pi h_T R = 2 \times 3.14 \times 100 \times 6.4 \times 10^6$$

$$= 40.192 \times 10^8\ m^2$$

The number of people can observe the TV programme,

$= 10^{-3} \times 40.192 \times 10^8$

$= 40.192 \times 10^5$ (= 40.192 laks)

Illustration 4 : A transmitting antenna at the top of a tower has a height of 50 m and the height of the receiving antenna is 32 m. What is the maximum distance between them for satisfactory communication in LOS mode ? Given radius of earth R = 6400 km.

Solution : $h_R = 32$ m, $h_T = 50$ m, $R = 6400 \times 10^3$ m

$$d_M = \sqrt{2h_T R} + \sqrt{2h_R R} = \sqrt{2 \times 50 \times 6400 \times 10^3} + \sqrt{2 \times 32 \times 6400 \times 10^3}$$

$$= 25.29 \times 10^3 + 20.23 \times 10^3 = 45.5\ \text{km}$$

SUMMARY

1. **Main elements of the communication system are :**
   (1) Transmitter (2) Transmission channel and (3) Receiver.
2. **Transducer :** The device which transforms one form of energy into another form of energy is called a transducer.
3. **Transmission Channel :** Transmission channel is a link or medium between transmitter and receiver. The signals of information are propagating through this medium and received at receiver.
4. **Noise :** Noise is an unwanted signal. They get admixed with the information signal in the transmission channel and distort the information signal.
5. **Signal :** For a transmission, the information is transformed into electric signal is called a signal. There are two types of signals. (1) Analog signal and (2) Digital signal.

6. **Bandwidth :** In communication system the difference of maximum frequency and minimum frequency efficiently transmitted by the system is called the bandwidth of the system. The bandwidth of audio signal is 20 kHz and bandwidth of video signal is 4.2 MHz.

7. **Modulation :** The process of superposing low frequency audio signals on high frequency is called modulation. Low frequency is called the modulating signal and high frequency wave is called the carrier waves. The resultant wave is known as modulated wave.

There are three types of the modulation.

(1) Amplitude modulation (AM), (2) Frequency modulation (FM) and (3) Phase modulation (PM).

8. **Amplitude Modulation :** A modulation in which the amplitude of the carrier wave is changed according to instantaneous value of the modulating wave is called amplitude modulation. Here, phase and frequency of carrier wave remain unchanged.

Modulation Index : The ratio of the amplitude of modulating wave and amplitude of carrier wave is called a modulation index.

$$m_a = \frac{E_m}{E_C}$$

Value of m_a is always less than 1.

9. **Demodulation :** The process of separating information signal from carrier wave is called demodulation. This process is performed in the receiver.

The circuit which performs the process of demodulation is called a detector circuit.

10. **Propagation of an Electromagnetic Waves :** (1) The wave propagating along the surface of the earth are called ground wave or surface wave. The frequencies up to 2 MHz is propagating through ground waves. (2) The waves returned to the earth after getting reflected by the ionosphere at a height of about 60-400 km is known as sky waves. 2 MHz to 30 MHz frequencies are propagating through the sky waves. (3) The waves from the transmitting antenna, reaching a receiver in a straight line after the reflection from the ground are called space waves.

EXERCISE

For the following statements choose the correct option from the given options :

1. For an efficient transmission of 100 MHz frequency, the minimum required length of antenna should be ....

(A) 3 m (B) $\frac{3}{4}$ m (C) 10 m (D) 100 m

2. Which of the following is not a transmission channel ?

(A) Coaxial cable (B) Optical fiber (C) free space (D) Receiver

3. If the frequency of 3 kHz signal has to be transmitted through amplitude modulation. Which of the following frequency should use as a carrier frequency ?

(A) 30 Hz (B) 300 Hz (C) 3000 Hz (D) 3 MHz

4. In which section of the communication system, the noise signal get admixed with the information signal ?

(A) Source of information (B) Transmitter

(C) Communication channel (D) Receiver

5. The bandwidth of an optical fibre communication system is approximately

(A) 10 GHz (B) 100 GHz (C) 1 GHz (D) 1 THz

6. If the height of a TV transmitter tower is doubled, then the region covered by this transmitter

(A) becomes double (B) becomes four times

(C) becomes three times (D) no change

7. In order to cover a circular region of radius 16 km, by a TV transmitter what must be the height of the transmitting antenna ? (Radius of the earth = 6400 km)

(A) 0.02 km (B) 0.2 km (C) 0.1 km (D) 2 km

8. In communication system the modulation process is performed in .... section and demodulation process is performed in .... section.

(A) Transmitter, receiver (B) receiver, transmitter

(C) Transmission channel, receiver (D) Transmitter, transmision channel

9. The range of audio signal is ....

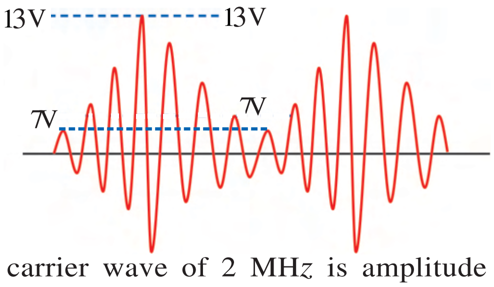
(A) 0 to 20 kHz (B) 20 Hz to 20 kHz

(C) 2 Hz to 20 kHz (D) 20 Hz to 200 kHz

10. The energy radiated from the antenna is proportional to ....

(A) λ (B) $\frac{1}{\lambda}$ (C) $\frac{1}{\lambda^2}$ (D) λ^2

11. What will be the modulation index of the following AM wave ?



(A) 6 % (B) 20 %

(C) 30 % (D) 50 %

12. A carrier wave of 2 MHz is amplitude modulated by a modulating wave of 2 kHz. Which of the following frequencies will be present in AM wave ?

(A) 2 MHz, 2 kHz (B) 2 kHz, 2.002 MHz, 1.998 MHz

(C) 2 MHz, 2.002 MHz, 1.998 MHz (D) 2.002 MHz, 1.998 MHz

ANSWERS

1. (B)	**2.** (D)	**3.** (D)	**4.** (C)	**5.** (B)	**6.** (A)
7. (A)	**8.** (A)	**9.** (B)	**10.** (C)	**11.** (C)	**12.** (C)

Answer the following questions in brief :

1. What is a transducer ? Give an example of it.
2. What is noise ? Give an example of natural noise ?
3. Name the types of the modulation.
4. Give the bandwidth of telephone communication system.
5. What is the frequency range of video signal ?
6. What is a demodulation process? In which section of communication system this process is performed ?
7. Give the name of four layers of the ionosphere.
8. What is space wave ?
9. What is a communication range ?
10. Which range of frequency can propagate through a ground wave ?

Answer the following questions :

1. Draw the block diagram of communication system and explain each block of it.
2. What is modulation ? Explain the importance of communication system in modulation.
3. Explain amplitude modulation. Draw the wave from of amplitude modulated wave.
4. How AM wave can be produced ? Discuss one of the methods to produce AM wave.
5. Write short note on ionosphere.
6. Obtain the relation between transmitter height h_T and communication range (d)

Solve the following examples :

1. What must be the height of the antenna of FM radio station so that people in a circular region of 3140 km^2 can enjoy the programme of an FM radio station ? [R = 6400 km]

   [**Ans. :** 78.125 m]

2. A TV transmitting antenna is 81 m tall. How much area can it cover to receive the programme of this station ? [**Ans. :** 3255.552 km^2]

3. The maximum amplitude of carrier wave is 12 V. What should be the amplitude of modulating signal in order to have a modulation index of 75 % ? [**Ans. :** 9 V]

4. The equation of AM wave is, $e = 100(1 + 0.6 \sin 6280t) \sin 2\pi \times 10^6 t$. Calculate, (i) Modulation index, (ii) frequency of carrier wave, (iii) frequency of modulating wave and (iv) frequency of LSB and USB.

   [**Ans. :** (i) 0.6 (ii) 1MHz (iii) 1 kHz (iv) 0.999 MHz, 1.001 MHz]

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SOLUTION

CHAPTER 1

1. The network of resistors is a balanced Wheatstone's bridge. The equivalent resistance of the network is $R^1 = 3\ \Omega$

   As the resistance of the loop is 1 Ω, the effective resistance of the circuit,

   $R = 3 + 1 = 4\ \Omega$

   emf produced in the loop, $\varepsilon = Blv$

   Current in the loop, $I = \frac{\varepsilon}{R} = \frac{Blv}{R}$

   $$\Rightarrow v = \frac{IR}{Bl}$$

2. Magnetic field is perpendicular to the coil.

   $\therefore\ \theta = 0^o$

   $\therefore$ Magnetic flux $\phi = AB\cos 0 = AB$

   Initial flux when $B_1 = 0.1\ Wbm^{-2}$ is, $\phi_1 = AB_1$

   Final flux when $B_2 = 0.2\ Wbm^{-2}$ is, $\phi_2 = AB_2$

   $\therefore$ Change in flux $\Delta\phi = \phi_2 - \phi_1 = A(B_2 - B_1)$

   Calculate average induced emf by using,

   $<\varepsilon> = N\frac{\Delta\phi}{\Delta t}$

3. (i) During the rotation from 0^o to 90^o.

   $\phi_1 = BA\cos 0^o = BA$

   $\phi_2 = BA\cos 90^o = 0$

   time $t = \frac{T}{4}$

   $\therefore$ Average induced emf,

   $$<\varepsilon> = -\frac{N\Delta\phi}{\Delta t} = -\frac{N(\phi_2 - \phi_1)}{t}$$

   $$= -\frac{N(0 - BA)}{\left(\frac{T}{4}\right)} = \frac{4NBA}{T}$$

   (ii) During the rotation from 90^o to 180^o

   $\phi_1 = BA\cos 90^o = 0,\ \phi_2 = BA\cos 180^o = -BA,\ t = \frac{T}{4}$

   $$<\varepsilon> = -\frac{N\Delta\phi}{\Delta t} = -\frac{N(0 - BA)}{\left(\frac{T}{4}\right)},$$

   $$<\varepsilon> = \frac{+4NBA}{T}$$

   Similarly, the induced emf will be $<\varepsilon> = \frac{-4NBA}{T}$ in the cases (iii) and (iv)

4. Consider an area element of length b and breadth dx at a distance x from the wire. Magnetic field produced at a distance x due to a straight current carrying long wire is,

$$B = \frac{\mu_0 I}{2\pi x}$$

∴ Magnetic flux linked with the area element is,

$$d\phi = AB = \frac{\mu_0 I}{2\pi x}(bdx)$$

Now, integrate $d\phi$ in the limits $x = a$ and $x = L + a$ to find the total magnetic flux linked with the loop l.

5. $l = 2$ m, $d = 20$ m, $B = 0.7 \times 10^{-4}$ T

Angle of dip $\phi = 60^o$

Calculate the velocity of the bar using the equation of motion $v^2 = 2gd$

$B_h = B\cos\phi = (0.7 \times 10^{-4})\cos 60^o = 0.35 \times 10^{-4}$ T

Now, use equation $\varepsilon = B_h v l$ to find the induced emf in a bar.

6. When the velocity of the rod is v, the induced emf in the rod $\varepsilon = Bvl$.

∴ Induced current in rod $I = \frac{\varepsilon}{R} = \frac{Bvl}{R}$

The force acting on the rod opposite to its motion is,

$$F_B = BIl = \frac{B^2 v_t l^2}{R}$$

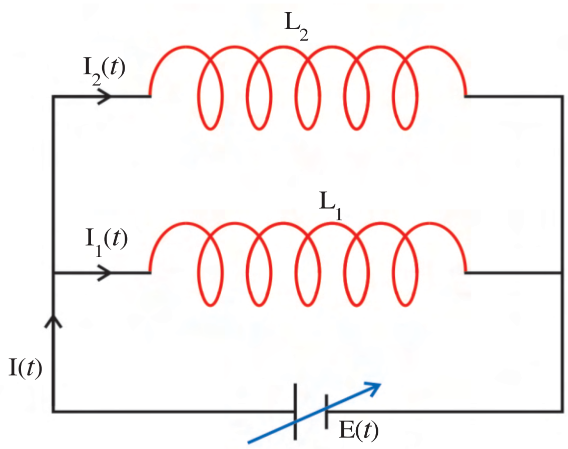
When this force becomes equal to the weight of the rod, acceleration of the rod will become zero and then after the rod continues to move with constant terminal velocity (v_t)

$$\therefore\ mg = \frac{B^2 v_t l^2}{R}$$

Calculate v_t from this equation,

7. Suppose the currents passing through inductors L_1 and L_2 at time t are I_1 and I_2, and the rates of change of currents through it are $\left(\frac{dI_1}{dt}\right)$ and $\left(\frac{dI_2}{dt}\right)$ respectively.

The p.d. across two ends of inductor L_1 is,



$$\varepsilon = -L_1\frac{dI_1}{dt} \Rightarrow \frac{dI_1}{dt} = -\frac{\varepsilon}{L_1}$$

p.d. across two ends of inductor L_2 is,

$$\varepsilon = -L_2\frac{dI_2}{dt} \Rightarrow \frac{dI_2}{dt} = -\frac{\varepsilon}{L_2}$$

If the equivalent inductance of the system of two inductors is L, then,

$\varepsilon = -L\frac{dI}{dt}$ where, I = current flowing through the main circuit at time t.

$$\rightarrow \varepsilon = -L\frac{d}{dt}(I_1 + I_2)$$

$$\rightarrow \varepsilon = -L\left(\frac{dI_1}{dt} + \frac{dI_2}{dt}\right)$$

by putting values of $\frac{dI_1}{dt}$ and $\frac{dI_2}{dt}$ find L.

8. **A-coil**

$N_A = 600$

$I_A = 3.0$ A

$\phi_A = 1.2 \times 10^{-4}$ Wb

B-coil

$N_B = 300$

$\phi_B = 9 \times 10^{-5}$ Wb

(i) Self-inductance of coil-A is,

$$L_A = \frac{\Phi_A}{I_A} = \frac{N_A\Phi_A}{I_A} = \frac{600 \times 1.2 \times 10^{-4}}{3} = 2.4 \times 10^{-2}\text{H} = 24\ m\text{H}$$

(ii) Mutual inductance of the system formed by coils A and B is,

$$M_{BA} = \frac{\Phi_B}{I_A} = \frac{9 \times 10^{-5}}{3} = 30\ \mu\text{H}$$

9. The radius of circular toroidal ring, $r_1 = 10 \times 10^{-2}$ m

The radius of cross-section of ring, $r_2 = 2 \times 10^{-2}$ m

Number of turns in the winging $N = 1.5 \times 10^4$

The magnetic field in a toroidal ring is, $B = \mu_0 nI = \frac{\mu_0 NI}{2\pi r_1}$

Total flux linked with the toroidal ring,

$$\Phi = NAB$$

$$\Phi = N(\pi r_2^2)\left(\frac{\mu_0 NI}{2\pi r_1}\right)$$

Calculate the inductance of the ring using equation $L = \frac{\phi}{I}$.

10. Consider a current I flowing through the larger loop of radius R.

The magnetic field at the center of this loop due to current is,

$$B = \frac{\mu_0 I}{2R}$$

The flux linked with the smaller loop of radius r is,

$$\Phi = AB$$

$$\Phi = (\pi r^2)\left(\frac{\mu_0 I}{2R}\right)$$

Calculate mutual inductance of the system of two loops using equation.

$$M = \frac{\Phi}{I}.$$

•

CHAPTER 2

1. Calculate the resistance of the bulb R by substituting the values of V and P (of the ratting of the bulb) in $R = \frac{V^2}{P}$ calculate maximum current, which can pass through the bulb using the values P and V (from ratting) in the expression $I = \frac{P}{V}$. This maximum current has to be passed through the bulb for safe full glowing of the bulb when it is connected with source of 220 V. For this purpose a choke coil should be connected in series with the bulb. Ideally there is no loss of power in choke (inductor) and current can be controlled. Thus, this becomes L–R A.C. circuit.

$$\therefore I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + \omega^2 L^2}}$$

Now L can be calculated from above formula by substituting the values of R, $\omega = 2\pi f$ where $f = 50$ Hz, $V_{rms} = 220$ V, I_{rms} = Maximum current found

2. For L-C-R series circuit, $|Z| = \sqrt{R^2 + (X_L - X_C)^2}$

Where $X_L = \omega L = 2\pi f L$ and $X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$

Now calculate voltage between two ends of resistance $= I_{rms}R$.

3. To tune means, to obtain resonance condition for the circuit. In this condition,

$\omega L = \frac{1}{\omega C}$ i.e. $\omega = \frac{1}{\sqrt{LC}}$

$$\therefore f = \frac{1}{2\pi}\frac{1}{\sqrt{LC}}$$

$\therefore\ C = \frac{1}{4\pi^2 f^2}\frac{1}{L}$

Now substitute $f = 800 \times 10^3$ Hz in this and find capacitance. Similarly substitute $f = 1200 \times 10^3$ Hz and find capacitance. The above two values of capacitance gives the range of the variable capacitor for tuning.

4. (1) $I_{max} = \sqrt{2}\, I_{rms} = \sqrt{2}\frac{V_{rms}}{|Z|}$ by substitution V_{rms}

$Z = \sqrt{R^2 + \omega^2 L^2}$ and $\omega = 2\pi f$ calculate I_{max}

(2) From $\tan\delta = \frac{\omega L}{R}$, find δ and using this time lag $= \frac{\delta(\text{in rad})}{\omega}$ calculate time lag.

5. (1) Calculate ε_s from $\frac{\varepsilon_s}{\varepsilon_p} = \frac{N_s}{N_p}$

(2) Find I_p using $\varepsilon_p I_p = \varepsilon_s I_s \Rightarrow I_p = \frac{\varepsilon_s}{\varepsilon_p} I_s = \frac{N_s}{N_p} I_s$

(3) Output power $= \varepsilon_s I_s$, Input power $= \varepsilon_p I_p$

6. Power $= V_{rms} I_{rms} \cos\delta$

but $I_{rms} = \frac{V_{rms}}{|Z|}$ moreover, $\cos\delta = \frac{R}{|Z|}$

$\therefore$ Power $= \frac{V_{rms}^2}{|Z|}\cos\delta$, where $|Z|^2 = R^2 + (X_L - X_C)^2$

$\therefore$ Power factor $\cos\delta = \frac{R}{|Z|}$

7. Average value of V over half cycle of periodic time

$$= \frac{1}{T/2}\int_0^{T/2} V\,dt = \frac{2}{T}\int_0^{T/2} V_m \sin\omega t\,dt$$

$$= \frac{2V_m}{T}\left[-\frac{\cos\omega t}{\omega}\right]_0^{T/2} = \frac{2V_m}{T\omega}\left[-\cos\frac{2\pi}{T}\frac{T}{2} + \cos\left(\frac{2\pi}{T}0\right)\right]$$

$$= \frac{2V_m}{T\frac{2\pi}{T}}(1 + 1)$$

$$= \frac{2V_m}{\pi}$$

8. Here $t = 0$, $V = 0$ shows voltage can be given by sin function $\therefore V = V_m \sin\omega t$ where $V_m = 100$ V is given at $t = \frac{1}{100\pi}$ s, $V = 2$ V and $\omega = 2\pi f$. From this calculate f.

9. Here $V = V_m \cos\omega t$............(i) and $I = \frac{V_m}{|Z|}\cos(\omega t - \delta)$ ....(ii) for A.C. circuit containing only inductor $|Z| = \omega L$, $\delta = \frac{\pi}{2}$ and $\omega = 2\pi f$ substituting these values in equation (iii) calculate I.

10. In $P = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos\delta$ where, $\cos\delta = \frac{R}{\sqrt{R^2 + X_C^2}}$

Substitute $X_C = 30\ \Omega$, $R = 40\ \Omega$, $V_m = 220$ V and $I_m = 4.4$ A

Calculate power and power factor.

11. Maximum current $I_m = \frac{V_m}{|Z|}$ where $|Z| = \sqrt{R^2 + (\omega L)^2}$

By $\omega = 2\pi f$ substituting corresponding values in above expressions calculate $I_{m.}$

12. Here I^2 will be

$I^2 = (I_1 \sin\omega t + I_2 \cos\omega t)^2$

On expending this and taking average

$$\therefore \langle I^2 \rangle = I_1^2 \langle \sin^2 \omega t \rangle + I_2^2 \langle \cos^2 \omega t \rangle + 2I_1I_2 \langle \sin \omega t \cos \omega t \rangle$$

Now $\langle \cos^2 \omega t \rangle = \langle \sin^2 \omega t \rangle = \frac{1}{2}$ and $\langle \sin \omega t \cos \omega t \rangle = 0$

$$\therefore \langle I^2 \rangle = \frac{I_1^2}{2} + \frac{I_2^2}{2} + 0$$

$$\therefore I_{rms} = \sqrt{\frac{I_1^2 + I_2^2}{2}}$$

13. For L-C free oscillation,

Natural angular frequency $\omega_0 = \frac{1}{\sqrt{LC}}$

Substituting the values of L and C calculate ω_0.

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CHAPTER 3

1. (a) Use $\lambda = \frac{2\pi}{k}$, $f = \frac{\omega}{2\pi}$

(b) Use $E_0 = B_0 c$

Velocity (c) of the wave is along negative X-axis, and magnetic field is positive along Y-axis. As the direction of $\vec{c}$ is according to the direction of $\vec{E} \times \vec{B}$, we get $\vec{E} = E_0 \hat{k} \equiv E_z \hat{k}$.

2. Use $I = \frac{\text{Power}}{\text{Area}} = \frac{\text{Power}}{4\pi r^2}$, $r = 1$ m

3. (a) $B_0 = \frac{E_0}{c}$,

(b) $I = \varepsilon_0 c E_{rms}^2 = \varepsilon_0 c \left(\frac{E_0}{\sqrt{2}}\right)^2$,

(c) Power = I × Area = I × $4\pi r^2$

4. $I = \varepsilon_0 c E_{rms}^2 = \frac{P_S}{4\pi r^2}$

$\therefore \; E_{rms} = \sqrt{\frac{P_S}{4\pi r^2 \varepsilon_0 c}}$

$B_{rms} = \frac{E_{rms}}{c}$

5. Intensity of wave $I = \varepsilon_0 c E_{rms}^2$

Power = I × Area

∴ Energy = Power × time (t = 1 s)

Momentum transferred (per second)

$\Delta p = \frac{\Delta U}{c}$

Radiation pressure $= \frac{\Delta P}{\text{Area}}$

6. (a) use equation $\rho_E = \frac{1}{2}\varepsilon_0 E_0^2$ for energy density.

(b) use equation $\Delta U = \rho_E \times V$ for calculating the energy contained in cylinder.

(c) For intensity of radiation use equation $I = \rho_E c$.

(d) use equation $\Delta p = \frac{\Delta U}{c}$ for momentum transformed per second.

(e) For radiation pressure use equation $p = \frac{\Delta p}{A}$.

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CHAPTER 4

1. For dark fringes, using $\frac{x_n d}{D} = (2n - 1)\frac{\lambda}{2}$, we can calculate λ.

2. For bright fringe, $\frac{x_n d}{D} = n\lambda$

For dark fringe, $\frac{x_m d}{D} = (2n - 1)\frac{\lambda}{2}$

$\therefore$ Distance between these fringes is $x_m - x_n$.

3. Using an equation $\frac{x_n d}{D} = n\lambda$.

4. $\bar{x}_1 = \frac{\lambda D}{d}$ and $\bar{x}_2 = \frac{\lambda(D + 50)}{d}$ $\therefore \lambda = \frac{(\bar{x}_2 - \bar{x}_1)d}{50}$

5. $t_2 - t_1 = nT$ (given) $= n\left(\frac{1}{f}\right)$

Path difference $= r_2 - r_1 = c(t_2 - t_1) = c \times \frac{n}{f} = n\lambda$

6. From the figure, path difference $= SS_2P - SS_1P = (SS_2 - SS_1) + (r_2 - r_1)$

$$= 0.25\lambda + \frac{xd}{D} = \frac{\lambda}{4} + \frac{xd}{D}$$

(i) For constructive interference

$$\frac{\lambda}{4} + \frac{xd}{D} = n\lambda \Rightarrow \lambda\left(n - \frac{1}{4}\right) = \frac{xd}{D}$$

(ii) For destructive interference

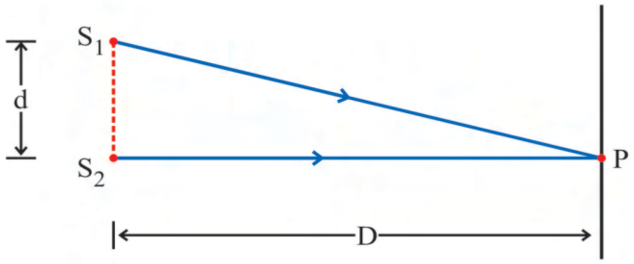
$$\frac{\lambda}{4} + \frac{xd}{D} = (2n - 1)\frac{\lambda}{2} \Rightarrow \frac{\lambda}{2}\left(2n - \frac{3}{2}\right) = \frac{xd}{D}$$

7. Using $d\sin\theta = n\lambda$, we have $\sin\theta = \frac{n}{2}$ ($\because d = 2\lambda$).

Since $\sin\theta \leq 1 \Rightarrow n \leq 2$, i.e. $n = 0$, 1 and 2.

8. Use $\frac{xd}{D} = n\lambda$

9. Here, $d << D$.



Path Difference $= (D^2 + d^2)^{\frac{1}{2}} - D$

$$= D\left(1 + \frac{d^2}{D^2}\right)^{\frac{1}{2}} - D$$

$$= D\left(1 + \frac{d^2}{2D^2}\right) - D \qquad (\because d >> D)$$

$$= \frac{d^2}{2D}$$

For destructive interferences,

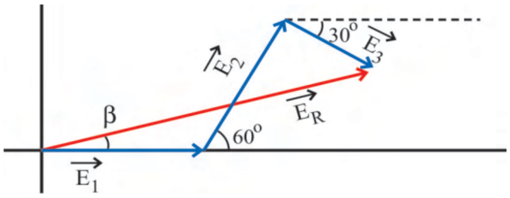
$$\frac{d^2}{2D} = (2n - 1)\frac{\lambda}{2}$$

$$\therefore \ \lambda = \frac{d^2}{D(2n-1)}$$

For, $n = 1 \Rightarrow \lambda = \frac{d^2}{D}$

$n = 2 \Rightarrow \lambda = \frac{d^2}{3D}$ etc.

10. Sum of horizontal components, $E_0 + E_0\cos60^o + E_0\cos(-30^o) = 2.37E_0$



Sum of vertical components, $0 + E_0\sin60^o + E_0\sin(-30^o) = 0.366E_0$

Now, $E_R = \sqrt{(2.37E_0)^2 + (0.366E_0)^2} = 2.4\ E_0$

and phase, $\beta = \tan^{-1}\left(\frac{0.366E_0}{2.37E_0}\right) = 8.8^o$

11. For diffraction maxima, $d\sin\theta = (2n + 1)\frac{\lambda}{2}$

12. Width of the second maxima = distance between second and third minima.
For diffraction minima, $d\sin\theta = n\lambda$
Also, for small θ (in rad), $\sin\theta \approx \tan\theta$

13. (i) Refractive index of liquid, $n_1 = \frac{\lambda}{\lambda'}$

where λ = Wavelength of light in air

λ' = Wavelength of light in liquid.

$$\therefore \ \lambda' = \frac{\lambda}{n_1} = \frac{6300\ \mathring{A}}{1.33}$$

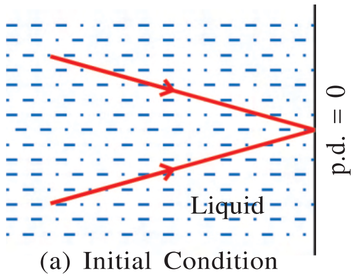
Now, $\bar{x} = \frac{\lambda' D}{d} = \frac{6300 \times 10^{-10}}{1.33 \times 10^{-3}} \times 1.33$

$\therefore \ \bar{x} = 0.63 \times 10^{-3}$ m

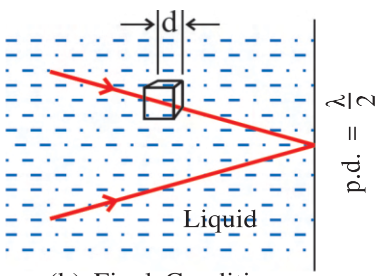
(ii) Using figure (*a*), $d = v_1 t_1$;
Here, n_1 = refractive index of liquid,
v_1 = speed of wave in liquid.
But $v_1 n_1 = C$
$\therefore \ v_1 n_1 t_1 = ct_1 = r_1$ (1)
Similarly, using figure (*b*),
$v_2 n_2 t_2 = ct_2 = r_2$ (2)
To observe first order minimum, path difference

$r_2 - r_1 = \frac{\lambda}{2}$

Using equation (1) and (2), $d = \frac{\lambda}{2(n_2 - n_1)}$.



(a) Initial Condition



(b) Final Condition

CHAPTER 5

1. For H-atom, radius of an orbit with principal quantum number n is,

$$r = \frac{n^2 h^2 \in_0}{\pi m e^2}$$

and speed of an electron in this orbit is,

$$v = \frac{e^2}{2\in_0 nh}$$

Frequency of orbiting electron,

$$f = \frac{1}{T} = \frac{v}{2\pi r} = \frac{me^4}{4\in_0^2 n^3 h^2}$$

or

$$f = \frac{me^4}{8\in_0^2 ch^3} \times \frac{2c}{n^3}$$

$$= \frac{2Rc}{n^3}$$

For $n = 2$, $f = 8.23 \times 10^{14}$ sec^{-1}

$\therefore$ Number of revolution during its life time is,

$(8.23 \times 10^{14}) \times (10^{-8}) = 8.23 \times 10^6$

2. (i) $\frac{1}{\lambda} = R\left(\frac{1}{n_k^2} - \frac{1}{n_i^2}\right)$

(ii) $E = \frac{hc}{\lambda}$

3. Using $\frac{1}{\lambda} = R = \left(\frac{1}{2^2} - \frac{1}{3^2}\right)$

4. Using $\frac{1}{\lambda} = R\left(\frac{1}{2^2} - \frac{1}{n^2}\right)$ for Balmer series, and then for Lyman series. Note that R is not given.

5. (i) Obtain dimension of fine structure constant, α using given expression.

(ii) Find its value.

(iii) For H-atom, $E_n = -\frac{me^2}{8\in_0{}^2 h^2 n^2}$. Multiplying and dividing by $4\pi^2 c^2$,

we get $E_n = \frac{-mc^2\alpha^2}{2n^2}$.

(iv) Angular momentum, $l = mvr = \frac{h}{2\pi}$

$$\therefore \; v = \frac{\hbar}{\left(\frac{mn^2h^2\epsilon_0}{\pi Ze^2 m}\right)} \quad (\because \; r = \frac{n^2h^2\epsilon_0}{\pi Ze^2 m})$$

(Using the equation of α, we get $v = \alpha c$).

6. B.E. of H-atom, $|E| = + 21.76 \times 10^{-19}$ J

Average kinetic energy of H-gas $= \frac{3}{2}k_B T$

$$\therefore \frac{3}{2}k_B T = 21.76 \times 10^{-19}$$

$$\therefore \; T = 1.05 \times 10^5 \text{ K}$$

7. $E = \frac{-me^4}{8\epsilon_0^2 h^2} \frac{Z^2}{n^2}$

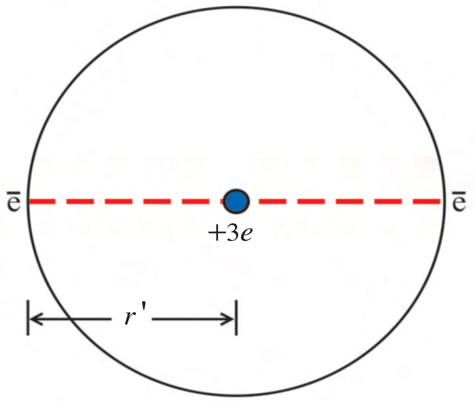
For He^+ ion, Z = 2, n = 1.

Also, $\frac{-me^4}{8\epsilon_0^2 h^2} = 13.6 \; eV$ (known).

8. In order to emit H_β-line (i.e., $n = 4 \rightarrow n = 2$), electron should be first excited to $n = 4$ orbit.

Thus energy difference of these two orbits should be supplied.

9. Total energy of Li^+ ion is given as,



$$E_{tot} = 2 \times \left[\frac{1}{2}mv^2 - \frac{3e^2}{4\pi\epsilon_0 r'}\right] + \frac{e^2}{4\pi\epsilon_0 (2r')}$$

Using $\frac{1}{2}mv^2 = \frac{1}{8\pi\epsilon_0}\frac{3e^2}{r'}$

and

$$r' = \frac{n^2h^2\epsilon_0}{\pi m Ze^2}$$

$$\therefore \; E_{tot} = \frac{-15}{n^2}\left(\frac{me^4}{8\epsilon_0^2 h^2}\right) = -204 \; eV \; (\because \; n = 1 \text{ and } \frac{me^4}{8\epsilon_0^2 h^2} = 13.6 \; eV)$$

$E_{tot}^{exp} = 198.09 \; eV$ (given) $\therefore$ % error = 2.98%

10. Total energy, $E = \frac{1}{2}mv^2 + \frac{1}{2}m\omega^2 r^2 \; (\because \; k = m\omega^2) = mv^2 \; (\because \; v = r\omega)$

Angular momentum, $mvr = n\frac{h}{2\pi}$ $\therefore$ $mv^2 = n$ २ $\frac{v}{r}$ $\therefore$ E = n २ $\frac{v}{r}$ = n २ ω.

11. Using Moseley's law, for K_α line, $\frac{1}{\lambda} = R(Z - 1)^2\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$

12. K_α line corresponds to electron transition from L-shell to K-shell. Corresponding wavelength

$$\frac{hc}{\lambda_{k_\alpha}} = (78 \times 10^3 - 12 \times 10^3) \times (1.6 \times 10^{-19}) \text{ J} \quad \therefore \lambda_{k\alpha} = 0.188 \text{ Å}$$

K_β line corresponds to electron transition from M-shell to K-shell which gives $\lambda_{k_\beta} = 0.165$ Å.

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CHAPTER 6

1. In a process if the total binding energy is increased then energy is produced (released !) in that process.

   (*a*) For Y $\rightarrow$ 2Z process,

   total binding energy of Y = 8.5 × 60 = 510 M*e*V

   total binding energy of 2Z = 2(5.0 × 30) = 300 M*e*V.

   Here, total binding energy is decreased. Hence energy is not emitted.

   (*b*) For W $\rightarrow$ 2Y process,

   total binding energy of W = 8 × 120 = 960 M*e*V

   total binding energy of 2Y = 2(8.5 × 60) = 1020 M*e*V.

   Here, total binding energy is increased, hence energy is emitted.

2. The total decay constant corresponding to both types of emission is $\lambda_t = \lambda_\alpha + \lambda_\beta$

$$\therefore \lambda_t = \frac{1}{1600} + \frac{1}{400} = \frac{1}{320} \text{ Yr}^{-1} \quad \therefore \tau_{\frac{1}{2}}(\text{total}) = \frac{0.693}{\lambda_t} = \frac{0.693}{1/320} = 221.76 \text{ Yr}$$

When 75% of nuclei disintegrate, 25% remain

$$\therefore \frac{N}{N_0} = \frac{25}{100} = \frac{1}{4} \qquad \therefore \frac{1}{4} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$$

$$\frac{N}{N_0} = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}} \qquad \left(\frac{1}{2}\right)^2 = \left(\frac{1}{2}\right)^{\frac{t}{\tau_{1/2}}}$$

$$\therefore \frac{t}{\tau_{1/2}} = 2 \quad \therefore t = 2(\tau_{\frac{1}{2}}) = 443.52 \text{ Yr}$$

3. $$\begin{matrix}\text{Total energy} \\ \text{at far distance}\end{matrix} \left(\begin{matrix}\text{Kinetic} \\ \text{energy}\end{matrix} + \begin{matrix}\text{Potential} \\ \text{energy}\end{matrix}\right) = \begin{matrix}\text{Total energy at} \\ \text{minimum distance}\end{matrix} \left(\begin{matrix}\text{Kinetic} \\ \text{energy}\end{matrix} + \begin{matrix}\text{Potential} \\ \text{energy}\end{matrix}\right)$$

$$2\left(\frac{1}{2}mv^2\right) + 0 = 0 + \frac{kq^2}{r_0}$$

$$2(1.8 \times 10^3 \times 1.6 \times 10^{-19}) = \frac{(9 \times 10^9)(1.6 \times 10^{-19})^2}{r_0} \quad \therefore r_0 = 4 \times 10^{-14} \text{ m}$$

4. If half life is x hr; then

at 0 time activity	=	16000	counts/min
after x hr activity	=	8000	counts/min
after $2x$ hr activity	=	4000	counts/min
after $3x$ hr activity	=	2000	counts/min
after $4x$ hr activity	=	1000	counts/min
after $5x$ hr activity	=	500	counts/min

$\therefore 5x = 240$ min $\therefore x = 48$ min

5. In 226 g Radium 6.02×10^{23} atoms present.

$\therefore$ in 1 g $\quad \dfrac{6.02\times10^{23}}{226} = N$ atoms present

$$\tau_{\frac{1}{2}} = \frac{0.693}{\lambda} \qquad \therefore \lambda = \frac{0.693}{\tau_{\frac{1}{2}}} = \frac{0.693}{4.98\times10^{10}} \text{ s}^{-1}$$

$$I = \lambda N = \left(\frac{0.693}{4.98\times10^{10}} \times \frac{6.02\times10^{23}}{226}\right) = 3.7 \times 10^{10} \text{ disintegration / s} = 1 \text{ C}i$$

6. Total mass of all nucleons in the free state $= Zm_p + Nm_n = 17 \times 1.00783 + 18 \times 1.00866 = 35.28899\ u$.

mass defect $= \Delta m = Zm_p + Nm_n - M_{nucleus} = 35.28899 - 34.9800 = 0.30899\ u$.

$\therefore$ Binding energy $= 0.30899 \times 931 = 287.66$ MeV

$\therefore$ Binding energy per nucleon $= \dfrac{287.66}{35} = 8.219 \dfrac{\text{M}e\text{V}}{\text{nucleon}}$

7. $R = R_0 A^{\frac{1}{3}} \quad \therefore (6.6\ fm) = (1.1\ fm) A^{\frac{1}{3}} \quad \therefore A = 216$

$\therefore$ Mass of nucleus $= 216 \times 1.0088\ u = 216 \times 1.0088 \times 1.66 \times 10^{-27}$ kg

Volume of nucleus $= \dfrac{4}{3}\pi R^3 = \dfrac{4}{3}(3.14)(6.6 \times 10^{-15})^3$ m^3

$\therefore$ Density of nucleus $\rho = \dfrac{\text{mass}}{\text{volume}} = \dfrac{(216)(1.008)(1.66\times10^{-27})}{\left(\frac{4}{3}\right)(3.14)(6.6\times10^{-15})^3} = 3 \times 10^{17}$ kg/m^3

8. $I = \lambda N \Rightarrow 8000 = \lambda(8 \times 10^7) \quad \therefore \lambda = 10^{-4}$ s^{-1},

$$\tau_{\frac{1}{2}} = \frac{0.693}{\lambda} = \frac{0.639}{10^{-4}} = 6930 \text{ s.}$$

9. Number of nuclei in 2 g of ${}_1H^2 = 6.02 \times 10^{23}$

$\therefore$ Number of nuclei in 1000 g of ${}_1H^2 = \dfrac{6.02\times10^{23}\times1000}{2} = 3.01 \times 10^{26}$

By fusion of 2 ${}_1H^2$ nuclei $3.27 \times 10^6 \times 1.6 \times 10^{-19}$ J energy is obtained.

$\therefore$ By fusion of 3.01×10^{26} ${}_1H^2$ nuclei the energy obtained $= \dfrac{3.27\times10^6\times1.6\times10^{-19}\times3.01\times10^{26}}{2}$ J

If a bulb of 100 W glows for t second, then energy consumed is $= (100)(t)$ J

$$\therefore 100t = \frac{3.27\times10^6\times1.6\times10^{-19}\times3.01\times10^{26}}{2} \quad \therefore t = \frac{7.874\times10^{11}\text{ s}}{3.16\times10^7\ s/year} = 24917 \text{ Yr}$$

CHAPTER 7

1. $n_e = 6 \times 10^{19} m^{-3}$, volume $= 10^{-2} \times 10^{-2} \times 2 \times 10^{-2} = 2 \times 10^{-6}$ m^{-3}

For intrinsic semiconductor, $n_e = n_h = 6 \times 10^{19}$ m^{-3}

$\therefore$ no. of holes in volume of $= 2 \times 10^{-3}$ m^{-3} $= n_h \times$ volume $= 6 \times 10^{19} \times 2 \times 10^{-6}$

$= 12 \times 10^{13}$

2. $n_i = 1.5 \times 10^{16}$ m^{-3}

Majority charge carrier $n_h = 4.5 \times 10^{22}$ m^{-3}, Minority charge carrier n_e = ?

Now use, $n_i^2 = n_e n_h$ and calculate n_e.

3. Use $E_g = \frac{hc}{\lambda}$ **4.** Use $E_g = \frac{hc}{\lambda}$

5. d = 400 nm, E = 5×10^5 V/m,

Potential barrier $V_0 = Ed = 5 \times 10^5 \times 4 \times 10^{-7} = 0.2$ V

Minimum energy of free electron = V_0 = 0.2 eV.

6. (1) For $V_A > V_B$, diode D_1 will be in forward bias and D_2 will be in reverse bias. Therefore, resistance between A and B will be $R_{AB} = 50\ \Omega$.

(2) For $V_B > V_A$, D_1 will be in reverse bias and D_2 will be in forward bias.

$\therefore$ Resistance between A and B will be $R_{AB} = 50\ \Omega$.

7. $R_L = 10\ k\ \Omega$, $A_V = 200$, $r_i = 10\ k\ \Omega$

(1) From $A_V = -g_m R_L$, calculate g_m. (2) Use $g_m = \frac{\beta ac}{r_i} = \frac{A_i}{r_i}$ and calculate A_i

8. I_C = 18.6 mA, I_C = ?, α = ?. From $I_C = 0.93\ I_E$, calculate I_E and from $I_B = I_E - I_C$ calculate I_B then use $\alpha = \frac{I_C}{I_E}$ to calculate α.

9. $\Delta V_{BE} = 200 \times 10^{-3}$ V, $\Delta V_{CE} = 200\ \mu A$, r_i = ?, A_V = ?

$$r_i = \frac{\Delta V_{BE}}{\Delta I_B} = \frac{200 \times 10^{-3}}{200 \times 10^{-6}} = 1000\ \Omega \qquad A_V = \frac{\Delta V_{CE}}{\Delta V_{BE}} = \frac{2}{200 \times 10^{-3}} = 10$$

10. Power gain $A_P = A_V A_i = (-g_m R_L)\ A_i = \left(\frac{\Delta I_C}{\Delta V_{BE}}\right) R_L A_i$. Now calculate R_L.

11. For input circuit, $V_{BB} = I_B R_B + V_{BE}$. Use this equation to calculate I_B

For output circuit, $V_{CC} = V_{CE} + I_C R_L$. Use this equation to calculate I_C.

Now, current gain $A_i = \frac{I_C}{I_B}$.

12. For input circuit, $V_{CC} - I_B R_B - V_{BE} = 0$

$\therefore V_{BE} = V_{CC} - I_B R_B = 6 - 5 \times 10^{-6} (1 \times 10^6) = 1$ V

For output circuit $V_{CC} - I_C R_L - V_{CE} = 0$

$\therefore V_{CE} = V_{CC} - I_C R_L = 6 - (5 \times 10^{-3} \times 1.1 \times 10^3) = 0.5$ V

13. $A_P = A_V A_i = g_m R_L A_i \;\; \therefore \; A_P = \dfrac{\beta ac}{r_i} R_L A_i \;\;\; \therefore \; R_L = \dfrac{A_P r_i}{A_i A_i} = \dfrac{2000 \times 1000}{100 \times 100} = 200 \; \Omega$

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CHAPTER 8

1. Area $A = \pi d_T^2 = \pi(2h_T R)$

Height of antenna $h_T = \dfrac{A}{2\pi R} = \dfrac{3140}{2 \times 3.14 \times 6400} = 0.078125$ km $= 78.125$ m

2. Area $A = \pi d_T^2 = \pi(2h_T R) = 3.14 \times 2 \times 81 \times 6400 \times 10^3$

$= 3255552 \times 10^3 \text{ m}^2 = 3255.552 \text{ km}^2$

3. $E_C = 12$ V, $m_a = 0.75$, $E_m = ?$, $m_a = \dfrac{E_m}{E_C} \;\; \therefore \; E_m = m_a \times E_C = 0.75 \times 12 = 9$ V

4. $e = 100(1 + 0.6 \sin 6280t)\sin 2\pi 10^6 t$ compares with $e = E_C(1 + m_a \sin \omega_m t)\sin\omega_c t$

$m_a = 0.6$, $\omega_m = 6280$ rad/s, $\omega_c = 2\pi \times 10^6$ rad/s

$\therefore f_m = \dfrac{\omega_m}{2\pi} = \dfrac{6280}{2\pi} = 10^3 \text{ Hz} = 1 \text{ kHz}$

$f_c = \dfrac{\omega_C}{2\pi} = \dfrac{2\pi \times 10^6}{2\pi} = 10^6 \text{ Hz} = 1000 \text{ kHz}$

Frequency of LSB $= f_c - f_m = 1000 - 1 = 999$ kHz

Frequency of LSB $= f_c + f_m = 1000 + 1 = 1001$ kHz

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LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference 1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5566	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6345	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

LOGARITHMS

	0	1	2	3	4	5	6	7	8	9	Mean Difference 1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7769	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9843	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Antilogarithms

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	2	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	2	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1289	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Antilogarithms

	0	1	2	3	4	5	6	7	8	9	Mean Difference								
											1	2	3	4	5	6	7	8	9
50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	6	6	7
54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	7	7
56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	9	10
67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
82	6607	6622	6637	6653	6668	6683	6699	6715	6730	6745	2	3	5	6	8	9	11	12	14
83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	3	5	7	9	10	12	14	16
88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	12	14	16
90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	15	17	19
96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20
	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

NATURAL SINES

Degree	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Deifferences				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1'	2'	3'	4'	5'
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0366	0384	0401	0419	0436	0454	0471	0488	0506	3	6	9	12	15
3	.0523	0541	0558	0576	0593	0610	0628	0645	0663	0680	3	6	9	12	15
4	.0698	0715	0732	0750	0767	0785	0802	0819	0837	0854	3	6	9	12	14
5	.0872	0889	0906	0924	0941	0958	0976	0993	1011	1028	3	6	9	12	14
6	.1045	1063	1080	1197	1115	1132	1149	1167	1184	1201	3	6	9	12	14
7	.1219	1236	1253	1271	1288	1305	1323	1340	1357	1374	3	6	9	12	14
8	.1392	1409	1426	1444	1461	1478	1495	1513	1530	1547	3	6	9	12	14
9	.1564	1582	1599	1616	1633	1650	1668	1685	1702	1719	3	6	9	12	14
10	.1736	1754	1771	1788	1805	1822	1840	1857	1874	1891	3	6	9	11	14
11	.1908	1925	1942	1959	1977	1994	2011	2028	2045	2062	3	6	9	11	14
12	.2079	2096	2113	2130	2147	2164	2181	2198	2215	2233	3	6	9	11	14
13	.2250	2267	2284	2300	2317	2334	2351	2368	2385	2402	3	6	8	11	14
14	.2419	2436	2453	2470	2487	2504	2521	2538	2554	2571	3	6	8	11	14
15	.2588	2605	2622	2639	2656	2672	2689	2706	2723	2740	3	6	8	11	14
16	.2756	2773	2790	2807	2823	2840	2857	2874	2890	2907	3	6	8	11	14
17	.2924	2940	2957	2974	2990	3007	3024	3040	3057	3074	3	6	8	11	14
18	.3090	3107	3123	3140	3156	3173	3190	3206	3223	3239	3	6	8	11	14
19	.3256	3272	3289	3305	3322	3338	3355	3371	3387	3404	3	5	8	11	14
20	.3420	3437	3453	3469	3486	3502	3518	3535	3551	3567	3	5	8	11	14
21	.3584	3600	3616	3633	3649	3665	3681	3697	3714	3730	3	5	8	11	14
22	.3746	3762	3778	3795	3811	3827	3843	3859	3875	3891	3	5	8	11	14
23	.3907	3923	3939	3955	3971	3987	4003	4019	4035	4051	3	5	8	11	14
24	.4067	4083	4099	4115	4131	4147	4163	4179	4195	4210	3	5	8	11	13
25	.4226	4242	4258	4274	4289	4305	4321	4337	4352	4368	3	5	8	11	13
26	.4384	4399	4415	4431	4446	4462	4478	4493	4509	4524	3	5	8	10	13
27	.4540	4555	4571	4586	4602	4617	4633	4648	4664	4679	3	5	8	10	13
28	.4695	4710	4726	4741	4756	4772	4787	4802	4818	4833	3	5	8	10	13
29	.4848	4863	4879	4894	4909	4924	4939	4955	4970	4985	3	5	8	10	13
30	.5000	5015	5030	5045	5060	5075	5090	5105	5120	5135	3	5	8	10	13
31	.5150	5165	5180	5195	5210	5225	5240	5255	5270	5284	2	5	7	10	12
32	.5299	5314	5329	5344	5358	5373	5388	5402	5417	5432	2	5	7	10	12
33	.5446	5461	5476	5490	5505	5519	5534	5548	5563	5577	2	5	7	10	12
34	.5592	5606	5621	5635	5650	5664	5678	5693	5707	5721	2	5	7	10	12
35	.5736	5750	5764	5779	5793	5807	5821	5835	5850	5864	2	5	7	9	12
36	.5878	5892	5906	5920	5934	5948	5962	5976	5990	6004	2	5	7	9	12
37	.6018	6032	6046	6060	6074	6088	6101	6115	6129	6143	2	5	7	9	12
38	.6157	6170	6184	6198	6211	6225	6239	6252	6266	6280	2	5	7	9	11
39	.6293	6307	6320	6334	6347	6361	6374	6388	6401	6414	2	4	7	9	11
40	.6428	6441	6455	6468	6481	6494	6508	6521	6534	6547	2	4	7	9	11
41	.6561	6574	6587	6600	6613	6626	6639	6652	6665	6678	2	4	7	9	11
42	.6691	6704	6717	6730	6743	6756	6769	6782	6794	6807	2	4	6	9	11
43	.6820	6833	6845	6858	6871	6884	6896	6909	6921	6934	2	4	6	8	11
44	.6947	6959	6972	6984	6997	7009	7022	7034	7046	7059	2	4	6	8	10

NATURAL SINES

Degree	0'	6'	12'	18'	24'	30'	36'	42'	48'	54'	Mean Deifferences				
	0.0°	0.1°	0.2°	0.3°	0.4°	0.5°	0.6°	0.7°	0.8°	0.9°	1'	2'	3'	4'	5'
45	.7071	7083	7096	7108	7120	7133	7145	7157	7169	7181	2	4	6	8	10
46	.7193	7206	7218	7230	7242	7254	7266	7278	7290	7302	2	4	6	8	10
47	.7314	7325	7337	7349	7361	7373	7385	7396	7408	7420	2	4	6	8	10
48	.7431	7443	7455	7466	7478	7490	7501	7513	7524	7536	2	4	6	8	10
49	.7547	7559	7570	7581	7593	7604	7615	7627	7638	7649	2	4	6	8	9
50	.7660	7672	7683	7694	7705	7716	7727	7738	7749	7760	2	4	6	7	9
51	.7771	7782	7793	7804	7815	7826	7837	7848	7859	7869	2	4	5	7	9
52	.7880	7891	7902	7912	7923	7934	7944	7955	7965	7976	2	4	5	7	9
53	.7986	7997	8007	8018	8028	8039	8049	8059	8070	8080	2	3	5	7	9
54	.8090	8100	8111	8121	8131	8141	8151	8161	8171	8181	2	3	5	7	8
55	.8192	8202	8211	8221	8231	8241	8251	8261	8271	8281	2	3	5	7	8
56	.8290	8300	8310	8320	8329	8339	8348	8358	8368	8377	2	3	5	6	8
57	.8387	8396	8406	8415	8425	8434	8443	8453	8462	8471	2	3	5	6	8
58	.8480	8490	8499	8508	8517	8526	8536	8545	8554	8563	2	3	5	6	8
59	.8572	8581	8590	8599	8607	8616	8625	8634	8643	8652	1	3	4	6	7
60	.8660	8669	8678	8686	8695	8704	8712	8721	8729	8738	1	3	4	6	7
61	.8746	8755	8763	8771	8780	8788	8796	8805	8813	8821	1	3	4	6	7
62	.8829	8838	8846	8854	8862	8870	8878	8886	8894	8902	1	3	4	5	7
63	.8910	8918	8926	8934	8942	8949	8957	8965	8973	8980	1	3	4	5	6
64	.8988	8996	9003	9011	9018	9026	9033	9041	9048	9056	1	3	4	5	6
65	.9063	9070	9078	9085	9092	9100	9107	9114	9121	9128	1	2	4	5	6
66	.9135	9143	9150	9157	9164	9171	9178	9184	9191	9198	1	2	3	5	6
67	.9205	9212	9219	9225	9232	9239	9245	9252	9259	9265	1	2	3	4	6
68	.9272	9278	9285	9291	9298	9304	9311	9317	9323	9330	1	2	3	4	5
69	.9336	9342	9348	9354	9361	9367	9373	9379	9385	9391	1	2	3	4	5
70	.9397	9403	9409	9415	9421	9426	9432	9438	9444	9449	1	2	3	4	5
71	.9455	9461	9466	9472	9478	9483	9489	9494	9500	9505	1	2	3	4	5
72	.9511	9516	9521	9527	9532	9537	9542	9548	9553	9558	1	2	3	3	4
73	.9563	9568	9573	9578	9583	9588	9593	9598	9603	9608	1	2	2	3	4
74	.9613	9617	9622	9627	9632	9636	9641	9646	9650	9655	1	2	2	3	4
75	.9659	9664	9668	9673	9677	9681	9686	9690	9694	9699	1	1	2	3	4
76	.9703	9707	9711	9715	9720	9724	9728	9732	9736	9740	1	1	2	3	3
77	.9744	9748	9751	9755	9759	9763	9767	9770	9774	9778	1	1	2	3	3
78	.9781	9785	9789	9792	9796	9799	9803	9806	9810	9813	1	1	2	2	3
79	.9816	9820	9823	9826	9829	9833	9836	9839	9842	9845	1	1	2	2	3
80	.9848	9851	9854	9857	9860	9863	9866	9869	9871	9874	0	1	1	2	2
81	.9877	9880	9882	9885	9888	9890	9893	9895	9898	9900	0	1	1	2	2
82	.9903	9905	9907	9910	9912	9914	9917	9919	9921	9923	0	1	1	2	2
83	.9925	9928	9930	9932	9934	9936	9938	9940	9942	9943	0	1	1	1	2
84	.9945	9947	9949	9951	9952	9954	9956	9957	9959	9960	0	1	1	1	2
85	.9962	9963	9965	9966	9968	9969	9971	9972	9973	9974	0	0	1	1	1
86	.9976	9977	9978	9979	9980	9981	9982	9983	9984	9985	0	0	1	1	1
87	.9986	9987	9988	9989	9990	9990	9991	9992	9993	9993	0	0	0	1	1
88	.9994	9995	9995	9996	9996	9997	9997	9997	9998	9998	0	0	0	0	0
89	.9998	9999	9999	9999	9999	1.000	1.000	1.000	1.000	1.000	0	0	0	0	0

NATURAL TANGENTS

Degree	0 0.0°	6 0.1°	12 0.2°	18 0.3°	24 0.4°	30 0.5°	36 0.6°	42 0.7°	48 0.8°	54 0.9°	Main Deifferences 1	2	3	4	5
0	.0000	0017	0035	0052	0070	0087	0105	0122	0140	0157	3	6	9	12	15
1	.0175	0192	0209	0227	0244	0262	0279	0297	0314	0332	3	6	9	12	15
2	.0349	0367	0384	0402	0419	0437	0454	0472	0489	0507	3	6	9	12	15
3	.0524	0542	0559	0577	0594	0612	0629	0647	0664	0682	3	6	9	12	15
4	.0699	0717	0734	0752	0769	0787	0805	0822	0840	0857	3	6	9	12	15
5	.0875	0892	0910	0928	0945	0963	0981	0998	1016	1033	3	6	9	12	15
6	.1051	1069	1086	1104	1122	1139	1157	1175	1192	1210	3	6	9	12	15
7	.1228	1246	1263	1281	1299	1317	1334	1352	1370	1388	3	6	9	12	15
8	.1405	1423	1441	1459	1477	1495	1512	1530	1548	1566	3	6	9	12	15
9	.1584	1602	1620	1638	1655	1673	1691	1709	1727	1745	3	6	9	12	15
10	.1763	1781	1799	1817	1835	1853	1871	1890	1908	1926	3	6	9	12	15
11	.1944	1962	1980	1998	2016	2035	2053	2071	2089	2107	3	6	9	12	15
12	.2126	2144	2162	2180	2199	2217	2235	2254	2272	2290	3	6	9	12	15
13	.2309	2327	2345	2364	2382	2401	2419	2438	2456	2475	3	6	9	12	15
14	.2493	2512	2530	2549	2568	2686	2605	2623	2642	2661	3	6	9	12	16
15	.2679	2698	2717	2736	2754	2773	2792	2811	2830	2849	3	6	9	13	16
16	.2867	2886	2905	2924	2943	2962	2981	3000	3019	3038	3	6	9	13	16
17	.3057	3076	3096	3115	3134	3153	3172	3191	3211	3230	3	6	10	13	16
18	.3249	3269	3288	3307	3327	3346	3365	3385	3404	3424	3	6	10	13	16
19	.3443	3463	3482	3502	3522	3541	3561	3581	3600	3620	3	7	10	13	16
20	.3640	3659	3679	3699	3719	3739	3759	3779	3799	3819	3	7	10	13	17
21	.3839	3859	3879	3899	3919	3939	3959	3979	4000	4020	3	7	10	13	17
22	.4040	4061	4081	4101	4122	4142	4163	4183	4204	4224	3	7	10	14	17
23	.4245	4265	8286	4307	4327	4748	4369	4390	4411	4431	3	7	10	14	17
24	.4452	4473	4494	4515	4536	4557	4578	4599	4621	4642	4	7	11	14	18
25	.4663	4684	4706	4727	4748	4770	4791	4813	4834	4856	4	7	11	14	18
26	.4877	4899	4921	4942	4964	4986	5008	5029	5051	5073	4	7	11	15	18
27	.5095	5117	5139	5161	5184	5206	5228	5250	5272	5295	4	7	11	15	18
28	.5317	5340	5362	5384	5407	5430	5452	5475	5498	5520	4	8	11	15	19
29	.5543	5566	5589	5612	5635	5658	5681	5704	5727	5750	4	8	12	15	19
30	.5774	5797	5820	5844	5867	5890	5914	5938	5961	5985	4	8	12	16	20
31	.6009	6032	6056	6080	6104	6128	6152	6176	6200	6224	4	8	12	16	20
32	.6249	6273	6297	6322	6346	6371	6395	6420	6445	6469	4	8	12	16	20
33	.6494	6519	6544	6569	6594	6619	6644	6669	6694	6720	4	8	13	17	21
34	.6745	6771	6796	6822	6847	6873	6899	6924	6950	6976	4	9	13	17	21
35	.7002	7028	7054	7080	7107	7133	7159	7186	7212	7239	4	9	13	18	22
36	.7265	7292	7319	7346	7373	7400	7427	7454	7481	7508	5	9	14	18	23
37	.7536	7563	7590	7618	7646	7673	7701	7729	7757	7785	5	9	14	18	23
38	.7813	7841	7869	7898	7926	7954	7983	8012	8040	8069	5	9	14	19	24
39	.8098	8127	8156	8185	8214	8243	8273	8302	8332	8361	5	10	15	20	24
40	.8391	8421	8451	8481	8511	8541	8571	8601	8632	8662	5	10	15	20	25
41	.8693	8724	8754	8785	8816	8847	8878	8910	8941	8972	5	10	16	21	26
42	.9004	9036	9067	9099	9131	9163	9195	9228	9260	9293	5	11	16	21	27
43	.9325	9358	9391	9424	9457	9490	9523	9556	9590	9623	6	11	17	22	28
44	.9657	9691	9725	9759	9793	9827	9861	9896	9930	9965	6	11	17	23	29

NATURAL TANGENTS

Degree	0 0.0°	6 0.1°	12 0.2°	18 0.3°	24 0.4°	30 0.5°	36 0.6°	42 0.7°	48 0.8°	54 0.9°	Main Deifferences 1	2	3	4	5
45	1.0000	0035	0070	0105	0141	0176	0212	0247	0283	0319	6	12	18	24	30
46	1.0355	0392	0428	0464	0501	0538	0575	0612	0649	0686	6	12	18	25	31
47	1.0724	0761	0799	0837	0875	0913	0951	0990	1028	1067	6	13	19	25	32
48	1.1106	1145	1184	1224	1263	1303	1343	1383	1423	1463	7	13	20	27	33
49	1.1504	1544	1585	1626	1667	1708	1750	1792	1833	1875	7	14	21	28	34
50	1.1918	1960	2002	2045	2088	2131	2174	2218	2261	2305	7	14	22	29	36
51	1.2349	2393	2437	2482	2527	2572	2617	2662	2708	2753	8	15	23	30	38
52	1.2799	2846	2892	2938	2985	3032	3079	3127	3175	3222	8	16	24	31	39
53	1.3270	3319	3367	3416	3465	3514	3564	3613	3663	3713	8	16	25	33	41
54	1.3764	3814	3865	3916	3968	4019	4071	4124	4176	4229	9	17	26	34	43
55	1.4281	4335	4388	4442	4496	4550	4605	4659	4715	4770	9	18	27	36	45
56	1.4826	4882	4938	4994	5051	5108	5166	5224	5282	5340	10	19	29	38	48
57	1.5399	5458	5517	5577	5637	5697	5757	5818	5880	5941	10	20	30	40	50
58	1.6003	6066	6128	6191	6255	6319	6383	6447	6512	6577	11	21	32	43	53
59	1.6643	6709	6775	6842	6909	6977	7045	7113	7182	7251	11	23	34	45	56
60	1.7321	7391	7461	7532	7603	7675	7747	7820	7893	7966	12	24	36	48	60
61	1.8040	8115	8190	8265	8341	8418	8495	8572	8650	8728	13	26	38	51	64
62	1.8807	8887	8967	9047	9128	9210	9292	9375	9458	9542	14	27	41	55	68
63	1.9626	9711	9797	9883	9970	2.0057	2.0145	2.0233	2.0323	2.413	15	29	44	58	73
64	2.0503	0594	0686	0778	0872	0965	1060	1155	1251	1348	16	31	47	63	78
65	2.1445	1543	1642	1742	1842	1943	2045	2148	2251	2355	17	34	51	68	85
66	2.2460	2566	2673	2781	2889	2998	3109	3220	3332	3445	18	37	55	73	92
67	2.3559	3673	3789	3906	4023	4142	4262	4383	4504	4627	20	40	60	79	99
68	2.4751	4876	5002	5129	5257	5386	5517	5649	5782	5916	22	43	65	87	108
69	2.6051	6187	6325	6464	6605	6746	6889	7034	7179	7326	24	47	71	95	119
70	2.7475	7625	7776	7929	8083	8239	8397	8556	8716	8878	26	52	78	104	131
71	2.9042	9208	9375	9544	9714	9887	3.0061	3.0237	3.0415	3.0595	29	58	87	116	145
72	3.0777	0961	1146	1334	1524	1716	1910	2106	2305	2506	32	64	96	129	161
73	3.2709	2914	3122	3332	3544	3759	3977	4197	4420	4646	36	72	108	144	180
74	3.4824	5105	5339	5576	5816	6059	6305	6554	6806	7062	41	81	122	163	204
75	3.7321	7583	7848	8118	8391	8667	8947	9232	9520	9812	46	93	139	186	232
76	4.0108	0408	0713	1022	1335	1653	1976	2303	2635	2972	53	107	160	213	267
77	4.3315	3662	4015	4374	4737	5107	5483	5864	6252	6646					
78	4.7046	7453	7867	8288	8716	9152	9594	5.0045	5.0504	5.0970					
79	5.1446	1929	2422	2924	3465	3955	4486	5026	5578	6140					
80	5.6713	7297	7894	8502	9124	9758	6.0405	6.1066	6.1742	6.2432					
81	6.3138	3859	4596	5350	6122	6912	7720	8548	9395	7.0264	Mean Differences no longer sufficiently accurate				
82	7.1154	2066	3002	3962	4997	5958	6996	8062	9158	8.0285					
83	8.1443	2636	3863	5126	6427	7769	9152	9.0579	9.2052	9.3572					
84	9.514	9.677	9.845	10.02	10.20	10.39	10.58	10.78	10.99	11.20					
85	11.43	11.66	11.91	12.16	12.43	12.71	13.00	13.30	13.62	13.95					
86	14.30	14.67	15.06	15.46	15.89	16.35	16.83	17.34	17.89	18.46					
87	19.08	19.74	20.45	21.20	22.02	22.90	23.86	24.90	26.03	27.27					
88	28.64	30.14	31.82	33.69	35.80	38.19	40.92	44.07	47.74	52.08					
89	57.29	63.66	71.62	81.85	95.49	114.6	143.2	191.0	286.5	573.0					